End of Course Problem Set
Math 804T
Experimentation, Conjecture and Reasoning

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1. From problem 8, page 214, we know how to find a right triangle with consecutive integer sides, that is, in an arithmetic progression. Is it possible to find a right triangle with integer sides which are in an increasing geometric progression, that is integer sides of length a, ar and ar^2? with $r > 1$? Explain why or why not.

2. Problem 18, page 247 ("Do we get gold this time?") in Section 4.3

3. The cube, one of the five platonic solids can be inscribed in a sphere, that is, a sphere which touches the 8 vertices of the cube. What is the radius of that sphere which has an inscribed cube of side length 1?

4. Consider the sequence of Lucas numbers L_n, $2, 1, 3, 4, 7, 11, \ldots$ defined in Section 2.2, page 59, problem 10, and the sequence of Fibonacci numbers F_n, $1, 1, 2, 3, 5, 8, \ldots$. Can you discover a simple expression for $(F_n + L_{n+1})/2$, the average of L_n and F_n? Can you prove it with the methods of this chapter?

5. At a teaching council 4 math teachers, 3 English teachers and 3 foreign language teachers are to be seated in a row. How many seating arrangements are possible when teachers of the same subject are required to sit together?

6. A woman has 8 friends, of whom she will invite 5 to a tea party. How many choices has she if the 2 of the friends are feuding and will not attend together?

7. "Poker dice" is a game played by simultaneously rolling 5 dice. (It's also like the popular kid's game Yahtzee, but the rules are simpler, since you only roll once.) Find the probability of:

   (a) Pr[no two alike]

   (b) Pr[one pair]

   (c) Pr[two pair]

   (d) Pr[full house], where a "full house" is three of one kind and pair of a different kind.

8. Suppose that 5% of men and 0.25% of women are color-blind. A color blind person is chosen at random What is the probability of the person being male? What assumptions are you making in this problem to compute the probability?