

Question of the Day

If you flip a coin 100 times and you see heads only 41 times, how confident are you that your coin is fair? How would you quantify your confidence, that is, attach a number to your estimation of fairness?

Key Concepts

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1. Using statistical and probability methods can help distinguish between events that occur randomly and events that have underlying causes.
 2. Systematic thinking with probability concepts can illustrate seemingly confusing or even impossible situations, and sometimes has surprising results.
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Vocabulary

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1. A confidence interval is an interval in which a measurement or trial falls corresponding to a given probability.
 2. *Post hoc, ergo propter hoc* is the Latin term for the fallacy “after this, therefore because of this” which confuses correlated events for cause and effect.
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Mathematical Ideas

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Mendel's Snapdragons

This section is adapted from: *The Heart of Mathematics*, 1st Edition, by E. B. Burger and Michael Starbird, Key Curriculum Press, 2000, page 610, problems I. 1 - 4 and page 615,

One of Mendel's experiments involved red-flowered and white-flowered snapdragons. Through crossbreeding experiments he concluded that each flower had two genes. The red flowers had two red genes and the white had two white genes. We will call those parent plants the first generation, P_1 . When two flowers bred, the offspring received one gene from each parent. If the offspring inherited one red and one white gene, the genes combined to produce a pink flower. Now given Mendel's work, as described, suppose that a second generation plant is bred with a red parent plant. What is the probability that the offspring having a white flower? A red flower? A pink flower?

Answer: The offspring of the crossbreeding will receive one gene from the red parent or first-generation plant, which must necessarily be a red gene, since that is all the parent generation red plant can supply. However, the gene from the pink second-generation plant would be either a white gene, or a red gene. Hence the probability of a red offspring is $1/2$. The probability of a pink offspring is also $1/2$. The probability of a white offspring is necessarily 0.

Now suppose that a second generation plant is bred with a white parent plant. What is the probability of the offspring having a white flower? A red flower? A pink flower? What proportions of the various colors of flowers would you expect to see in a bed of such flowers?

Answer: The offspring of the crossbreeding will receive one gene from the white parent or first-generation plant, which must necessarily be a white gene, since that is all the parent generation white plant can supply. However, the gene from the pink second-generation plant would be either a white gene, or a red gene. Hence the probability of a white offspring is $1/2$. The probability of a pink offspring is also $1/2$. The probability of a red offspring is necessarily 0. The proportions of pink and white flowers in a bed of such cross-breeds would be about $1/2 : 1/2$ under the interpretation that the pink second-generation flowers are crossbred with only the white first-generation white flowers. If the proportions of available white flowers is changed, or the breeding is allowed between first-generation white and first-generation white as well as second-generation pink, and second-generation pink, then the proportions will change.

Suppose two second generation crossbred plants mate. What is the probability that the offspring will be white-flowered?

Now the one parent plant contributes either a white gene and a red gene with equal chance of $1/2$ each to the offspring. The other parent plant also contributes either a white gene and a red gene with equal chance of $1/2$ each to the offspring. The probability that the offspring has WW and shows white flowers is therefore, $1/4$. The probability the offspring has RR is also $1/4$. The probability the offspring shows pink flowers is therefore $1/2$.

Conditional Probabilities

This section is adapted from: *The Heart of Mathematics*, 1st Edition, by E. B. Burger and Michael Starbird, Key Curriculum Press, 2000, page 614, problems II. 6,

An eyewitness observes a hit-and-run accident taxi-cab accident in a city in which 95% of the cabs are green and 5% are blue. The witness is 80% sure the cab was blue. Given all this information, how likely is it that the cab actually was blue?

Answer: One way to answer this question is imagine a universe of 1000 randomly staged accidents observed by this hypothetical observer. In 950 of these accidents, the taxi-cab would be green, and in 50 of these accidents the taxicab would be blue. By the observer being 80% sure, let us interpret that as 80% of the time the observer observes and recalls the cab color correctly and in 20% cases the observer recalls incorrectly the cab color. Among the 50 accidents with a blue cab, the observer would recall 40 of them correctly, and 10 of them incorrectly as being a green cab. Furthermore, the observer would recall 760 of them correctly as occurring with a green cab, and 190 of those accidents as occurring with a blue cab! Hence, if the observer insists that the accident occurred with a blue, then it did occur with a blue cab in $40/(40 + 190) = 0.173$ or about 17% of the time. Adding an only somewhat reliable eyewitness adds only slightly to our belief about the color of the cab!

This problem is an application of Bayes' formula for inverting conditional probabilities: Let B be the event that a Blue cab staged the hit and run accident. Let EB be the probability that the Eyewitness reported a Blue cab. Then the complementary event is EG , the Eyewitness reports a Green taxicab, so that the entire universe

of event is contained in $EB \cup EG$. Then we know that $\Pr[EB|B] = 0.8$ and $\Pr[EG|G] = 0.8$, because the eyewitness is 80% accurate. Hence $\Pr[EB|G] = 0.2$, again because the eyewitness is 80% reliable.

$$\Pr[B|EB] = \frac{\Pr[EB|B] \cdot \Pr[B]}{\Pr[EB|B] \cdot \Pr[B] + \Pr[EB|G] \cdot \Pr[G]} = \frac{0.8 * 0.05}{0.8 * 0.05 + 0.2 * 0.95} = 0.173.$$

This is the same arithmetic as the previous example with 1000 cases.

Problems to Work for Understanding

1. Simplifying Ideas, page 610, 1,2,6,8,10
 2. New Ideas, page 613, 1,2
 3. Habits of Mind, page 615, 3
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