

End of Course Problem Set  
Math 804T  
Experimentation, Conjecture and Reasoning

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1. From problem 8, page 214, we know how to find a right triangle with consecutive integer sides, that is, in an arithmetic progression. Is it possible to find a right triangle with integer sides which are in an increasing geometric progression, that is integer sides of length  $a$ ,  $ar$  and  $ar^2$ ? with  $r > 1$ ? Explain why or why not.
2. Problem 18, page 247 (“Do we get gold this time?” ) in Section 4.3
3. The cube, one of the five platonic solids can be inscribed in a sphere, that is, a sphere which touches the 8 vertices of the cube. What is the radius of that sphere which has an inscribed cube of side length 1? (For bonus points, not required, just for fun and extra points if you get it right! What is the radius of a sphere which has an inscribed tetrahedron of side length 1?)
4. Consider the sequence of Lucas numbers  $L_n$ , defined in Section 2.2, page 59, problem 10, and the sequence of Fibonacci numbers  $F_n$ . Can you discover a simple expression for  $(F_n + L_n)/2$ , the average of  $L_n$  and  $F_n$ ? Can you prove it with the methods of this chapter?
5. Consider the sequence of Lucas numbers  $L_n$ , defined in Section 2.2, page 59, problem 10, and the sequence of Fibonacci numbers  $F_n$ . Can you discover what the ratio  $L_n/F_n$  seems to approach as  $n$  gets larger? Can you prove it with the methods of this chapter?

6. At a teaching council 4 math teachers, 3 English teachers and 3 foreign language teachers are to be seated in a row. How many seating arrangements are possible when teachers of the same subject are required to sit together?
7. A woman has 8 friends, of whom she will invite 5 to a tea party. How many choices has she if the 2 of the friends are feuding and will not attend together? How many choices has if 2 of her friends will only attend together?
8. “Poker dice” is a game played by simultaneously rolling 5 dice. (It’s also like the popular kid’s game Yahtzee, but the rules are simpler, since you only roll once.) Find the probability of:
  - (a)  $\Pr[\text{no two alike}]$
  - (b)  $\Pr[\text{one pair}]$
  - (c)  $\Pr[\text{two pair}]$
  - (d)  $\Pr[\text{full house}]$ , where a “full house” is three of one kind and pair of a different kind.
9. Suppose that 5% of men and 0.25% of women are color-blind. A color blind person is chosen at random What is the probability of the person being male? What assumptions are you making in this problem to compute the probability?
10. Suppose that an insurance company classifies people into one of three classes – good risks, average risks and bad risks. The company records indicate that probabilities that good, average and bad risk persons will be involved in an accident over a 1-year span are respectively 0.05, 0.15 and 0.30. If 20% of the population are good risks, 50% are average risks and 30% are bad risks, what proportion of people have accidents in a fixed year? If a policy holder Steve had no accidents in 2005, what is the probability that Steve is a good risk?