End of Course Problem Set

(1) Take the four right triangles in the kit and arrange them into a square of side length $a + b$ with a square hole of side length c.
a. Sketch an outline of your figure.
b. Show that we can compute the area of the big square in two ways to obtain: $(a + b)^2 = 4\frac{ab}{2} + c^2$
c. Simplify the expression from part (a) to obtain the Pythagorean Theorem.

(2) In the previous problem, you were given another proof to the Pythagorean Theorem. Compare this new proof to the proof presented in the workshop in February. How were these proofs similar? How are they different? Which did you prefer? Which proof would work better in your classroom? Why is it valuable to see multiple proofs to the same theorem?

(3) Consider a rectangle with base of length 1 and height of length x. Suppose that when you remove the largest square, you are left with a smaller rectangle of the same proportions as the original rectangle. Show that the original rectangle was a Golden Rectangle.

(4) (problem 1, page 286) Take each regular solid and slice off its vertices to produce new solids that have two different types of sides. Fill in the chart (see your book) by counting or computing the number of vertices, edges, and faces of each. Also compute the value $V - E + F$ for each truncated solid in your chart. (Here V is the number of vertices on the truncated solid, E is the number of edges on the truncated solid, and F is the number of faces on the truncated solid.) Finally,describe how many faces of each type the truncated solid has.

(5) Consider the expression $5F_n^2 + 4(-1)^n$.
a. Evaluate this expression for $n = 1, 2, 3, 4, 5$.
b. Using your data from part (a), make a conjecture about what type of number you would expect for $n = 6, 7, 8$.
c. Test your conjecture for $n = 6, 7, 8$. Does this pattern hold for all Fibonacci numbers? If it does not, can you find a counterexample?

(6) (problem 9, page 59) By experimenting with numerous examples in search of a pattern, determine a formula for $F_n + L_n$. That is, find a formula for the sum of a Fibonacci number and the corresponding Lucas number.

(7) Assume that the probability of having a boy or a girl is the same.
a. What is the probability of having all boys in a family of five children?
b. How many children must a couple have in order to have more than a 0.95 probability that at least one of the children will be a girl?

(8) (problem 5, page 565) Your math prof says there will be a 15-question test on probability. She also reports that the test will be made up of problems from the Mindscapes of this chapter as follows: No questions will be selected from the In Your Own Words Category. She will select two questions each from sections 7.2, 7.3, and 7.7 and 3 questions each from sections 7.4, 7.5, 7.6. How many different possible exams could she make up? What is the probability that this very question is on your exam?

(9) You own a \$5000 car. The probability that your car will be stolen next year is 0.02, but the probability that your car will be broken into and the radio stolen is 0.10. The damage of such a break-in and theft is \$200. Cheatem's Insurance Company offers you a policy that would cover both of the above thefts for a cost of only \$150.
a. What is the expected value of this insurance policy?
b. What is a fair price for such a policy?

(10) (problem 3, page 614) You apply for a national scholarship. 100,000 students apply and 200 scholarships will be awarded. You call to find out if you are one of the winners. The absent-minded professor in charge of the program reports that the letters have just been sent out, but not list of all the winners is available. The professor also recalls that you were selected. The professor correctly recalls information 90% of the time. Assuming that the scholarships are awarded randomly, what is the probability that you have won?