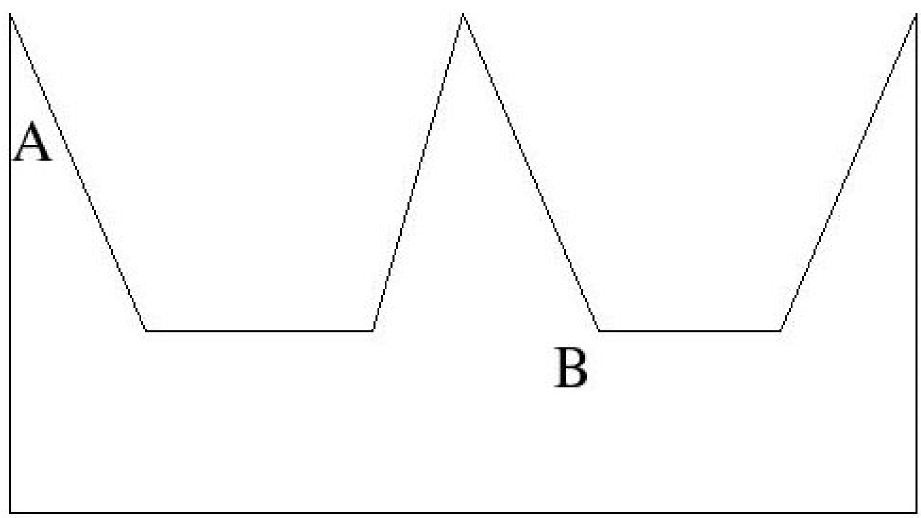
QuestionofDay

Question of the Day

Here’s a floor plan for an art gallery. If you stand in the corner marked A, which walls and parts of the gallery can you see? What if you stand in the corner B? How many guards would you need, each standing at a corner, so that each wall can be seen by at least one guard? (The guards can turn their heads in a full circle, but cannot leave their corners.)



Key Concepts

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1. It is possible to triangulate a polygonal closed curve by adding “diagonal edges” from one vertex to another until the interior is divided into triangles.
2. It is possible to “three-color” the vertices of the resulting triangles so that each triangle has vertices of three different colors.
3. Because the color used least occurs at no more than $v/3$ vertices of a polygonal closed curve with v vertices, guard placed at those corners will suffice.

Vocabulary

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1. *Triangulation* is division of a plane polygon into a set of triangles, usually with the restriction that each triangle side is entirely shared by two adjacent triangles. (Definition adapted from Weisstein, Eric W. ”Triangulation.” From MathWorld-A Wolfram Web Resource, http://mathworld.wolfram.com /Triangulation.html)
2. A *polygonal closed curve* is a plane figure made up of straight line segments connected end to end to form a

loop which does not cross itself at any point.

Mathematical Ideas

Mathematical Ideas

This section is adapted from: Instructor Resources and Adjunct Guide for the second edition of *The Heart of Mathematics* by E. Burger, M. Starbird, and D. Bergstrand.

Sample Activities to Get Started

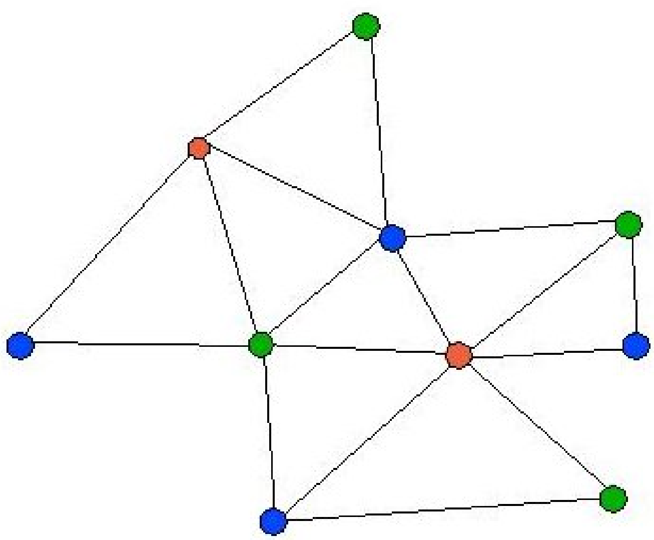
Draw a polygonal closed curve and find a segment that spans across the interior and joins two vertices. Continue to find such spanning spanning segments as long as possible without crossing segments. Observe that the polygon is divided into triangles with vertices at the original vertices of the polygonal curve.

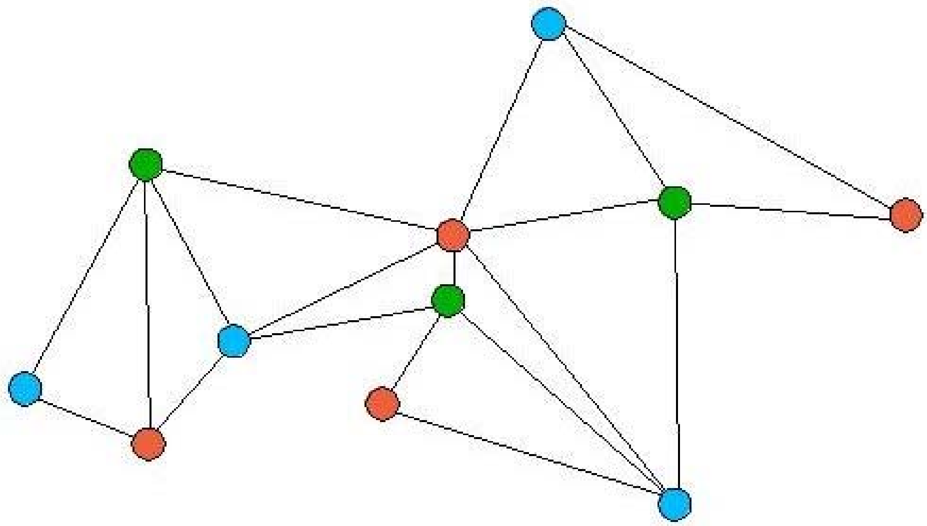
On the triangulated polygonal curve, label each vertex as *R* (for red) *B* (for blue) and *G* (for green) so that each triangle has one vertex of each color. Use the method described in the text to successively mark the colors, so that guessing and back-tracking is not necessary. Find the color that appears least frequently.

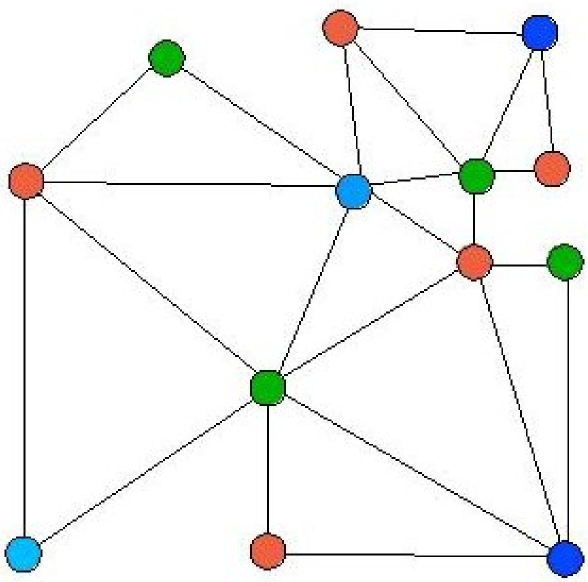
Sample Worked Problems

Page 229, Problem 11, Tricolor me

For each triangulation, color the vertices red, blue, or green so that ever triangle has all three colors.

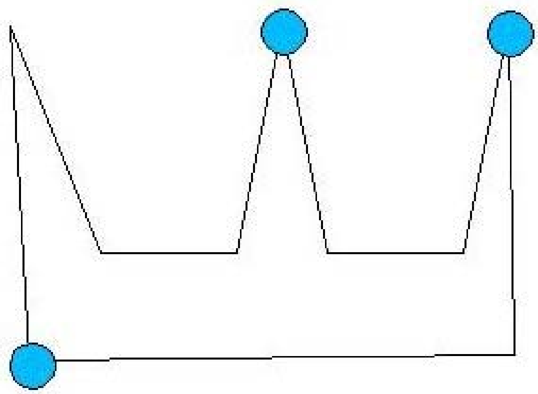






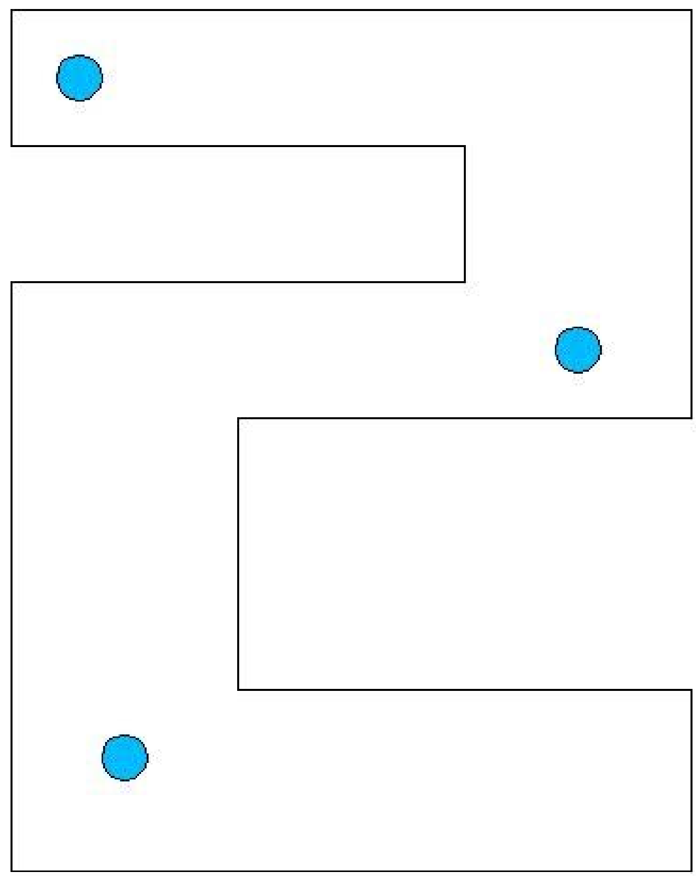
Page 230, Problem 17, Nine needs three

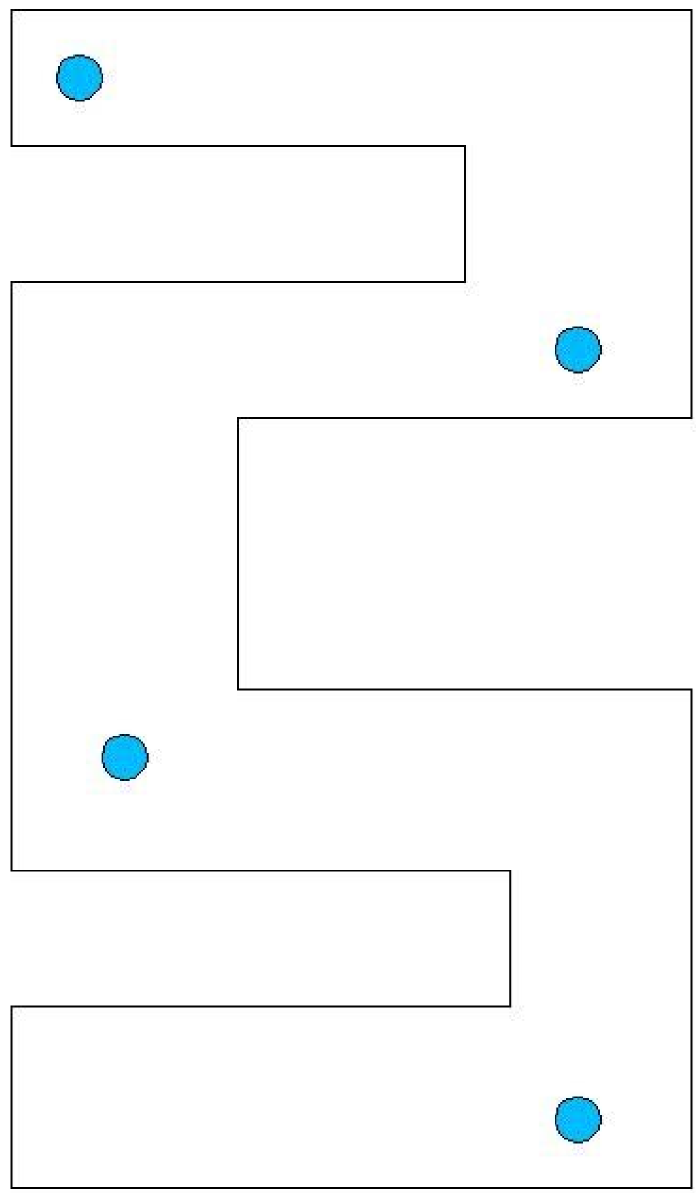
Draw a floor plan for a museum having nine sides that needs exactly three guards to watch the entire gallery. Show the placement of the guards in your drawing.

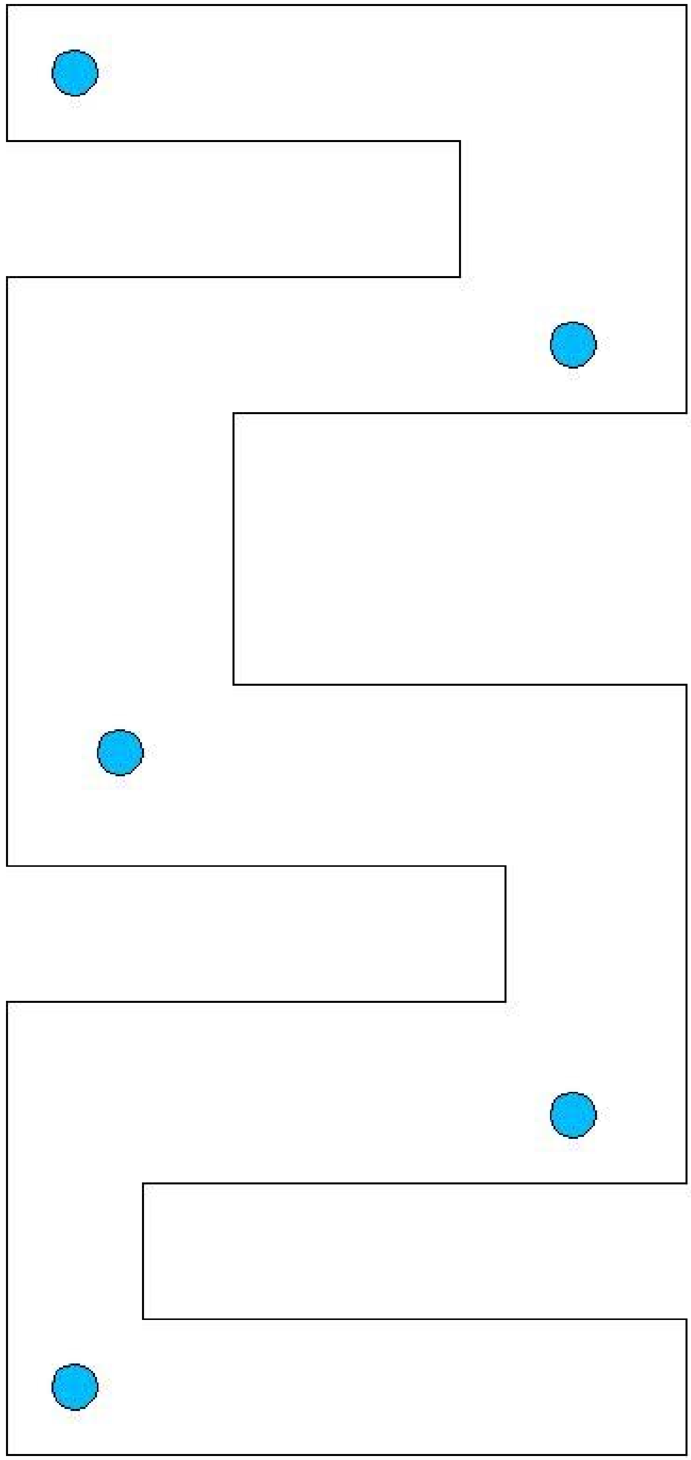


Page 230, Art Gallery Theorem, Problem 20

Draw examples of museums with only right-angled corners having 12 sides, 16 sides, and 20 sides that require three, four, and five guards respectively.







Problems to Work for Understanding

1. Solidifying Ideas, pp. 228-229, Problems 7,8,10,12,15
2. New Ideas, pp. 229-230, Problems 16,19
3. Habits of Mind, page 230, Problem 21

Reading Suggestion:

1. Section 4.2, *Heart of Mathematics*, E. Burger, M. Starbird, Key Curriculum Press.
2.

Outside Readings and Links:

1. Weisstein, Eric W. "Art Gallery Theorem." From MathWorld-A Wolfram Web Resource, http://mathworld.wolfram.com/ArtGalleryTheorem.html
2. Weisstein, Eric W. "Illumination Problem." From MathWorld-A Wolfram Web Resource. http://mathworld.wolfram.com/IlluminationProblem.html
3. Chvatal's Art Gallery Theorem, http://www.cut-the-knot.org/Curriculum/Combinatorics/Chvatal.shtml

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