

Question of the Day

There are 31 educators from the state of Nebraska currently enrolled in Experimentation, Conjecture, and Reasoning. What is the probability that two participants in our class share the same birthday?

Key Concepts

Key Concepts

1. Probability provides us with a quantitative method to analyze the uncertain and the unknown.
2. We develop the ideas behind probability by examining simple familiar situations - rolling dice, flipping coins, drawing cards from a deck.
3. Consider an experiment where the set of all possible outcomes, the *sample space* S is a finite and most important: all of the outcomes are equally likely. The probability that of an *event* E occurs is the proportion of points in the sample space that are in E ,

$$\Pr[E] = \frac{\text{number of points in } E}{\text{number of points in } S}$$

Vocabulary

Vocabulary

1. The *sample space* of an experiment is the set of all possible outcomes of an experiment. It is especially convenient if the sample space is finite in extent, and all outcomes are equally likely.
 2. Definition: The *probability* of the event E occurring is $\frac{N}{S}$, where S is the total number of equally likely outcomes in the sample space and N is the number of different outcomes giving E .
 3. Definition: The *relative frequency* of an outcome E occurring is $\frac{Q}{T}$, where T is the number of times your experiment was repeated and Q is the number of times event E occurred in your experiment.
-

Mathematical Ideas

Mathematical Ideas

Law of Large Numbers

What is the connection between probability and relative frequency? If an experiment is repeated a large number of times, then the relative frequency of a particular outcome will tend to be close to the probability of that particular outcome. We call this the *Law of Large Numbers* and one of the primary tasks of probability theory is to make this statement more precise and quantitative. In your courses on statistics, you saw one way to do this with the notion of confidence intervals.

It Either Happens or It Doesn't

Sometimes it is easier to figure out the probability that an event does *not* happen. Thus, it may be useful to observe: $P(E) = 1 - P(N)$, where $P(E)$ is the probability that the event E does happen and $P(N)$ is the probability that E does not happen. Can you think of three situations when it is easier to determine the probability of an event *not* happening?

Blackjack

Two cards are selected from an ordinary playing deck. What is the probability that they form a blackjack? That is, what is the probability that one of the cards is an ace, and the other is either a ten, jack, queen or king?

The sample space here consists of $\binom{52}{2}$ possible outcomes and all are assumed equally likely.

Now count the number of ways to get a blackjack. There are 4 ways to have an ace (one from each of the four suits) and then there are 16 ways to get one of the value-10 cards ten, jack, queen, king. Therefore there are 64 blackjack hands.

Hence the probability of a blackjack is

$$\frac{64}{52!/(50!2!)} = 0.048265$$

Section 7.2, page 537, Number 19, December 9

Choose two people at random. What is the probability that they were both born on December 9?

Here the sample space is the set of birthdays: the birthday of the first person chosen, then the birthday of the second person chosen. By the Fundamental principle of Counting, there are 365×365 elements in this sample space. We are assuming that all these choices are equally likely, an assumption based on the randomness of our selection process and the distribution of birthdays around the calendar. These are reasonable but not iron-clad assumptions. The event we are interested in has exactly one element in it, namely the December 9 and December 9 selection. Therefore the probability is $1/(365)^2$

$$\approx 7.506 \times 10^{-6}.$$

Section 7.2, Page 538, Number 33, Eight Flips

What is the probability of flipping a half-dollar eight times and a heads appearing at least once.

In this problem, the sample space is the set of all possible sequences of coin-flips, from HHHHHHHH, HHHHHHHT, . . . to TTTTTTTH, TTTTTTTT. There are 2^8 equally likely outcomes in this sample space. Now we are interested in the event that contains coin-flip sequences with 1, or 2, or , or even 8 heads. this is too many to count, so let's look at the complement, the set of coin-flip sequences that have no Heads. There is only one, namely TTTTTTTT, and so the probability of *no* heads is $1/2^8$. The probability of at least one

head is $1 - 1/2^8 = 255/256$

≈ 0.9961 .

Section 7.2, Page 539, Number 40, Three paradox

The probability of tossing three coins and having all three landing the same is $1/4$. What's wrong with the following dubious reasoning?

When we toss three coins, we know for a fact that two of the coins will land the same, therefore, we only have to get the third one to match. Thus, the probability is $1/2$

The fallacy here comes from believing that the sample space we are considering has sample points that are equally likely: Consider having two heads, and desiring to get a third head. There is only one way to get HHH. But consider the probability of having two heads and getting a tail. There are three ways to do this: HHT, HTH, and THH. The probability of getting a third head and not-a-third-head is not $1/2$ and $1/2$ respectively, it is $1/4$ and $3/4$ respectively.

This section is adapted from: Instructor Resources and Adjunct Guide for the second edition of *The Heart of Mathematics* by E. Burger, M. Starbird, and D. Bergstrand. Some of the problems and examples are from *A First Course in Probability*, by Sheldon Ross.

Problems to Work for Understanding

1. Solidifying Ideas: page 536: 10, 12, 14, 18, 19
 2. New Ideas: page 538: 31, 32
 3. Habits of Mind: page 539: 38
-

Last modified: