The National Council of Teachers of Mathematics produced *Principles and Standards for School Mathematics* (NCTM 2000) to update and extend the recommendations for learning and teaching mathematics that had appeared in *Curriculum and Evaluation Standards for School Mathematics* (NCTM 1989), *Professional Standards for Teaching Mathematics* (NCTM 1991), and *Assessment Standards for School Mathematics* (NCTM 1995). *Principles and Standards* enunciated the Curriculum Principle, which states, “A curriculum is more than a collection of activities; it must be coherent, focused on important mathematics, and well articulated across the grades” (p. 14). Specifically, “a well-articulated curriculum gives teachers guidance regarding important ideas or major themes, which receive special attention at different points in time. It also gives guidance about the depth of study warranted at particular times and when closure is expected for particular skills or concepts” (p. 16).

This definition of curriculum articulation echoes a central question that occupies state and local leaders in mathematics education: What mathematics should be the focus of instruction and learning at particular grade levels of the pre-K–12 educational system? As *Principles and Standards* states, “Those who design curriculum frameworks, assessments, instructional materials, and classroom instruction based on *Principles and Standards* will need to make their own decisions about emphasis and order” (p. 31). *Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics* provides one possible response to the question of how to organize curriculum standards within a coherent, focused curriculum, by showing how to build on important mathematical content and connections identified for each grade level, pre-K–8.

**Inconsistency in the Placement of Topics by Grade Level in U.S. Mathematics Curricula**

Analysis of curricula of countries participating in the Third International Mathematics and Science Study (TIMSS [1997]; now known as the Trends in International Mathematics and Science Study) led to the familiar description of school mathematics in the United States as “a mile wide and an inch deep” (Schmidt, McKnight, and Raizen 1997). In addition, research on the curricular expectations of states and school systems across the country indicates inconsistency in the grade placements of mathematics topics, as well as in how they are defined and what students are expected to learn.

State and local districts, with varying resources for providing leadership in mathematics education, have been working fairly independently to develop student learning expectations, as required by the federal law No Child Left Behind (2002). The result has been a wide variety of mathematics curriculum standards, with little consensus on the placement or emphasis of topics within specific grade levels (Reys et al. 2005). For example, in a study of the mathematics curriculum standards of ten states (Reys et al. 2006), the total number of grade-level expectations in mathematics for grade 4 ranged from 26 to 89 (see table 1).
Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics

tools that support our shared goal of a high-quality mathematics education for every student.

is curriculum. Take this opportunity to share the best that we know as we work together to produce improved

initial step in advancing collaborative discussion about what mathematics students should know and be able
to do. Use the focal points presented here to guide discussions as you review, refine, and revise material.

Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics thus represents an important,

the mathematics that we expect them to learn.

emerge as skills, process sense, and ways of thinking and can measure and communicate what students know about

ion, developing materials, and creating meaningful assessments that can help students learn critical math-

in the creation and eventual development of new models for defining curriculum, organizing instruction, and

vision in the development of the next generation of curriculum standards, textbooks, and tests. This work may

be the leaders of every level to use Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics to

important connections, understandings, and skills. The long-term opportunity, however, is for mathematics
Table 1. Number of Fourth-Grade Learning Expectations (LEs) per State by Content Strand (from Reys et al. 2006, p. 20)

<table>
<thead>
<tr>
<th>State</th>
<th>Number &amp; Operations</th>
<th>Geometry</th>
<th>Measurement</th>
<th>Algebra</th>
<th>Data Analysis, Prob &amp; Stat</th>
<th>Total Number of LEs</th>
</tr>
</thead>
<tbody>
<tr>
<td>California</td>
<td>16</td>
<td>11</td>
<td>4</td>
<td>7</td>
<td>5</td>
<td>43</td>
</tr>
<tr>
<td>Texas</td>
<td>15</td>
<td>7</td>
<td>3</td>
<td>4</td>
<td>3</td>
<td>32</td>
</tr>
<tr>
<td>New York</td>
<td>27</td>
<td>8</td>
<td>10</td>
<td>5</td>
<td>6</td>
<td>56</td>
</tr>
<tr>
<td>Florida</td>
<td>31</td>
<td>11</td>
<td>17</td>
<td>10</td>
<td>20</td>
<td>89</td>
</tr>
<tr>
<td>Ohio</td>
<td>15</td>
<td>8</td>
<td>6</td>
<td>6</td>
<td>13</td>
<td>48</td>
</tr>
<tr>
<td>Michigan</td>
<td>37</td>
<td>5</td>
<td>11</td>
<td>0</td>
<td>3</td>
<td>56</td>
</tr>
<tr>
<td>New Jersey</td>
<td>21</td>
<td>10</td>
<td>8</td>
<td>6</td>
<td>11</td>
<td>56</td>
</tr>
<tr>
<td>North Carolina</td>
<td>14</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>26</td>
</tr>
<tr>
<td>Georgia</td>
<td>23</td>
<td>10</td>
<td>5</td>
<td>3</td>
<td>4</td>
<td>45</td>
</tr>
<tr>
<td>Virginia</td>
<td>17</td>
<td>8</td>
<td>11</td>
<td>2</td>
<td>3</td>
<td>41</td>
</tr>
</tbody>
</table>

The Importance of Curricular Focus in Mathematics

Many factors have contributed to the need for a common mathematical focus for each grade level, pre-K–8. These include the increased emphasis on accountability testing, high levels of mobility of both students and teachers, and greater costs of curriculum development. A focused, coherent mathematics curriculum with a national scope has the potential to ease the impact of widely varying learning and assessment expectations on both students and teachers who relocate. In addition, a focused curriculum would allow teachers to commit more time each year to topics receiving special emphasis. At the same time, students would have opportunities to explore these topics in depth, in the context of related content and connected applications, thus developing more robust mathematical understandings.

In a survey of employees of forty-seven educational agencies—those responsible for improving curriculum and instruction in their states—85 percent of the respondents indicated that “national leadership is needed to assist in future articulation of learning expectations in mathematics, particularly from national professional organizations of mathematics teachers (K–12 and university) and mathematicians” (Reys et al. 2005, p. 17). This publication addresses that need.
Curriculum focal points are important mathematical topics for each grade level, Pre-K through Grade 8. These areas of instructional emphasis can serve as organizing structures for curriculum design and instruction at and across grade levels. The topics are central to mathematics: they convey knowledge and skills that are essential to educated citizens, and they provide the foundations for further mathematical learning. Because the focal points are core structures that lay a conceptual foundation, they can serve to organize content, connecting and bringing coherence to multiple concepts and processes taught at and across grade levels. They are important to all mathematics learning.

When instruction focuses on a small number of key areas of emphasis, students gain extended experience with core concepts and skills. Such experience can facilitate deep understanding, mathematical fluency, and an ability to generalize. The decision to organize instruction around focal points assumes that the learning of mathematics is cumulative, with work in the later grades building on and deepening what students have learned in the earlier grades, without repetitiveness and inefficient reteaching. A curriculum that focuses on focal points also has the potential to offer opportunities for the diagnosis of difficulties and immediate intervention, thus helping students who are struggling with important mathematics content.

- What characteristics qualify a concept or topic to be a curriculum focal point?
- For inclusion in Curriculum Focal Points for Kindergarten through Grade 8 Mathematics, a focal point had to pass three rigorous tests:

  1. Was it mathematically important, both for further study in mathematics and for use in applications in and outside of school?
  2. Does it "fit" with what is known about learning mathematics?
  3. Does it connect logically with the mathematics in earlier and later grade levels?

A curriculum focal point may draw on several connected mathematical content topics described in Principles and Standards for School Mathematics (NCTM 2000). It should be addressed by students in the context of the mathematical processes of problem solving, reasoning and proof, communication, connections, and representation. Without facility with these critical processes, a student's mathematical knowledge is likely to be fragile and limited in its usefulness.

A complete set of curriculum focal points, situated within the processes of mathematics, can provide an outline of an integrated mathematics curriculum that is different from the one created by a set of grade-level mastery objectives or a list of separated content and process targets. In contrast with grade-level mastery emphases, calling for instruction that will help students learn content that gives them a foundation for increasing their understanding as they encounter richer and more challenging mathematics.
Instruction based on focal points would devote the vast majority of attention to the content identified for special emphasis in a grade. A curriculum for pre-K–8 based on a connected set of such focal points could provide a solid mathematical foundation for high school mathematics.
How Should Curriculum Focal Points Be Used?

To achieve the best results with students when teaching for the depth, understanding, and proficiency sought by the curriculum focal points, teachers themselves will need a deep understanding of the mathematics and facility with the relationships among mathematical ideas. Thus, effective instruction built on the curriculum focal points requires in-depth preparation of preservice teachers and ongoing professional development for in-service teachers.
Principles and Standards for School Mathematics (NCTM 2000) describes the foundational mathematical ideas on which the focal points in Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics rest and toward which they direct students’ learning. Principles and Standards remains the definitive reference on the development of mathematical content and processes across the grades. Since the publication of this influential work in 2000, ideas like coherence, focus, high expectations, computational fluency, representation, and important mathematics have become regular elements in discussions about improving school mathematics, and thinking about these ideas has evolved considerably. As the next step in devising resources to support the development of a coherent curriculum, NCTM now offers a new publication, with a set of curriculum focal points and connections for mathematics education in prekindergarten through grade 8.

Principles and Standards includes a thorough discussion of the necessity for learning mathematical content through the processes of problem solving, reasoning and proof, communication, connections, and representation. Although some of these processes may be evident in the descriptions of particular focal points, this new publication primarily targets content. Its presentation of curriculum focal points assumes that the mathematical processes described in Principles and Standards will be implemented in instruction that requires students to discuss and validate their mathematical thinking; create and analyze a variety of representations that illuminate the connections within the mathematics; and apply the mathematics that they are learning in solving problems, judging claims, and making decisions.

Curriculum Focal Points for Prekindergarten through Grade 8 Mathematics identifies three focal points at each grade level. Each set of three focal points, together with the integrated content taught in the context of the processes, should encompass the major portion of instruction at that grade level. The presentations of the focal points for each grade level also identify “Connections to the Focal Points” in a column at the right. These connections serve two purposes:

1. They recognize the need for introductory and continuing experiences related to focal points identified for other grade levels.
2. They identify ways in which a grade level’s focal points can support learning in relation to strands that are not focal points at that grade level.

The “Connections to the Focal Points” column for each grade level brings in other important topics in meaningful ways. For example, the grade 2 “Connections” highlight the fact that the Measurement Focal Point for grade 2 (“Developing an understanding of linear measurement and facility in measuring lengths”) includes work with applications and models using the shapes from the Geometry Focal Point for grade 1 (“Composing and decomposing geometric shapes”). At the same time, students in grade 2 continue to use vocabulary and spatial reasoning that will be essential for learning the content specified in the Geometry Focal Point for grade 3 (“Describing and analyzing properties of two-dimensional shapes”). Because a curriculum that is integrated
Principles and Standards expects instruction to address in the corresponding grade bands.
Three curriculum focal points are identified and described for each grade level, pre-K–8, along with connections to guide integration of the focal points at that grade level and across grade levels, to form a comprehensive mathematics curriculum. To build students’ strength in the use of mathematical processes, instruction in these content areas should incorporate—

- the use of the mathematics to solve problems;
- an application of logical reasoning to justify procedures and solutions; and
- an involvement in the design and analysis of multiple representations to learn, make connections among, and communicate about the ideas within and outside of mathematics.

The purpose of identifying these grade-level curriculum focal points and connections is to enable students to learn the content in the context of a focused and cohesive curriculum that implements problem solving, reasoning, and critical thinking.

These curriculum focal points should be considered as major instructional goals and desirable learning expectations, not as a list of objectives for students to master. They should be implemented with the intention of building mathematical competency for all students, bolstered by the pedagogical understanding that not every student learns at the same rate or acquires concepts and skills at the same time.

Those who are involved in curriculum planning for grades 6–8 should note that this set of curriculum focal points has been designed with the intention of providing a three-year middle school program that includes a full year of general mathematics in each of grades 6, 7, and 8. Those whose programs offer an algebra course in grade 8 (or earlier) should consider including the curriculum focal points that this framework calls for in grade 8 in grade 6 or grade 7. Alternatively, these topics could be incorporated into the high school program. Either way, curricula would not omit the important content that the grade 7 and grade 8 focal points offer students in preparation for algebra and for their long-term mathematical knowledge.
Prekindergarten Curriculum Focal Points

Number and Operations: Developing an understanding of whole numbers, including concepts of correspondence, counting, cardinality, and comparison. Children develop an understanding of the meanings of whole numbers and recognize the number of objects in a set. They use one-to-one correspondence to count objects and use their understanding of numbers to solve problems by matching sets and comparing number amounts. They use counting to determine number amounts and compare quantities using language such as “more than” and “less than.”

Measurement: Identifying measurable attributes and comparing objects by using these attributes. Children identify objects as the same, different, and then “more” or “less,” on the basis of attributes that they can measure. They identify measurable attributes such as length and weight and solve problems by making direct comparisons of objects on the basis of those attributes.

Geometry: Identifying shapes and describing spatial relationships. Children develop spatial reasoning by working from two perspectives on space as they examine the shapes of objects and the space around them. They build pictures and designs by combining two- and three-dimensional shapes, and they solve such problems as deciding whether pieces will fit into a space or into a puzzle. They discuss the relative positions of objects using vocabulary such as “above,” “below,” and “next to.”

Data Analysis: Children learn the foundations of data analysis by using objects, attributes that they have identified in relation to geometry and measurement, e.g., size, quantity, orientation, number of sides, or vertices, or comparing. For example, children sort geometric figures by shape, “circle,” “square,” “triangle,” or describe sets of objects by the number of objects in each set.
Curriculum Focal Points and Connections for Kindergarten

The set of three curriculum focal points and related connections for mathematics in kindergarten follow. These topics are the recommended content emphases for this grade level. It is essential that these focal points be addressed in contexts that promote problem solving, reasoning, communication, making connections, and designing and analyzing representations.

<table>
<thead>
<tr>
<th>Kindergarten Curriculum Focal Points</th>
<th>Connections to the Focal Points</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Number and Operations:</strong> Representing, comparing, and ordering whole numbers and joining and separating sets</td>
<td><strong>Data Analysis:</strong> Children sort objects and use one or more attributes to solve problems. For example, they might sort solids that roll easily from those that do not. Or they might collect data and use counting to answer such questions as, “What is our favorite snack?” They re-sort objects by using new attributes (e.g., after sorting solids according to which ones roll, they might re-sort the solids according to which ones stack easily).</td>
</tr>
</tbody>
</table>

Children use numbers, including written numerals, to represent quantities and to solve quantitative problems, such as counting objects in a set, creating a set with a given number of objects, comparing and ordering sets or numerals by using both cardinal and ordinal meanings, and modeling simple joining and separating situations with objects. They choose, combine, and apply effective strategies for answering quantitative questions, including quickly recognizing the number in a small set, counting and producing sets of given sizes, counting the number in combined sets, and counting backward.

**Geometry:** Describing shapes and space

Children interpret the physical world with geometric ideas (e.g., shape, orientation, spatial relations) and describe it with corresponding vocabulary. They identify, name, and describe a variety of shapes, such as squares, triangles, circles, rectangles, (regular) hexagons, and (isosceles) trapezoids presented in a variety of ways (e.g., with different sizes or orientations), as well as such three-dimensional shapes as spheres, cubes, and cylinders. They use basic shapes and spatial reasoning to model objects in their environment and to construct more complex shapes.

**Measurement:** Ordering objects by measurable attributes

Children use measurable attributes, such as length or weight, to solve problems by comparing and ordering objects. They compare the lengths of two objects both directly (by comparing them with each other) and indirectly (by comparing both with a third object), and they order several objects according to length.
Geometry: Composing and decomposing geometric shapes:

- Children compose and decompose plane and solid figures (e.g., by putting two congruent shapes to make a composite shape).

Number and Operations: Developing an understanding of whole number relationships:

- Children compare and order whole numbers (e.g., at least to 100) to develop an understanding of order and one-to-one correspondence.

Connections to the Focal Points:

- Describing and analyzing representations of such properties as congruence and similarity.
## Curriculum Focal Points and Connections for Grade 2

The set of three curriculum focal points and related connections for mathematics in grade 2 follow. These topics are the recommended content emphases for this grade level. It is essential that these focal points be addressed in contexts that promote problem solving, reasoning, communication, making connections, and designing and analyzing representations.

### Grade 2 Curriculum Focal Points

**Number and Operations:** Developing an understanding of the base-ten numeration system and place-value concepts

Children develop an understanding of the base-ten numeration system and place-value concepts (at least to 1000). Their understanding of base-ten numeration includes ideas of counting in units and multiples of hundreds, tens, and ones, as well as a grasp of number relationships, which they demonstrate in a variety of ways, including comparing and ordering numbers. They understand multidigit numbers in terms of place value, recognizing that place-value notation is a shorthand for the sums of multiples of powers of 10 (e.g., 853 as 8 hundreds + 5 tens + 3 ones).

**Number and Operations and Algebra:** Developing quick recall of addition facts and related subtraction facts and fluency with multidigit addition and subtraction

Children use their understanding of addition to develop quick recall of basic addition facts and related subtraction facts. They solve arithmetic problems by applying their understanding of models of addition and subtraction (such as combining or separating sets or using number lines), relationships and properties of number (such as place value), and properties of addition (commutativity and associativity). Children develop, discuss, and use efficient, accurate, and generalizable methods to add and subtract multidigit whole numbers. They select and apply appropriate methods to estimate sums and differences or calculate them mentally, depending on the context and numbers involved. They develop fluency with efficient procedures, including standard algorithms, for adding and subtracting whole numbers, understand why the procedures work (on the basis of place value and properties of operations), and use them to solve problems.

**Measurement:** Developing an understanding of linear measurement and facility in measuring lengths

Children develop an understanding of the meaning and processes of measurement, including such underlying concepts as partitioning (the mental activity of slicing the length of an object into equal-sized units) and transitivity (e.g., if object A is longer than object B and object B is longer than object C, then object A is longer than object C). They understand linear measure as an iteration of units and use rulers and other measurement tools with that understanding. They understand the need for equal-length units, the use of standard units of measure (centimeter and inch), and the inverse relationship between the size of a unit and the number of units used in a particular measurement (i.e., children recognize that the smaller the unit, the more iterations they need to cover a given length).

### Connections to the Focal Points

**Number and Operations:** Children use place value and properties of operations to create equivalent representations of given numbers (such as 35 represented by 35 ones, 3 tens and 5 ones, or 2 tens and 15 ones) and to write, compare, and order multidigit numbers. They use these ideas to compose and decompose multidigit numbers. Children add and subtract to solve a variety of problems, including applications involving measurement, geometry, and data, as well as nonroutine problems. In preparation for grade 3, they solve problems involving multiplicative situations, developing initial understandings of multiplication as repeated addition.

**Geometry and Measurement:** Children estimate, measure, and compute lengths as they solve problems involving data, space, and movement through space. By composing and decomposing two-dimensional shapes (intentionally substituting arrangements of smaller shapes for larger shapes or substituting larger shapes for many smaller shapes), they use geometric knowledge and spatial reasoning to develop foundations for understanding area, fractions, and proportions.

**Algebra:** Children use number patterns to extend their knowledge of properties of numbers and operations. For example, when skip counting, they build foundations for understanding multiples and factors.
Connections and Analyzing Properties of Two-Dimensional Shapes

Number and Operations: Developing an Understanding of Rational Numbers and Fraction

Measurement: Understanding and Applying Concepts of Multiplication and Division as Inverse Operations

Application of Innovative and Analytic Approaches to Problem Solving, Reasoning, and Communication, Making Connections, and Using Representations (E.g., Graphs, Tables, Equations, Functions, Models, and Other Notations) in the Process of Solving Problems.

The goal of these curriculum focal points is to address concepts that are essential for students to achieve success in mathematics. These focal points are clustered around specific areas of focus, each with its own set of standards that guide the teaching and learning process.

Grade 3 Curriculum Focal Points and Connections for Grade 3
**Curriculum Focal Points and Connections for Grade 4**

The set of three curriculum focal points and related connections for mathematics in grade 4 follow. These topics are the recommended content emphases for this grade level. It is essential that these focal points be addressed in contexts that promote problem solving, reasoning, communication, making connections, and designing and analyzing representations.

<table>
<thead>
<tr>
<th>Grade 4 Curriculum Focal Points</th>
<th>Connections to the Focal Points</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Number and Operations and Algebra:</strong> Developing quick recall of multiplication facts and related division facts and fluency with whole number multiplication</td>
<td><strong>Algebra:</strong> Students continue identifying, describing, and extending numeric patterns involving all operations and nonnumeric growing or repeating patterns. Through these experiences, they develop an understanding of the use of a rule to describe a sequence of numbers or objects.</td>
</tr>
<tr>
<td>Students use understandings of multiplication to develop quick recall of the basic multiplication facts and related division facts. They apply their understanding of models for multiplication (i.e., equal-sized groups, arrays, area models, equal intervals on the number line), place value, and properties of operations (in particular, the distributive property) as they develop, discuss, and use efficient, accurate, and generalizable methods to multiply multidigit whole numbers. They select appropriate methods and apply them accurately to estimate products or calculate them mentally, depending on the context and numbers involved. They develop fluency with efficient procedures, including the standard algorithm, for multiplying whole numbers, understand why the procedures work (on the basis of place value and properties of operations), and use them to solve problems.</td>
<td><strong>Geometry:</strong> Students extend their understanding of properties of two-dimensional shapes as they find the areas of polygons. They build on their earlier work with symmetry and congruence in grade 3 to encompass transformations, including those that produce line and rotational symmetry. By using transformations to design and analyze simple tilings and tessellations, students deepen their understanding of two-dimensional space.</td>
</tr>
<tr>
<td><strong>Number and Operations:</strong> Developing an understanding of decimals, including the connections between fractions and decimals</td>
<td><strong>Measurement:</strong> As part of understanding two-dimensional shapes, students measure and classify angles.</td>
</tr>
<tr>
<td>Students understand decimal notation as an extension of the base-ten system of writing whole numbers that is useful for representing more numbers, including numbers between 0 and 1, between 1 and 2, and so on. Students relate their understanding of fractions to reading and writing decimals that are greater than or less than 1, identifying equivalent decimals, comparing and ordering decimals, and estimating decimal or fractional amounts in problem solving. They connect equivalent fractions and decimals by comparing models to symbols and locating equivalent symbols on the number line.</td>
<td><strong>Data Analysis:</strong> Students continue to use tools from grade 3, solving problems by making frequency tables, bar graphs, picture graphs, and line plots. They apply their understanding of place value to develop and use stem-and-leaf plots.</td>
</tr>
<tr>
<td><strong>Measurement:</strong> Developing an understanding of area and determining the areas of two-dimensional shapes</td>
<td><strong>Number and Operations:</strong> Building on their work in grade 3, students extend their understanding of place value and ways of representing numbers to 100,000 in various contexts. They use estimation in determining the relative sizes of amounts or distances. Students develop understandings of strategies for multidigit division by using models that represent division as the inverse of multiplication, as partitioning, or as successive subtraction. By working with decimals, students extend their ability to recognize equivalent fractions. Students’ earlier work in grade 3 with models of fractions and multiplication and division facts supports their understanding of techniques for generating equivalent fractions and simplifying fractions.</td>
</tr>
</tbody>
</table>

Students recognize area as an attribute of two-dimensional regions. They learn that they can quantify area by finding the total number of same-sized units of area that cover the shape without gaps or overlaps. They understand that a square that is 1 unit on a side is the standard unit for measuring area. They select appropriate units, strategies (e.g., decomposing shapes), and tools for solving problems that involve estimating or measuring area. Students connect area measure to the area model that they have used to represent multiplication, and they use this connection to justify the formula for the area of a rectangle.
Connections to the Focal Points

Curriculum Focal Points for Grade 5

Number and Operations: Developing an understanding of and fluency with addition and subtraction

- Problem is extended to solve the most usual form of the problem for the solution, and they interpret the answer in the context of the situation. They recognize the context in which a problem occurs, and they determine whether the problem requires the use of addition or subtraction. They understand why addition or subtraction is appropriate and apply the appropriate operations to solve problems.

- Students apply their understanding of models for division, place value, and the relationships among operations. They develop fluency in operations with multi-digit whole numbers and decimals and develop and discuss strategies for multiplication. They develop, evaluate, and apply algorithms for addition and subtraction of multi-digit numbers. They develop and evaluate strategies for multiplication and division of multi-digit numbers. They develop and evaluate strategies for division of whole numbers.
# Curriculum Focal Points and Connections for Grade 6

The set of three curriculum focal points and related connections for mathematics in grade 6 follow. These topics are the recommended content emphases for this grade level. It is essential that these focal points be addressed in contexts that promote problem solving, reasoning, communication, making connections, and designing and analyzing representations.

<table>
<thead>
<tr>
<th>Grade 6 Curriculum Focal Points</th>
<th>Connections to the Focal Points</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Number and Operations:</strong> Developing an understanding of and fluency with multiplication and division of fractions and decimals</td>
<td><strong>Number and Operations:</strong> Students’ work in dividing fractions shows them that they can express the result of dividing two whole numbers as a fraction (viewed as parts of a whole). Students then extend their work in grade 5 with division of whole numbers to give mixed number and decimal solutions to division problems with whole numbers. They recognize that ratio tables not only derive from rows in the multiplication table but also connect with equivalent fractions. Students distinguish multiplicative comparisons from additive comparisons.</td>
</tr>
<tr>
<td>Students use the meanings of fractions, multiplication and division, and the inverse relationship between multiplication and division to make sense of procedures for multiplying and dividing fractions and explain why they work. They use the relationship between decimals and fractions, as well as the relationship between finite decimals and whole numbers (i.e., a finite decimal multiplied by an appropriate power of 10 is a whole number), to understand and explain the procedures for multiplying and dividing decimals. Students use common procedures to multiply and divide fractions and decimals efficiently and accurately. They multiply and divide fractions and decimals to solve problems, including multistep problems and problems involving measurement.</td>
<td><strong>Algebra:</strong> Students use the commutative, associative, and distributive properties to show that two expressions are equivalent. They also illustrate properties of operations by showing that two expressions are equivalent in a given context (e.g., determining the area in two different ways for a rectangle whose dimensions are ( x + 3 ) by 5). Sequences, including those that arise in the context of finding possible rules for patterns of figures or stacks of objects, provide opportunities for students to develop formulas.</td>
</tr>
<tr>
<td><strong>Number and Operations:</strong> Connecting ratio and rate to multiplication and division</td>
<td><strong>Measurement and Geometry:</strong> Problems that involve areas and volumes, calling on students to find areas or volumes from lengths or to find lengths from volumes or areas and lengths, are especially appropriate. These problems extend the students’ work in grade 5 on area and volume and provide a context for applying new work with equations.</td>
</tr>
<tr>
<td>Students use simple reasoning about multiplication and division to solve ratio and rate problems (e.g., “If 5 items cost $3.75 and all items are the same price, then I can find the cost of 12 items by first dividing $3.75 by 5 to find out how much one item costs and then multiplying the cost of a single item by 12”). By viewing equivalent ratios and rates as deriving from, and extending, pairs of rows (or columns) in the multiplication table, and by analyzing simple drawings that indicate the relative sizes of quantities, students extend whole number multiplication and division to ratios and rates. Thus, they expand the repertoire of problems that they can solve by using multiplication and division, and they build on their understanding of fractions to understand ratios. Students solve a wide variety of problems involving ratios and rates.</td>
<td><strong>Algebra:</strong> Writing, interpreting, and using mathematical expressions and equations</td>
</tr>
<tr>
<td>Students write mathematical expressions and equations that correspond to given situations, they evaluate expressions, and they use expressions and formulas to solve problems. They understand that variables represent numbers whose exact values are not yet specified, and they use variables appropriately. Students understand that expressions in different forms can be equivalent, and they can rewrite an expression to represent a quantity in a different way (e.g., to make it more compact or to feature different information). Students know that the solutions of an equation are the values of the variables that make the equation true. They solve simple one-step equations by using number sense, properties of operations, and the idea of maintaining equality on both sides of an equation. They construct and analyze tables (e.g., to show quantities that are in equivalent ratios), and they use equations to describe simple relationships (such as ( 3x - y )) shown in a table.</td>
<td></td>
</tr>
</tbody>
</table>
Connections to the Focal Points

**Grade 7 Curriculum Focal Points**

- **Algebra and Geometry:** Developing an understanding of and applying proportionality, including similarity.
  - Measurement and Geometry: Students connect their work on proportionality to their work on area and volume by investigating similar objects. They understand that if a scale factor describes how corresponding lengths in two similar objects are related, then the square of the scale factor describes how corresponding areas are related, and the cube of the scale factor describes how corresponding volumes are related. Students apply proportionality involving units of measurement to solve problems involving similar units of measurement, including the circumference, radius, and diameter of a circle, when they find the area of a sector of a circle, and when they scale drawings.

- **Data Analysis:** Students use proportions to make and interpret histograms, circle graphs, and other data displays. They apply percentages to make approximate predictions.

- **Number and Operations:** Students use numbers involving decimals, fractions, and percents to make and interpret histograms, circle graphs, and other data displays. They apply percentages to make approximate predictions.

**Number and Operations:** Developing an understanding of operations on all rational numbers and solving linear equations.

- As students develop a sense of number, they develop an understanding of the properties of operations and how they apply to numbers. They develop a sense of the meaning of the equal sign as a balance point. They extend their understanding of rational numbers, including negative integers, by applying properties of arithmetic and considering the order of operations. They use the arithmetic of rational numbers as they solve linear equations in one variable. Students formulate and solve linear equations in one variable, and use linear equations to solve problems involving proportional relationships.

- As students extend understandings of addition, subtraction, multiplication, and division, together with their properties, to all rational numbers, including negative integers, by applying properties of arithmetic and considering the order of operations, they extend their understanding of the equal sign as a balance point. They use the arithmetic of rational numbers as they solve linear equations in one variable. Students formulate and solve linear equations in one variable, and use linear equations to solve problems involving proportional relationships.

**Algebra and Geometry:** Developing an understanding of and applying proportionality, including similarity.

- By decomposing two- and three-dimensional shapes into smaller component shapes, students find surface areas and volumes of prisms and cylinders. They develop and understand formulas for decomposing a circle into a number of wedges and rearranging them into a shape that approximates a parallelogram. They select appropriate two- and three-dimensional problems involving surface areas, areas and circumferences of circles, and volumes of prisms and cylinders.

- Students extend understandings of addition, subtraction, multiplication, and division, together with their properties, to all rational numbers, including negative integers, by applying properties of arithmetic and considering the order of operations. They use the arithmetic of rational numbers as they solve linear equations in one variable. Students formulate and solve linear equations in one variable, and use linear equations to solve problems involving proportional relationships.

**Number and Operations:** Developing an understanding of operations on all rational numbers and solving linear equations.

- As students develop a sense of number, they develop an understanding of the properties of operations and how they apply to numbers. They develop a sense of the meaning of the equal sign as a balance point. They extend their understanding of rational numbers, including negative integers, by applying properties of arithmetic and considering the order of operations. They use the arithmetic of rational numbers as they solve linear equations in one variable. Students formulate and solve linear equations in one variable, and use linear equations to solve problems involving proportional relationships.
Curriculum Focal Points and Connections for Grade 8

The set of three curriculum focal points and related connections for mathematics in grade 8 follow. These topics are the recommended content emphases for this grade level. It is essential that these focal points be addressed in contexts that promote problem solving, reasoning, communication, making connections, and designing and analyzing representations.

<table>
<thead>
<tr>
<th>Grade 8 Curriculum Focal Points</th>
<th>Connections to the Focal Points</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Algebra: Analyzing and representing linear functions and solving linear equations and systems of linear equations</strong></td>
<td><strong>Algebra:</strong> Students encounter some nonlinear functions (such as the inverse proportions that they studied in grade 7 as well as basic quadratic and exponential functions) whose rates of change contrast with the constant rate of change of linear functions. They view arithmetic sequences, including those arising from patterns or problems, as linear functions whose inputs are counting numbers. They apply ideas about linear functions to solve problems involving rates such as motion at a constant speed.</td>
</tr>
</tbody>
</table>

  When students use linear functions, linear equations, and systems of linear equations to represent, analyze, and solve a variety of problems. They recognize a proportion \((y/x = k, \text{ or } y = kx)\) as a special case of a linear equation of the form \(y = mx + b\), understanding that the constant of proportionality \(k\) is the slope and the resulting graph is a line through the origin. Students understand that the slope \((m)\) of a line is a constant rate of change, so if the input, or \(x\)-coordinate, changes by a specific amount, \(a\), the output, or \(y\)-coordinate, changes by the amount \(ma\). Students translate among verbal, tabular, graphical, and algebraic representations of functions (recognizing that tabular and graphical representations are usually only partial representations), and they describe how such aspects of a function as slope and \(y\)-intercept appear in different representations. Students solve systems of two linear equations in two variables and relate the systems to pairs of lines that intersect, are parallel, or are the same line, in the plane. Students use linear equations, systems of linear equations, linear functions, and their understanding of the slope of a line to analyze situations and solve problems.

| **Geometry and Measurement:** Analyzing two- and three-dimensional space and figures by using distance and angle | **Geometry:** Given a line in a coordinate plane, students understand that all "slope triangles"—triangles created by a vertical "rise" line segment (showing the change in \(y\)), a horizontal "run" line segment (showing the change in \(x\)), and a segment of the line itself—are similar. They also understand the relationship of these similar triangles to the constant slope of a line. |

  Students use fundamental facts about distance and angles to describe and analyze figures and situations in two- and three-dimensional space and to solve problems, including those with multiple steps. They prove that particular configurations of lines give rise to similar triangles because of the congruent angles created when a transversal cuts parallel lines. Students apply this reasoning about similar triangles to solve a variety of problems, including those that ask them to find heights and distances. They use facts about the angles that are created when a transversal cuts parallel lines to explain why the sum of the measures of the angles in a triangle is 180 degrees, and they apply this fact about triangles to find unknown measures of angles. Students explain why the Pythagorean theorem is valid by using a variety of methods—for example, by decomposing a square in two different ways. They apply the Pythagorean theorem to find distances between points in the Cartesian coordinate plane to measure lengths and analyze polygons and polyhedra.

| **Data Analysis and Number and Operations and Algebra:** Analyzing and summarizing data sets | **Data Analysis:** Building on their work in previous grades to organize and display data to pose and answer questions, students now see numerical data as an aggregate, which they can often summarize with one or several numbers. In addition to the median, students determine the 25th and 75th percentiles (1st and 3rd quartiles) to obtain information about the spread of data. They may use box-and-whisker plots to convey this information. Students make scatterplots to display bivariate data, and they informally estimate lines of best fit to make and test conjectures. |

  Students use descriptive statistics, including mean, median, and range, to summarize and compare data sets, and they organize and display data to pose and answer questions. They compare the information provided by the mean and the median and investigate the different effects that changes in data values have on these measures of center. They understand that a measure of center alone does not thoroughly describe a data set because very different data sets can share the same measure of center. Students select the mean or the median as the appropriate measure of center for a given purpose. |
Readers should note that colors repeat in sequence from grade band to grade band:

- Red indicates Grade 6+
- Green indicates Grade 7+
- Yellow indicates Grade 4+
- For the Grade 6 band:
  - Red indicates Grade 5+
  - Green indicates Grade 4
  - Yellow indicates Kindergarten
- For the Pre-K-2 grade band:
  - Blue indicates Grade 1
  - Red indicates Grade 2

Local points of particular grade levels:

The tables use the following colors to indicate content in Principles and Standards that appears in the grade band.

Refer to the Grade Level. This color falls within a group of colors that represent all the grade levels in the relevant band. The color is located on a local point of connection on the left if it is indicated by a dot in a color that specifies the right-hand column of each table. A link between an expectation on Standards for the Grade Band or Grade Band appears in the right-hand column of each table. A link between an expectation on the Common Core State Standards (CCSSM) and the Curriculum Roadmap Grades 8 Mathematics for the band of Grade Band or Grade Band. The right-hand column in each grade-band table shows the curriculum related points for the Common Core State Standards (CCSSM) and the Curriculum Roadmap Grades 8 Mathematics for the band of Grade Band or Grade Band.


Appendix