

Mathematical Ideas

End of Course Problem Set

1. Take the four right triangles in the kit that came with the book (we used the kit at the September workshop) and arrange them into a square of side length $a + b$ with a square hole of side length c .
 - (a) Sketch an outline of your figure.
 - (b) Show that we can compute the area of the big square in two ways to obtain: $(a + b)^2 = (4ab)/2 + c^2$
 - (c) Simplify the expression from part (b) and describe what the sequence of activities in (a) and (b) has established.
 - (d) Compare this new proof to the proof presented in the workshop in September. Which did you prefer? Which proof would work better in your classroom? Why?
2. Consider a rectangle with base of length 1 and height of length x . Suppose that when you remove the largest square, you are left with a smaller rectangle of the same proportions as the original rectangle. Show that the original rectangle was a Golden Rectangle.
3. (problem 21, page 287) Take each regular solid and slice off its vertices to produce new solids that have two different types of sides. Fill in the chart (see your book) by counting or computing the number of vertices, edges, and faces of each. Also compute the value $V - E + F$ for each truncated solid in your chart. (Here V is the number of vertices on the truncated solid, E is the number of edges on the truncated solid, and F is the number of faces on the truncated solid.) Finally, describe how many faces of each type the truncated solid has.
4. Consider the shape made from a putting together three squares (of the same size) together in a symmetric L-shape. Show that you can use this shape to tile the plane. Use the idea of super-tiles to express your demonstration. (This problem is a specific instance of problem 22, on page 268. Use that problem to guide your answer.)
5. Consider the expression $5F_n^2 + 4(-1)^n$.

- (a) Evaluate this expression for $n = 1, 2, 3, 4, 5$.
 - (b) Using your data from part (a), make a conjecture about what numbers you would expect for $n = 6, 7, 8$. (You do not need a formula here.) Explain how you made your conjecture.
 - (c) Test your conjecture for $n = 8, 9, 10$ by creating another formula (or a step-by-step procedure may be easier to describe) that generates the value of $5F_n^2 + 4(-1)^n$. (Of course, the formula $5F_n^2 + 4(-1)^n$ itself is a way of generating the numbers, but that's not what I am looking for. I'm looking for an alternative that does not use the Fibonacci numbers explicitly.)
6. (a) Find the greatest common divisor of F_9 and F_{12} .
- (b) Find the greatest common divisor of F_{15} and F_{20} .
- (c) Find the greatest common divisor of F_{24} and F_{36} .
- (d) Can you make a conjecture about the greatest common divisor of F_n and F_M ?
7. Assume that the probability of having a boy or a girl is the same.
- (a) What is the probability of having all boys in a family of five children?
 - (b) How many children must a couple have in order to have more than a 0.95 probability that at least one of the children will be a girl?
8. (problem 30, page 568) Your math prof says there will be a 15-question test on probability. She also reports that the test will be made up of problems from the Mindscapes of our book's chapter 7 as follows: No questions will be selected from the In Your Own Words Category. She will select two questions each from sections 7.2, 7.3, and 7.7 and 3 questions each from sections 7.4, 7.5, 7.6. How many different possible exams could she make up? What is the probability that this very question is on your exam?
9. Suppose that two fair dice are tossed. What is the probability that the sum equals 10, given that it exceeds 8?
10. Suppose that 0.5% of all students seeking treatment with the school nurse have mononucleosis. Of those who do have mono, 90% complain

of a sore throat. But 30% of those who do not have mono also claim to have sore throats. If a student goes to the school nurse and says he or she has a sore throat, what is the probability the student has mono?