Sicherman Dice

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Many different types of games involve the use of numbered dice. The most common type of die used is the standard six-sided die with the numbers one through six used on its six sides. What if there was another way of numbering a set of dice (using only positive integers) that would create the same probability outcomes as those of a standard set of dice? The February 1978 issue of Scientific American reports that George Sicherman discovered such a numbering (Broline, 1979). Sicherman discovered that two cubic die numbered 1-2-2-3-3-4 and 1-3-4-5-6-8 have the same sum probabilities as do the standard set of cubic dice (both labeled 1-2-3-4-5-6). This paper will explore his findings.

Comment [U1]: You want to avoid the confusion that a die has multiple numbers on each side—a ridiculous idea for game players, but maybe not for people reading a mathematics paper.

To help answer this question, we begin with a simpler situation involving two two-sided dice. Suppose we have a pair of two-sided dice, each with the numbers 1 and 2 on its faces. The following table shows the possible sums achieved.

		Die #1	
	sum	**1**	**2**
Die #2	**1**	2	3
	2	3	4

Sum	2	3	4
# of ways	1	2	1

Another way to represent this information is with the use of polynomials. Each term of the polynomial represents the number on the side of the die and how many sides have that number. In general, the term tx^k means that there are t sides labeled k. So, to represent a two-sided die with a polynomial, we write $p(x) = 1x^1 + 1x^2$, where the term $1x^1$ means 1 side has a 1 and $1x^2$ means 1 side has a 2. Since we are using two dice, we can calculate the distribution of the sums by computing $p(x)^2$.

$$p(x)^2 = \underbrace{\left(1x^1 + 1x^2\right)}^{\text{die \#1}}\overbrace{\left(1x^1 + 1x^2\right)}^{\text{die \#2}}$$

$$p(x)^2 = x^2 + x^3 + x^3 + x^4$$

$$p(x)^2 = x^2 + 2x^3 + x^4$$

Note that the terms of the resulting polynomial gives the distribution of the sums in the following way: the term $1x^2$ means there is one way to obtain a sum of 2, the term $2x^3$ means there are two ways to obtain a sum of 3, and the term $1x^4$ means there is one way to obtain a sum of 4. The number of ways to obtain each sum found here match the table above.

This type of general representation works because of the distribution property that is being used when multiplying polynomial expressions. This method is the same as the first table by taking each number of dice #1 and adding it to each number of dice #2. So, to create the table of sums, we do the following: (1+1), (1+2), (2+1), (2+2). So the multiplication of the polynomials shows the distribution and all of the different ways that the two dice will achieve each sum.

Comment [U2]: Both tables have numbers.

Now, look at the distribution of the sums when using three two-sided dice. Using the previous knowledge about the polynomial expression and $p(x) = 1x^1 + 1x^2$, then we want to compute $p(x)^3$.

$$\begin{aligned}
p(x)^3 &= \overset{\text{die \#1}}{\left(1x^1 + 1x^2\right)}\overset{\text{die \#2}}{\left(1x^1 + 1x^2\right)}\overset{\text{die \#3}}{\left(1x^1 + 1x^2\right)} \\
p(x)^3 &= p(x)^2 \bullet p(x) \\
p(x)^3 &= \left(1x^2 + 2x^3 + 1x^4\right)\left(1x^1 + 1x^2\right) \\
p(x)^3 &= x^3 + x^4 + 2x^4 + 2x^5 + x^5 + x^6 \\
p(x)^3 &= x^3 + 3x^4 + 3x^5 + x^6
\end{aligned}$$

From this distribution we can see that we will have one way to obtain a sum of 3, three ways to obtain a sum of 4, three ways to obtain a sum of 5, and one way to obtain a sum of 6. We can justify this result by using the following table showing all the different results possible when three two-sided are rolled.

Die #1	**Die #2**	**Die #3**	**Sum Obtained**
1	1	1	3
1	1	2	4
1	2	1	4
1	2	2	5
2	1	1	4
2	1	2	5
2	2	1	5
2	2	2	6

Now look at the distribution of the sums when using four two-sided dice. Using the previous knowledge about the polynomial expression and $p(x) = 1x^1 + 1x^2$, then we want to compute $p(x)^4$.

$$\begin{aligned}
&\qquad\quad \text{die \#1} \qquad \text{die \#2} \qquad \text{die \#3} \qquad \text{die \#4} \\
p(x)^4 &= \left(1x^1 + 1x^2\right)\left(1x^1 + 1x^2\right)\left(1x^1 + 1x^2\right)\left(1x^1 + 1x^2\right) \\
p(x)^4 &= p(x)^3 \bullet p(x) \\
p(x)^4 &= \left(x^3 + 3x^4 + 3x^5 + x^6\right)\left(x^1 + x^2\right) \\
p(x)^4 &= x^4 + x^5 + 3x^5 + 3x^6 + 3x^6 + 3x^7 + x^7 + x^8 \\
p(x)^4 &= x^4 + 4x^5 + 6x^6 + 4x^7 + x^8
\end{aligned}$$

From this distribution we can see that we will have one way to obtain a sum of 4, four ways to obtain a sum of 5, six ways to obtain a sum of 6, four ways to obtain a sum of 7, and one way to obtain a sum of 8.

From the previous calculations, we begin to notice a pattern develop with the distribution of the sums and the number of two-sided dice being used.

1 die	$p(x) = x^1 + x^2$
2 dice	$p(x)^2 = x^2 + 2x^3 + x^4$
3 dice	$p(x)^3 = x^3 + 3x^4 + 3x^5 + x^6$
4 dice	$p(x)^4 = x^4 + 4x^5 + 6x^6 + 4x^7 + x^8$

The coefficients of the terms follow the pattern of Pascal’s Triangle, and so combinations can be used to calculate each of these coefficients. A combination represents *r*-member subsets of a set with *n* members in which order is not important. A combination of *n* objects taken *r* at a time is found by ${}_nC_r = \dfrac{n!}{r!(n-r)!}$. The exponent of the first term is the same as the number of dice (*n*), and the exponent of each consecutive term increases by 1 with the exponent of the last term being 2 times the total number of dice (2*n*). So, a general expression to compute the distribution of the sums if there were *n* two-sided dice can be written as:

$$p(x)^n = C(n,0)x^n + C(n,1)x^{n+1} + C(n,2)x^{n+2} + ... + C(n,n-2)x^{2n-2} + C(n,n-1)x^{2n-1} + C(n,n)x^{2n}$$

We can use this general expression, for example, to write the distribution of sums for ten two-sided dice:

$$p(x)^{10} = C(10,0)x^{10} + C(10,1)x^{11} + C(10,2)x^{12} + ... + C(10,8)x^{18} + C(10,9)x^{19} + C(10,10)x^{20}$$

$$p(x)^{10} = x^{10} + 10x^{11} + 45x^{12} + 120x^{13} + 210x^{14} + 252x^{15} + 210x^{16} + 120x^{17} + 45x^{18} + 10x^{19} + x^{20}$$

From this distribution, we can see that when using ten two-sided dice, we will have one way to obtain a sum of 10, ten ways to obtain a sum of 11, forty-five ways to obtain a sum of 12, and so on. We also learn from this that when rolling ten two-sided dice, it is not possible, for example, to obtain a sum of 7.

We can now expand the use of the polynomial expressions to represent a six-sided die. Let $q(x) = x^1 + x^2 + x^3 + x^4 + x^5 + x^6$ represent one standard six-sided die. To compute the distribution of sums for two standard six-sided dice, we want to compute $q(x)^2$. Let $f(x) = q(x)^2$. Then

$$\begin{array}{l}
f(x) = (x^1 + x^2 + x^3 + x^4 + x^5 + x^6)(x^1 + x^2 + x^3 + x^4 + x^5 + x^6) \\
\qquad x^2 + x^3 + x^4 + x^5 + x^6 + x^7 \\
\qquad\qquad x^3 + x^4 + x^5 + x^6 + x^7 + x^8 \\
\qquad\qquad\qquad x^4 + x^5 + x^6 + x^7 + x^8 + x^9 \\
\qquad\qquad\qquad\qquad x^5 + x^6 + x^7 + x^8 + x^9 + x^{10} \\
\qquad\qquad\qquad\qquad\qquad x^6 + x^7 + x^8 + x^9 + x^{10} + x^{11} \\
\qquad\qquad\qquad\qquad\qquad\qquad x^7 + x^8 + x^9 + x^{10} + x^{11} + x^{12} \\
f(x) = x^2 + 2x^3 + 3x^4 + 4x^5 + 5x^6 + 6x^7 + 5x^8 + 4x^9 + 3x^{10} + 2x^{11} + x^{12}
\end{array}$$

This distribution shows one way to obtain a sum of 2, two ways to obtain a sum of 3, three ways to obtain a sum of 4, four ways to obtain a sum of 5, and so on.

A general math activity for upper elementary and middle school classrooms (see Appendix B – Classroom activity) is to look at the number of ways any given sum can be achieved by rolling a standard pair of fair six-sided dice. The following table shows the results obtained from this activity.

Die #1

		1	**2**	**3**	**4**	**5**	**6**
Die #2	**1**	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

Sum	2	3	4	5	6	7	8	9	10	11	12
# of ways	1	2	3	4	5	6	5	4	3	2	1

Note that the polynomial expression for f(x) and the table activity obtain the same results for each possible sum.

Recall our original question: can there be other pairs of six-sided dice that have the same sum distribution as a pair of standard six-sided dice? Suppose there were such a pair of six-sided dice. Let $p_1(x)$ and $p_2(x)$ be the polynomials representing these dice. Then $p_1(x)p_2(x) = f(x)$. Therefore, all possible $p_1(x)$ and $p_2(x)$ can be found by factoring $f(x)$ into two terms, where the sum of the coefficient in each term add up to six. With the help of a computer program called Sage, the factoring of $f(x)$ is easily done: $x^2(x+1)^2(x^2-x+1)^2(x^2+x+1)^2$.

Notice that each of the factors of $f(x)$ occur twice, so it is reasonable to assume that each factor used once would create a pair of standard six-sided dice. Therefore, to produce a pair of standard six-sided dice with positive integer entries on each side, we would combine the factors $x(x+1)(x^2-x+1)(x^2+x+1)=x^1+x^2+x^3+x^4+x^5+x^6$.

George Sicherman discovered a different formation of the factors, which led to another pair of six-sided dice with positive integer entries on each side. He used the factors $x(x+1)(x^2+x+1)$ for one die and the factors $x(x+1)(x^2+x+1)(x^2-x+1)^2$ for the other die.

$$\begin{aligned} &x(x+1)(x^2+x+1)\\ &=(x^2+x)(x^2+x+1)\\ &=x+2x^2+2x^3+x^4 \end{aligned}$$

$$(x^4+2x^3+2x^2+x)(x^4-2x^3+3x^2-2x+1)$$

$$\begin{aligned} &x(x+1)(x^2+x+1)(x^2-x+1)^2\\ &=x(x+1)(x^2+x+1)(x^4-2x^3+3x^2-2x+1)\\ &=(x^2+x)(x^2+x+1)(x^4-2x^3+3x^2-2x+1)\\ &=\\ &=x+x^3+x^4+x^5+x^6+x^8 \end{aligned}$$

This arrangement of the factors is the creation of the Sicherman dice, which has a die with the numbers 1-2-2-3-3-4 and another die with the numbers 1-3-4-5-6-8. Using the same activity as above can be another way to show how the distribution of the sums can be the same as a standard pair of dice. The following chart shows Sicherman’s dice sums.

Die #1

Die #2

	1	**2**	**2**	**3**	**3**	**4**
1	2	3	3	4	4	5
3	4	5	5	6	6	7
4	5	6	6	7	7	8
5	6	7	7	8	8	9
6	7	8	8	9	9	10
8	9	10	10	11	11	12

Sum	2	3	4	5	6	7	8	9	10	11	12
# of ways	1	2	3	4	5	6	5	4	3	2	1

According to Wikipedia, there is no other rearrangement of the factors that will produce two six-sided dice with positive integer entries on each side. The polynomial expression created from the distribution of the factors for each die represents the six sides. The sum of the coefficients of the terms equals six, representing the six sides. For example, the polynomial expression for a standard die is $x^1 + x^2 + x^3 + x^4 + x^5 + x^6$, and by substituting 1 for x, the calculated value equals six. For each of the Sicherman die, the same calculated value of six is obtained. For Sicherman die #1: If $r(x) = x + 2x^2 + 2x^3 + x^4$, then $r(1) = 1 + 2(1)^2 + 2(1)^3 + (1)^4 = 1 + 2 + 2 + 1 = 6$. For Sicherman die #2: $x + x^3 + x^4 + x^5 + x^6 + x^8 = 1 + 1^3 + 1^4 + 1^5 + 1^6 + 1^8 = 1 + 1 + 1 + 1 + 1 + 1 = 6$. The same calculated value of six is also obtained if we let x = 1 and substitute into the factor expressions of each die.

Standard die: $x(x+1)(x^2 - x + 1)(x^2 + x + 1) = 1(1+1)(1^2 - 1 + 1)(1^2 + 1 + 1) = 6$
Sicherman die #1: $x(x+1)(x^2 + x + 1) = 1(1+1)(1^2 + 1 + 1) = 6$
Sicherman die #2: $x(x+1)(x^2 + x + 1)(x^2 - x + 1)^2 = 1(1+1)(1^2 + 1 + 1)(1^2 - 1 + 1)^2 = 6$

Can we find other arrangements of the factors that will create a pair of dice whose evaluated sum when x = 1 is also six? An important part of creating the pair of dice using the factors is that x must be a factor of each die. If x is not part of the factoring for each die, then after multiplying the factors for each die, one of the polynomial expressions will have 1 as a term. This term of 1 can be written as $1x^0$, which represents that the die has a face labeled with a zero, which is not a positive integer. The factor of x + 1, when letting x = 1, has a value of 2, and the factor of $x^2 + x + 1$, when letting x = 1, has a value of 3. Therefore, the polynomials for each die must have a factor of x + 1 and $x^2 + x + 1$. If one of the polynomials had both factors of $x^2 + x + 1$, for example, and since (1 +1+1)(1+1+1) = 9, then it would be impossible for that polynomial to equal six when evaluated at one. This leaves only the factor of $x^2 - x + 1$ to work with. By letting x = 1, the value of the factor is 1, which means that it doesn’t matter where the factor is placed. If each die has $x^2 - x + 1$ as a factor, then we have the standard pair, and if both factors of $x^2 - x + 1$ are put together in one die, then we have the Sicherman dice pair. Therefore, Sicherman discovered the only other pair of dice that will produce the same distribution of sums as a standard pair of six-sided dice.

How is the probability of rolling a specific sum affected if you use a pair of Sicherman dice instead of a standard pair? At first glance, a person may think that the probability of rolling a sum of five should be different. The truth is that the probabilities are the same no matter which pair of dice you use. With a standard pair of dice, you have four different ways to obtain a sum of five (1-4, 2-3, 3-2, 4-1), which means that the probability of rolling a sum of five is 4/36, or 1/9. With a pair of Sicherman dice, you still have four different ways to obtain a sum of five (1-4, 2-3, 2-3, 4-1), which still gives the same probability of 4/36, or 1/9. Because the sum distributions are the same for both pair of dice, the probability of rolling any particular sum is the same, no matter which pair of dice you use.

Would these Sicherman dice be an effective substitute for the traditional standard pair of dice for board games? For those games that require only a sum to move on the board, then the answer is: Yes! If the game has some unique characteristic where rolling a double is special, such as Monopoly, then it depends on how badly the player wants to keep the set of doubles from the standard pair of dice. With a standard pair of six-sided dice, there are six different ways to roll doubles (1-1, 2-2, 3-3, 4-4, 5-5, 6-6), whereas with the Sicherman dice, there are only four ways to roll doubles (1-1, 3-3, 3-3, 4-4). Rolling a double is useful in monopoly if you are in jail and do not want to pay the $50 cost to be released, for example. Rolling a double also allows you to have another turn, but if you roll too many doubles in a row, then you are sent to jail. So, if you want a better chance of rolling doubles, then the Sicherman dice are not a good replacement for a standard pair of dice.

What if the die has more than six sides? Suppose we have a die with sides numbered $n_1, n_2, n_3, ... n_k$. Similar polynomial expressions can be used here as were previously used with the six-sided dice. The creation of the polynomials can become very lengthy and sometimes difficult to factor, but the same principles hold true. Broline (1979) discovered a way to use Platonic solids to extend a relationship between a die with k sides and Sicherman's ideas for six-sided dice. He created tables (Appendix A) that show the different factoring relationships that will lead to the same distributions of sums as a standard pair of tetrahedral, cubic, octahedral, and dodecahedral dice.

A single, standard octahedral (8-sided) die has a polynomial expression of $c(x) = x^1 + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8$. Computing the distribution of sums for a pair of standard octahedral dice creates the following polynomial expansion:

$$
\begin{aligned}
c(x)^2 &= (x^1 + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8)(x^1 + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8) \\
&\quad x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + x^9 \\
&\quad x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + x^9 + x^{10} \\
&\quad x^4 + x^5 + x^6 + x^7 + x^8 + x^9 + x^{10} + x^{11} \\
&\quad x^5 + x^6 + x^7 + x^8 + x^9 + x^{10} + x^{11} + x^{12} \\
&\quad x^6 + x^7 + x^8 + x^9 + x^{10} + x^{11} + x^{12} + x^{13} \\
&\quad x^7 + x^8 + x^9 + x^{10} + x^{11} + x^{12} + x^{13} + x^{14} \\
&\quad x^8 + x^9 + x^{10} + x^{11} + x^{12} + x^{13} + x^{14} + x^{15} \\
&\quad x^9 + x^{10} + x^{11} + x^{12} + x^{13} + x^{14} + x^{15} + x^{16} \\
c(x)^2 &= x^2 + 2x^3 + 3x^4 + 4x^5 + 5x^6 + 6x^7 + 7x^8 + 8x^9 + 7x^{10} + 6x^{11} + 5x^{12} + 4x^{13} + 3x^{14} + 2x^{15} + x^{16}
\end{aligned}
$$

Using Sage once again, this polynomial factors into the following product of sums: $x^2(x+1)^2(x^2+1)^2(x^4+1)^2$. The table from Appendix A shows three different representations of the factors that lead to the creation of dice different from a standard pair but having the same distribution of sums.

Example 1) die 1 : $x(x+1)^2(x^2+1) = x^1 + 2x^2 + 2x^3 + 2x^4 + x^5$

die 2 : $x(x^2+1)(x^4+1)^2 = x^1 + x^3 + 2x^5 + 2x^7 + x^9 + x^{11}$

In other words, die one has sides numbered 1-2-2-3-3-4-4-5, and die two has sides numbered 1-3-5-5-7-7-9-11. The following table shows another way to represent the sum distribution for this pair of dice:

Die #1

		1	**2**	**2**	**3**	**3**	**4**	**4**	**5**
Die #2	**1**	2	3	3	4	4	5	5	6
	3	4	5	5	6	6	7	7	8
	5	6	7	7	8	8	9	9	10
	5	6	7	7	8	8	9	9	10
	7	8	9	9	10	10	11	11	12
	7	8	9	9	10	10	11	11	12
	9	10	11	11	12	12	13	13	14
	11	12	13	13	14	14	15	15	16

Example 2) die 1 : $x(x+1)^2(x^4+1) = x^1+2x^2+x^3+x^5+2x^6+x^7$

die 2 : $x(x^2+1)^2(x^4+1) = x^1+2x^3+2x^5+2x^7+x^9$

Here, die one has sides numbered 1-2-2-3-5-6-6-7, and die two has sides numbered 1-3-3-5-5-7-7-9. The following table shows another way to represent the sum distribution for this pair of dice:

Die #1

		1	**2**	**2**	**3**	**5**	**6**	**6**	**7**
Die #2	**1**	2	3	3	4	6	7	7	8
	3	4	5	5	6	8	9	9	10
	3	4	5	5	6	8	9	9	10
	5	6	7	7	8	10	11	11	12
	5	6	7	7	8	10	11	11	12
	7	8	9	9	10	12	13	13	14
	7	8	9	9	10	12	13	13	14
	9	10	11	11	12	14	15	15	16

Example 3) die 1 : $x(x+1)(x^2+1)^2 = x^1+x^2+2x^3+2x^4+x^5+x^6$

$$\text{die 2}: \; x(x+1)(x^4+1)^2 \; = \; x^1 + x^2 + 2x^5 + 2x^6 + x^9 + x^{10}$$

Here, one die has sides numbered 1-2-3-3-4-4-5-6 and the other die has sides numbered 1-2-5-5-6-6-9-10. The following table shows another way to represent the sum distribution for this pair of dice:

		Die #1							
		1	**2**	**3**	**3**	**4**	**4**	**5**	**6**
Die #2	**1**	2	3	4	4	5	5	6	7
	2	3	4	5	5	6	6	7	8
	5	6	7	8	8	9	9	10	11
	5	6	7	8	8	9	9	10	11
	6	7	8	9	9	10	10	11	12
	6	7	8	9	9	10	10	11	12
	9	10	11	12	12	13	13	14	15
	10	11	12	13	13	14	14	15	16

George Sicherman’s discovery of a new pair of dice that behave in the same manner as a standard pair has led many other mathematicians to discover other dice relationships. Not surprisingly, the calculations become more difficult as the number of sides of the dice increases. However, with some mathematical knowledge and computer programs, these calculations can be done.

References

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Appendix A

Tetrahedral Dice

factoring: $x(x+1)^a(x^2+1)^b$

a	b	Numbering
1	1	4 3 2 1 (standard pair)
2	0	3 2 2 1
0	2	5 3 3 1

Cubic Dice

factoring: $x(x+1)(x^2+x+1)(x^2-x+1)^a$

a	Numbering
1	6 5 4 3 2 1 (standard pair)
0	4 3 3 2 2 1
2	8 6 5 4 3 1

Octahedral Dice

factoring: $x(x+1)^a(x^2+1)^b(x^4+1)^c$

a	b	c	Numbering
1	1	1	8 7 6 5 4 3 2 1 (standard pair)
2	1	0	5 4 4 3 3 2 2 1
0	1	2	11 9 7 7 5 5 3 1
2	0	1	7 6 6 5 3 2 2 1
0	2	1	9 7 7 5 5 3 3 1
1	2	0	6 5 4 4 3 3 2 1
1	0	2	10 9 6 6 5 5 2 1

Dodecahedral Dice

factoring: $x(x^2+x+1)(x+1)^a(x^2+1)^b(x^2-x+1)^c(x^4-x^2+1)^d$

a	b	c	d	Numbering
1	1	1	1	12 11 10 9 8 7 6 5 4 3 2 1 (standard pair)
1	1	0	0	6 5 5 3 3 3 3 3 3 2 2 1
1	1	2	2	18 15 14 12 11 10 9 8 7 5 4 1
1	1	0	1	10 9 9 8 8 7 4 3 3 2 2 1
1	1	2	1	14 12 11 10 9 8 7 6 5 4 3 1
1	1	1	0	8 7 6 6 5 5 4 4 3 3 2 1
1	1	1	2	16 15 12 11 10 9 8 7 6 5 2 1
0	2	0	1	11 10 9 9 8 7 5 4 3 3 2 1
2	0	2	1	13 12 10 9 9 8 6 5 5 4 2 1
0	2	0	2	15 14 13 9 9 8 8 7 7 3 2 1
2	0	2	0	9 8 7 6 6 5 5 4 4 3 2 1
0	2	1	1	13 11 11 9 9 7 7 5 5 3 3 1
2	0	1	1	11 10 10 9 7 6 6 5 3 2 2 1
0	2	1	2	17 15 13 11 11 9 9 7 7 5 3 1
2	0	1	0	7 6 6 5 5 4 4 3 3 2 2 1

Classroom Activity

For a standard pair of dice (numbered 1-6), complete the sum table below:

Blue Die

	sum	1	2	3	4	5	6
Red Die	1	2					
	2						
	3						
	4						
	5						
	6						

George Sicherman discovered another pair of dice that are special in some ways. He numbered his dice 1-2-2-3-3-4 and 1-3-4-5-6-8. Complete the sum table below for the Sicherman dice.

Blue Die

	sum	1	2	2	3	3	4
Red Die	1	2					
	3						
	4						
	5						
	6						
	8						

To calculate probability, you want to find a ratio represented by two outcomes.

$$\text{Probability} = \frac{\#\,\text{of successful outcomes}}{\#\,\text{of total outcomes}}$$

1. Find the probability of rolling a 3:
    a) with the standard blue die.

    b) with the sicherman blue die.

2. Find the probability of rolling a 6:
    a) with the standard blue die.

    b) with the sicherman blue die.

3. Find the probability of rolling a sum of 4:
    a) with the standard pair of dice.

    b) with the sicherman pair of dice.

4. Find the probability of rolling a sum of 7:
    a) with the standard pair of dice.

    b) with the sicherman pair of dice.

5. Find the probability of rolling a sum of 12:
    a) with the standard pair of dice.

    b) with the sicherman pair of dice.

Hopefully you found the answers for parts a) and b) to be the same for each of the questions 3-5.
Do you think that this would be true for any of the other sums that could be obtained?
Choose a couple of different sums to check your answer.

What do you notice is special about a pair of Sicherman dice compared to a standard pair of dice?