

Experimentation, Conjecture and Reasoning

Math 804T Section 700

Steve Dunbar

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Instructors:

Steve Dunbar

308 Avery Hall

Phone: (402-)472-7236 (with Voice Mail)

email: sdunbar1@unl.edu

WWW: <http://www.math.unl.edu/~sdunbar1>

Math Department: (402-)472-3731 (M-F 8:00 AM – 5:00 PM)

Center for Science, Math & Computer Education: phone: (402-)472-8965,

fax: (402-)472-9311

Blackboard Web Page: my.unl.edu , then login, you should a link for

EXPR CONJECT&RESNING MATH804T SEC 700 FALL 2008

Prerequisites: Math 800 or Math 802 in the NU-Teach Program or equivalent.

Required Text: *The Heart of Mathematics, Second Edition*, by E. Burger and M. Starbird, Key College Publishing, 2005 (ISBN 1-931914-41-9) and the corresponding kit. (Should be available at the University Bookstore. Contact me or Shannon Parry at the Center for Math, Science and Computer Education office if you need assistance in obtaining the book.)

Strategic Objectives for the Course:

The overall goal for this course is to bring you to the next level in the development of your mathematical habits of mind. A productive mathematical thinker has a toolbox of skills and knowledge to experiment, conjecture, reason, and ultimately solve problems. “Mathematical habits of mind” means both understanding of and experience using these tools. Mathematical habits of mind are marked by great flexibility of thinking and the strong belief that precise exposition of solutions are important. Flexibility of thinking includes the use of indirect arguments as well as making connections between knowledge the mathematical thinker possesses and the problem being considered. A person with well developed habits of mind, when presented with a problem to solve, will collect information, assess it, find multiple pathways to the answer, and explain that answer clearly to others.

Although a complete mathematical toolbox includes algorithms, a person with well developed habits of mind knows both why algorithms work and under what circumstances an algorithm will be most effective. Mathematical habits of mind are also marked by ease of calculation and estimation as well as persistence in pursuing solutions to problems. A person with well developed habits of mind has a disposition to analyze all situations as well as the self-efficacy to believe that he or she can make progress toward a solution. Such a person also engages in metacognition by monitoring and reflecting on the processes of reasoning, conjecturing, proving, and problem solving.

This is a condensed Math in the Middle statement consistent with the 5 strands from the National Academy of Sciences report *Adding It Up*:

1. Conceptual Understanding,
2. Adaptive Reasoning,
3. Procedural Fluency,
4. Mathematical Problem Solving Competence,
5. Productive Disposition.

For further details about these strands, please consult the on-line version of *Adding It Up* at <http://www.nap.edu/books/0309069955/html>.

From another view, that of the *Principles and Standards for School Mathematics*, the course strategic objectives focus on problem solving, reasoning and proof and the associated processes:

1. Problem Solving
2. Reasoning and Proof
3. Communication
4. Connections
5. Representation

For further details about the NCTM Standards, please consult the on-line version of at <http://www.nctm.org/standards/>.

Three previous experiences teaching this course convince me that the content and approaches of this course meet these objectives.

To achieve the strategic goals of the course, I have divided the semester into four generally independent sessions. Each session will be 3-4 weeks in length, and will have two or three sub-sessions which will each take 1 to 2 weeks. In each sub-session you will have a pattern of activities to complete, which relate directly to the goals of the course:

1. Read the relevant section of the text book to fill your toolbox of skills and knowledge for reasoning, conjecturing, proving, and solving. The reading will also give you precise definitions and algorithms.
2. Complete the Pre-Test on the Web. The Pre-Test consists of 5 mathematics problems drawn from prior years' AMC contest problem sets related to the topic of the sub-session. The Pre-Test is not for grading, it is intended to assess where you start as a problem solver in this topic. Trying these problems will exercise your calculation and reasoning skills as well as persistence in pursuing solutions to problems.
3. Complete the five assigned "Solidifying Ideas" problems in the text and submit them for grading. Ultimately you will include these in your divided portfolio provided by the Center office. These five problems will exercise your calculation and reasoning skills as well as persistence in pursuing solutions to problems.
4. Complete the two assigned "New Ideas" problems in the text and submit for grading and ultimate inclusion in your portfolio. These problems will begin to exercise your a disposition to analyze all situations as well as the self-efficacy to believe that you can make progress toward a solution.

5. Complete the assigned “Habits of Mind” problem from the Further Challenges section of the text and submit for grading and ultimate inclusion in your portfolio. These problems will exercise your flexibility of thinking as well as making connections between knowledge you have and the problem being considered. A person with well developed habits of mind, when presented with a problem to solve, will collect information, assess it, find multiple pathways to the answer, and be able to explain that answer clearly to others.
6. A Post-Test, another 5 mathematics problems drawn from prior years’ AMC contest problem sets. It is drawn from the same database of problems related to the topic of the sub-session as the Pre-Test. You can take the Post-Test until you have reached mastery of the topic. The Post-test will measure your progress in understanding and applying the mathematical habits of mind gained in that sub-section or topic.

Session Reflections: Choose one problem from each of Session A, B, C, and D and write a one-page reflection about it. Select a problem that you had some mathematical insight on while you were working through the solution or describe a problem you worked that you shared with a colleague, friend, student, small group of students, or your entire class. Describe what happened in the interaction. Reflect on what mathematical ideas you learned from working on each problem.

Teaching and Learning Project:

This project explores learning and teaching from two perspectives. How can you embed the mathematics that you learn in your classroom? Does there exist a relationship between how well you understand the mathematics and how successful you can be as a teacher of that mathematics? For your Learning and Teaching Project this semester we would like everyone to use problems 6 (Baby Bunnies) on page 58 and 9 (Late Bloomers) on page 59 as the basis for your lesson. Over the last three years, we found these topics were adaptable to a wide range of student ages and courses. You should make the judgment where you want to start with your students, what you expect from your students, and how far you can push your students. More details about what to do in the Teaching and Learning Project on the page titled “Math 804T: Experimentation, Conjecture, and Reasoning, Math in the Middle, Learning and Teaching Project” in your Portfolio. The materials you turn in for your Teaching and Learning Project will be assessed according

to the rubric outlined on the pages labeled “Assessment Rubric for Written Reflections on Learning and Teaching Project” in your portfolio.

Detailed Course Schedule:

Session A: Numeric Patterns To learn to “Look for patterns”, “Find hidden, underlying structure” and “Understand simple things deeply”.

1. Fibonacci Numbers I (Chapter 2.2, pages 49-63)
2. Fibonacci Numbers II (Additional material)

Session B: Geometry To “Experience ideas in as many ways as possible”, to “Take ideas from one domain and explore them in another” and “Understand simple things deeply”.

1. Pythagorean Theorem (Chapter 4.1, Pages 208-217)
2. The Golden Rectangle (Chapter 4.3, pages 232-247)
3. The Platonic Solids (Chapter 4.5, pages 269-288)
4. GeoGebra Session I (Additional Material)
5. GeoGebra Session II (Additional Material)

Session C: Counting and Probability, “Simple clear cases let us apply principles that we can apply widely.”

1. General Counting (Chapter 7.4, pages 554-567)
2. Probability (Chapter 7.2, pages 523-540)

Session D: Conditional Probability, “Make it Quantitative” and “Beware of unintended consequences.”

1. Conditional Probability (Additional Material)
2. Bayes Theorem (Chapter 8.2, pages 645-662)

End of Course Review, A comprehensive look back at the course.

1. End of course review.

Session	Assignment	Pages	Problems	Due Date
Beginning Workshop				Sep 7-8
Session A				
Fibonacci Numbers I	Read Chap 2.2 Pre-Test Solidifying Ideas New Ideas Habits of Mind Post-Test Homework faxed Homework returned Last Day for Re-Do	49-56 Sequences 57 58 62 Sequences SD SD SD	2,5,6,7,8 26,28 38	Sep 3 Sep 3 Sep 10 Sep 17 Sep 25
Fibonacci Numbers II	Solidifying Ideas New Ideas Habits of Mind Homework faxed Homework returned Last Day for Re-Do	57 60 62 SD SD SD	9,10,11,12, 14 29,30 40	Sep 10 Sep 17 Sep 24 Oct 2
Teaching and Learning Activity	Activity received Activity returned	58 59	6 9	Dec 12 Jan 2

Session	Assignment	Pages	Problems	Due Date
Session B				
Pythagorean Theorem	Read Chap 4.1	208-212		Sep 17
	Pre-Test	Pythagorean		Sep 17
	Solidifying Ideas	213	6,8,9,14,15	
	New Ideas	215-216	16,20	
	Habits of Mind	216	22	
	Post-Test	Pythagorean		Sep 24
	Homework faxed	SD		Sep 24
	Homework returned	SD		Oct 1
Last Date for ReDo	SD		Oct 10	
Golden Rectangles	Read Chap 4.3	232-244		Oct 1
	Pre-Test	Rectangles		Oct 1
	Solidifying Ideas	244-245	3,5,9,11,12	
	New Ideas	247	17,19	
	Habits of Mind	248	22	
	Post-Test	Rectangles		Oct 10
	Homework faxed	SD		Oct 10
	Homework returned	SD		Oct 20
Last Date for ReDo	SD		Oct 29	
Platonic Solids	Read Chap 4.5	269-284		Oct 13
	Pre-Test	Solid Geom		Oct 13
	Solidifying Ideas	284-285	3,11,12,14,15	
	New Ideas	286	18,19	
	Habits of Mind	286	22	
	Post-Test	Solid Geom		Oct 22
	Homework faxed	SD		Oct 22
	Homework returned	SD		Oct 29
Last Date for ReDo	SD		Nov 7	
GeoGebra I	Read			Oct 22
	Solidifying Ideas			
	New Ideas			
	Habits of Mind			
	Homework faxed	SD		Oct 31
Homework returned	SD		Nov 10	
Last Date for ReDo	SD		Nov 17	
GeoGebra II	Read			Nov 3
	Solidifying Ideas			
	New Ideas			
	Habits of Mind			
	Homework faxed	SD		Nov 12
Homework returned	SD		Nov 19	
Last Date for ReDo	SD		Nov 28	

Session	Assignment	Pages	Problems	Due Date
Session C				
Counting	Pre-Test Solidifying Ideas New Ideas Habits of Mind Post-Test Homework faxed Homework returned Last Date for ReDo	Counting 566 568 569 Counting SD SD SD	6,7,8,10,20,21 34,35 37	Nov 12 Nov 12 Nov 19 Nov 26 Dec 3
Probability	Read 7.2 Pre-Test Solidifying Ideas New Ideas Habits of Mind Post-Test Homework faxed Homework returned Last Date for ReDo	523-534 Probability 563 538 286 Probability SD SD SD	10,12,14,18 19 31,32 38	Nov 19 Nov 19 Nov 26 Dec 10 Dec 5
Session D				
Conditional Prob	Read Pre-Test Solidifying Ideas New Ideas Habits of Mind Post-Test Homework faxed Homework returned	Data Dist On Web page On Web page On Web page Data Dist SD SD		Dec 1 Dec 1 Dec 8 Dec 15
Bayes Theorem	Read 8.2 Solidifying Ideas New Ideas Habits of Mind Homework faxed Homework returned	645-658 660 613 615 SD SD	6,7,8,10,3 16,17,18 21	Dec 8 Dec 15 Dec 22
End of Course	Problems Homework faxed Homework returned	my.unl.edu SD SD		Dec 15 Dec 19 Jan 2

Grading: We ask that you have a well-defined sense of professionalism, that you always give assignments your best effort, and that you develop a sense of responsibility to your educational community (school, district, ESU). We ask that you give this course your best effort and that you exhibit a persistent desire to learn. It is our goal to provide you with significant support and we strongly encourage you to ask for our assistance when it is needed. We are confident in your success.

Grading will be based on the work submitted for the portfolio the approximately weekly 8 Solidifying Ideas, New Ideas, and Habits of Mind solutions, the Post-Tests, and the Essays on Proof from each section. Reading and writing will serve as the basis for some of your learning in this course. When you are asked to read or write something, the following is expected: Read in a manner that enables you to be an active participant in discussions of text. Write in a manner that represents a significant response to a posed question and illustrates attention to grammar and the mechanics of writing. The course instructional team will offer comments that will assist you in learning to solve each problem assessed. As you are all self-motivated, we recognize that you are all putting forth your best effort. Our goal is to offer comments on how your solution can be improved. Your 3-ring binder has a divide for your homework solutions for each section. The binder preserves your work and gives a sense of completeness. Feel free to include personal reflections on the mathematical skills and concepts that are clear to you and those which may still be confusing. You may consider keeping all your work on a problem or only your final solution of a problem.

Grading Scale: Each of the 10 or 11 sub-sessions has eight problems, and these problems will be graded on a scale of “1” for a good or satisfactory solution, “2” for a better or noteworthy solution, and “3” for an excellent or exemplary solution. Therefore each of the 10 or 11 subs-sessions will be worth up to 24 points total, and each of these homework sub-sessions will count about 5% of your grade for a total of 50%. The 7 Post-Tests, one from most sub-sessions, will count about 3.5% each, making another 25% of the grade total. The Teaching and Learning Activity and the End of Course Problem set will count about 10% and your responses and critiques posted on the Blackboard discussion lists will count about 5% of your grade.

Late Homework, and Re-Do Policy Homework should be faxed to the Center Office on the due date noted in the schedule. Nevertheless, we understand that circumstances can arise in busy lives. However, we also need to complete our grading in a timely fashion, so contact us for special

consideration if it is really necessary. Any problem on which you receive a grade of “1” can be re-done, and resubmitted up to 1 week after the homework is returned for a re-grade. We would rather that you learn to do the problems correctly and completely than to hand out “punitive” grades.

Grading Policy

Part of our responsibility as the instructional staff is the assessment of each participant’s achievement in each Math in the Middle course. We recognize that teacher-participants are drawn from different grade levels, have different certifications to teach mathematics, and have different educational backgrounds with respect to previous opportunities to learn mathematics. Thus, we believe it is appropriate to have an assessment system that values effort, teamwork, progress in learning mathematics and the development of knowledge, skills, and dispositions for teaching and inquiry.

Expectations and typical characteristics of achievement at each grade level:

- A+** The grade of A+ is honorific and will be fairly rare. It is evidence that the instructors have special admiration for the participant’s achievements in the course.
- A** Achievement beyond the level needed to earn the grade of A-. Especially important will be evidence that the teacher has a good command of the mathematics studied in the course; the ability to transfer mathematics learned into the teacher’s classroom, and progress in developing the knowledge, skills and dispositions of educational inquiry.
- A-** Achievement beyond the level needed to earn a grade of B+. Especially important will be evidence that the teacher has a good command of the mathematics studied in the course; the ability to transfer mathematics learned into the teachers classroom, and progress in developing the knowledge, skills and dispositions of educational inquiry.
- B+** Regular class attendance, active participation, assignments submitted on-time, supportive and helpful to peers, admirable effort to complete assignments, evidence of good progress in learning mathematics and developing knowledge, skills, and dispositions of educational inquiry.
- B** Regular class attendance, reasonable participation, most assignments submitted on-time, cooperative with peers, reasonable effort to complete

assignments, to learn mathematics and to strengthen knowledge, skills, and dispositions of educational inquiry.

- B-** A grade below B is a statement that the instructors do not believe that the teacher (or lower) made a reasonable effort to use the opportunity provided by the Math in the Middle Institute to develop into a stronger teacher. Evidence may include one or more of the following traits: attendance problems, uncooperative behavior, failure to submit assignment, habitual tendency to submit assignments late, or performance on assignments that indicate an inadequate effort to learn mathematics and to develop knowledge, skills, and dispositions of educational inquiry.