

# End of Course Reflection: Math 804T: Experimentation, Conjecture and Reasoning, Fall 2007

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## 1 Math 804T, *Experimentation, Conjecture and Reasoning*, Fall 2007

I taught Math 804T, *Experimentation, Conjecture, and Reasoning* in Fall Semester 2007. The course had 34 students, all in-service teachers in middle schools in Nebraska. I conducted the course as a distance-education course with the Internet as a communication mode. We used BlackBoard for course documents, announcements, information, student discussion and question and answer. We used EDU for administering and scoring bi-weekly quizzes. We used Breeze (now known as Adobe Connect) for group tele-conferencing on a weekly basis. The text was *The Heart of Mathematics* by Burger and Starbird.

I divided the semester into four generally independent sessions. Three of the four sessions were about 3 weeks in length, and had two sub-sessions, each 1 to 2 weeks in length. I extended the session on Geometry to have 5 sub-sessions, each about a week in length. In most sub-sessions students completed a regular pattern of activities:

- Read the relevant material, mostly from the text-book. I supplemented with additional reading and study material in the Geometry and Probability sections.
- Complete the Pre-Test quiz on EDU. The Pre-Test consisted of 5 mathematics problems related to the topic of the sub-session. The problems were drawn from prior years' AMC contest problem sets.
- Complete 8 problems from the text and submit them for grading by faxing them on the due date to the Center office. The first five problems exercise basic calculation and reasoning skills. The next 2 problems exercise more challenging calculation and reasoning skills as well as persistence in pursuing solutions. The final problem exercises flexibility of thinking as well as making connections between knowledge and the problem.

- A Post-Test quiz from EDU, consisting of another 5 mathematics problems drawn from prior years' AMC contest problem sets.

At the end of the semester, the students completed an End of Course Problem set as a comprehensive take-home final on the entire content of the course.

The course also included a Learning and Teaching Project. For the Learning and Teaching Project this semester we required all the students to use problems 6 (Baby Bunnies) on page 58 and 9 (Late Bloomers) on page 59 from the text as the basis for the Learning and Teaching lesson. Over the last three years, we found these topics were adaptable to a wide range of student ages and courses.

The course started with a weekend workshop on September 7–8, 2007 and continued through December 21 when the End of Course Problem set was due. The following course had a weekend workshop on January 25–26, 2008 when I returned all the graded Teaching and Learning problems and final grades for the course.

## 2 Technical Problems with Breeze

Breeze did not work well for all the participants. Technical problems such as sound cutting out, no picture, loss of picture, and freezing of picture were common. The Breeze problems were unpredictable and inconsistent. For some teachers the combination of camera, microphone and Internet browser software accessing Breeze worked consistently well each week. On the other hand, sometimes sound and picture would work well for a student one week and the next week would not work at all. These Breeze problems seemed to get worse over the course of the semester. At the beginning of the semester we had most students connected and familiar with the system and the first few sessions worked reasonably well. Later in the semester, the problems increased in frequency and difficulty. A couple of times I resorted to calling on the phone and having another participant repeat and relay what was happening over the phone.

I could not determine what the real problems were. It is not clear whether problems are caused by bugs in the Breeze software itself, bugs in the student's software or installation, bugs in the hardware, or even network congestion at the student's Internet service provider. Of course, this uncertainty and the multiple levels of software makes debugging the system even more difficult and frustrating.

The technical problems prevented some students from participating in the group sessions. That in turn, greatly increased frustration and anxiety for those students. The general experience was frustrating for everyone. This was not a good learning environment and it caused some anguish and tears from the students. The Breeze experience was inconsistent in Cohort 2, when the system was new to everyone. The experience was better for Cohort 3, and showed promise as a distance-learning tool. However, the experience and service deteriorated this semester with Cohort 4. The frustration is enough to encourage me to seek an alternative tele-video conferencing solution.

### 3 Learning and Teaching Project is Valuable

The Learning and Teaching Project is valuable in encouraging the students to apply the mathematics that they have learned. Many students approached the lesson with some fear that the material would not be adaptable to the classroom, or would interfere with required teaching material. Most found it to be a valuable teaching experience and almost all of them learned something about themselves, their students and their teaching style. The structure of the Learning and Teaching Project based around its grading rubric is fine, and does not need change. The only addition I would make is to seek additional topics beyond the Baby Bunnies/ Fibonacci sequence to use in the future as the basis of the Learning and Teaching Project topics.

### 4 Homework and Grading

I introduced a new module in the Geometry section of the course centered on problems in classical Euclidean geometry using the GeoGebra software. The problems were taken from prior AMC 10 contests. To encourage the students, I structured the assignment so that for each problem the students did a construction of the problem with GeoGebra first. Then the students measured the solution numerically to a couple of decimal places using the algebraic part of GeoGebra. I asked the students to make a conjecture about the solution of the problem. The conjecture was usually expressing a geometric construct such as a length or area using mathematical constants. Then I asked the students to explicitly solve the problems symbolically, using similarity or area formulas. This structure turned out to be an excellent way to organize the problems. This pattern fit exactly with the title and goal of the course to be experimentation, conjecture and reasoning. The reasoning usually took the form of explicitly solving the problem, and expressing the answer analytically or symbolically, rather than a logic proof. After teaching the course four times, I am convinced that the students are ready to reason and explain a solution of a problem, but do not have the logic tools, experience and enough examples to structure a mathematical proof.

This pattern of software experiment, conjecture, and then reasoning out a symbolic solution turned out to be an excellent way to structure the exercises of the course. If I teach the course again in the future, I will re-structure all of the exercises with extensive, structured and directed experimentation with software. For the geometry sections, I will certainly use GeoGebra again. For the sequences and counting section, I may structure the problems to use either calculators, spreadsheets, or perhaps the new open computer algebra software Sage. Then the students will be explicitly asked to make a conjecture about what the final answer with symbolic constants involving  $\pi$ , square roots, Golden ratio, etc. Then finally, I will ask for a formal derivation, say by setting up and solving an equation to derive the symbolic answer. I will assign point values to each part of the solution. That means each part will say 10 to 20 points in value

and will allow for easy partial credit depending on what the problem is like. An idea that occurred in conversation with David Hartman would be to additionally require the students to do a Web search on some problems to find resources or applets that extend, illustrate or solve the problem. More points would be awarded for finding resources at a trusted site such as the NCTM Illustrations, or mathworld.com. Even better would be to structure problems with a worksheet format, where the students fill out the worksheet in a systematic format. The first few Getting Started problems would be highly structured, the second batch of New Ideas problems would be somewhat less structured, and then finally, the Habits of Mind problem would be just simply stated, not structured at all. I would make an effort to find related problems from the AMC 10 contests of mild difficulty as a problem source.

I made a small start toward this on the Fall 2007 End of Course Problem set, by structuring a few of the problems as giving 2 points for some experiments and a conjecture, and then a full 3 points for the full solution to a symbolic answer. The new idea would be to expand this grading scheme to a larger number of points for the experiment, even breaking the experiment into several parts. Then the conjecture would be some additional points, depending on how it was expressed. Then finally the symbolic solution would be worth some final points, making a total of 10 or 20 points for the whole solution.

This may resolve another problem that seemed to be more difficult for this Cohort 4 than in previous years. The simple 1–2–3 scoring I have used is just not fine enough to discriminate levels of solutions. A grade of 2 is often not enough for the teachers even though I intended it to indicate a good solution. Then the teachers complain to me about getting a grade of 2 on a problem for which substantial portions were correct but the final symbolic solution may not have been complete or correct. Now with large partial credit, I hope to avoid that frustration. I initially adopted the 1–2–3 or good-better-best grading system to encourage the teachers to avoid the “begging for partial credit” syndrome and to look at problems as a holistic unit, but it seems not to have that outcome. Structured, explicit partial credit would avoid frustration for the teachers and guide them in creating appropriate solutions at this early stage in their mathematical development.

This structure for the problems will work well for the parts of course built around the Fibonacci sequence, probability and of course the Geometry which I have already done. Many of the problems in Burger and Starbird can be rebuilt around this framework, allowing me to keep Burger and Starbird as a reading resource. I may want to use the new free and open source mathematical software Sage for computation, since it would be free to the teachers.

One good feature of using the AMC 10 problems was that many have alternate solutions. Then different students come to the same conclusion through different reasoning. That the same problem allowed multiple solutions was often surprising to the students. It is good to expose students to problems that have multiple solutions. I should do it deliberately as often as I can.

## 5 The Paper Shuffle

The homework paper shuffle seemed to be as inconsistent and frustrating as ever this semester. Papers came in late, the submission of the make-up solutions was late and inconsistent. Occasionally students submitted their papers to my personal email instead of faxing it in. This was in spite of adding much more explicit instructions in the syllabus and the beginning workshop.

Somehow I have to make the homework submission procedure simple, regular, consistent and on time.

## 6 Comments about the Class from Nathan Axvig

I had a conversation with Nathan Axvig, the teaching assistant for the course, about his impressions of the course on December 18, 2007. The conversation came after all the problems were submitted, but before the End of Course set was graded.

He believes the course content was appropriate, with topics adaptable to a college level course and also to middle school. He liked the topic of counting as his favorite topic, but that is his personal favorite topic. He was surprised and confused about what the students knew and did not know. He was also surprised about the inconsistent background geometry knowledge of the students. Nathan added later that at points, he was surprised by how much plane geometry most of the students knew. He said that it would have been nice to get a copy of their syllabus so he could have a rough idea of what they knew.

I asked him for his thoughts about Distance Education courses. He said that in an ideal world, he preferred face-to-face contact in a classroom. However, he understood that a distance education course is necessary for students who are place bound by family or job, and accepted that as a part of the course.

He said that the actual teaching time in a Breeze session was maybe 40 minutes out of an hour because of the start-up (“I can hear you, can you hear me now?”) and technical difficulty issues.

For course changes, Nathan would recommend more book-keeping and communication to keep track of the homework that is going back and forth between students and instructors. Because of the homework flow, grading the course takes more time and effort than a comparable course on campus. Nathan later added that he should have been a little less willing to accept late homework far past the due date. He expressed some uncertainty about how to handle the late homework, realizing that these aren’t the normal college students, that they have jobs, families and lives beyond the usual graduate student situation.

Nathan expressed some frustration with the grading practices and grading criteria for the course. Initially he thought the grading scale that I described in the syllabus was fine. Teacher complaints about grades, level of rigor expects and so on, forced him to move away from the original system and substitute a scale of “1 is wrong, 2 is okay with a few flaws, 3 is correct”. He believed the unfortunate aspect of this grading system was that he had

nowhere to go when someone would really get a problem correct with some really insight. He couldn't give them a 4, but he wanted to. He believed the grading system did not really distinguish between average and okay solutions to the truly exceptional. He felt that sometimes the teachers took grades of 1 and 2 very personally, Of course this leads to grading complaints, since individual students will believe that their effort was enough to earn a full score of 3, even though the problem was not solved rigorously. Nathan sated that he thought that providing a detailed description of exactly what is expected for a given problem would help greatly. A lot of the frustration came from the fact that the teachers thought that they had done a problem completely, yet Nathan thought otherwise, and graded correspondingly.

Nathan did like working with the teachers. He found it refreshing to work with adult learners already in the workplace. He said that the teachers had a genuine interest in learning the material. He did say that sometimes it felt awkward to being younger than any of the students in the class, yet still being a position of authority in the class.

## 7 Unstructured Additional Comments.

Sometimes I found explaining solving some equations and rationalizing some radical expressions and simplifying some factions to be frustrating. Sometimes, what seemed to me to be obvious or simple solutions techniques or approaches to be baffling to some of the teachers. This was not on the new concepts or ideas, where it would be natural, this was on basic mathematical, algebraic or logical approaches to the problems. I also found it difficult to explain to the teachers that it is preferable to not simplify multiplications, and to not simplify fractions or convert to decimals when doing counting or probability problems. I am not sure if the teachers just weren't as mathematically prepared this semester. More of the teachers this semester are 5th and 6th teachers, and so may not have encountered as much of this material as upper-middle or high school teachers. The difficulty in explanation may also be due to the difficulty in explaining mathematics at a distance over Breeze in contrast to explaining face-to-face.

Related to this was the efforts involved in the Group Sessions, On a few evenings, I spent as much as four straight hours in sessions, coaching students through problems that I did not believe were so hard that they required coaching. Spending that much time in the sessions working through the problems was wearing. It was additionally wearing as the technology was frustrating at every step, as explained above. Furthermore, explaining problems using tools on Breeze, the whiteboard tool and the chat box is not as easy as explaining problems face-to-face at a blackboard.

Somewhat related to that was the tendency of the students to get a decimal or numeric answer instead of a symbolic answer involving rationals, radicals, or constants such as  $\pi$ . Perhaps with some coaching and structured problems solutions as described above, and using the conjectures section explicitly to coax

a symbolic answer, I can overcome this tendency and encourage mathematical thinking.

Related to the above is the poor notational habits of some of the teachers, e.g. a failure to distinguish between  $F_{n-1}$  and  $F_n - 1$ .

Related to this was the anxiety of the teachers about their grades, as mentioned above. It may have just been a coincidental feature of this cohort of teachers, but I had to answer many questions about the recording of grades before the grades were ready, or when the problem sets would be ready, and so on. However, I recently found a message from a year ago about Cohort 3 expressing exactly the same sentiment, so perhaps I have forgotten that each Cohort behaves the same.

## 8 Final Exam

The final exam problems may have been too hard. After grading them, I think the problems may have been too notationally abstract and have too many steps. In particular the Fibonacci problems may have been too hard.

On the first problem it was hard for many students to recognize that  $L_n = F_{n-2} + F_n$  and then further to use that  $F_{n-2}/F_n$  converges to  $1/\phi^2$ . On the other hand, many students found the solution by expanding in continued fractions, a very creative solution. Many students failed to simplify  $1 + 1/\phi^2$  to an explicit radical expression.

The second problem which suggested using the principle of mathematical induction was too abstract for many of the students.

On the third problem about Pythagorean triples created from odd numbers, many students found making the leap to checking symbolically was hard. However, once coached to make the symbolic analysis, most students were able to finish the calculations.

Problems 4, 6, 8 and 9 were pretty easy for most of the students. This may have been because the topics of counting, probability, conditional probability and Bayes Theorem were fairly recent in the students minds. Problem 3 on inscribed cubes was just an easy problem.

Finally, Problem 7 on probability was hard because the counting of the denominator was different from many problems worked previously, although not inherently difficult.

The final problem on the Final exam asked students to name their favorite problem in the course. The one-page essay answer revealed a lot about the students' problem solving and learning style, e.g.. visual versus non-visual, systematic experimentation versus trial and error. Once again, this little essay or reflection turned out to be quite valuable. I should require a reflection on each section using this problem as a model. With the revised grading system, these little reflections would be worth relatively few points say 3 to 5 points compared to the problems having 10 to 20 points. This would give them the proper weight for consideration by the teachers. Disproportionate weight may have made the reflections too important in the past, causing the students to

expend too much effort or consideration. This may rectify the weight problem, while preserving the valuable aspects of introspection about problem solving.