Master of Arts in Teaching (MAT)
Masters Exam

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In middle school mathematics, students learn to use the formula "distance equals rate times time," usually expressed as $d = r \times t$. Why not consider the formula distance = *velocity* × time? Does the term *velocity* mean something different than the term *rate*? We could also consider the variations of these formulas: $distance \div time = rate$, or $distance \div rate = time$. We can examine the definitions of these words and words which are very similar. After looking at the definitions of these words, maybe we will have a better understanding of how to use the formulas and of the meaning behind them.

Distance is a description of how far apart objects are at any given time. Distance is usually given as a numerical figure with a label like feet, miles, meters or some type of unit of measurement. A rate is a special kind of ratio that compares two different measurements, like miles per hour or feet per second. Velocity is a specific type of rate that incorporates both speed and direction. Although velocity is labeled as miles per hour or feet per second, we also need to indicate a direction when we talk about velocity. Finally, time is a measurement in which events happen in an orderly fashion. For most middle-level purposes, time is measured in hours, minutes or seconds.

Now that we have defined all of the terms, we return to the formula $distance = velocity \times time$. If we drive 50 miles per hour for 2 hours we have traveled 100 miles ($50\frac{miles}{hour} \times 2$ hours = 100 miles). Notice that the *hour* labels cancel each other out which leaves us with 100 miles. This is the very basic formula for finding distance. We can use the alternate forms of the distance formula to find time or velocity. Consider the formula $time = distance \div velocity$ using the same example as I did above. Knowing we drove

100 miles at a velocity of 50 $\frac{miles}{hour}$ we can take 100 miles and divide by 50 $\frac{miles}{hour}$, or calculate 100 miles $\frac{1}{50}\frac{hour}{miles}$ = 2 hours. Notice that the units of miles cancel out and we are left with 2 hours. These formulas are of significant importance when figuring distance, velocity and time.

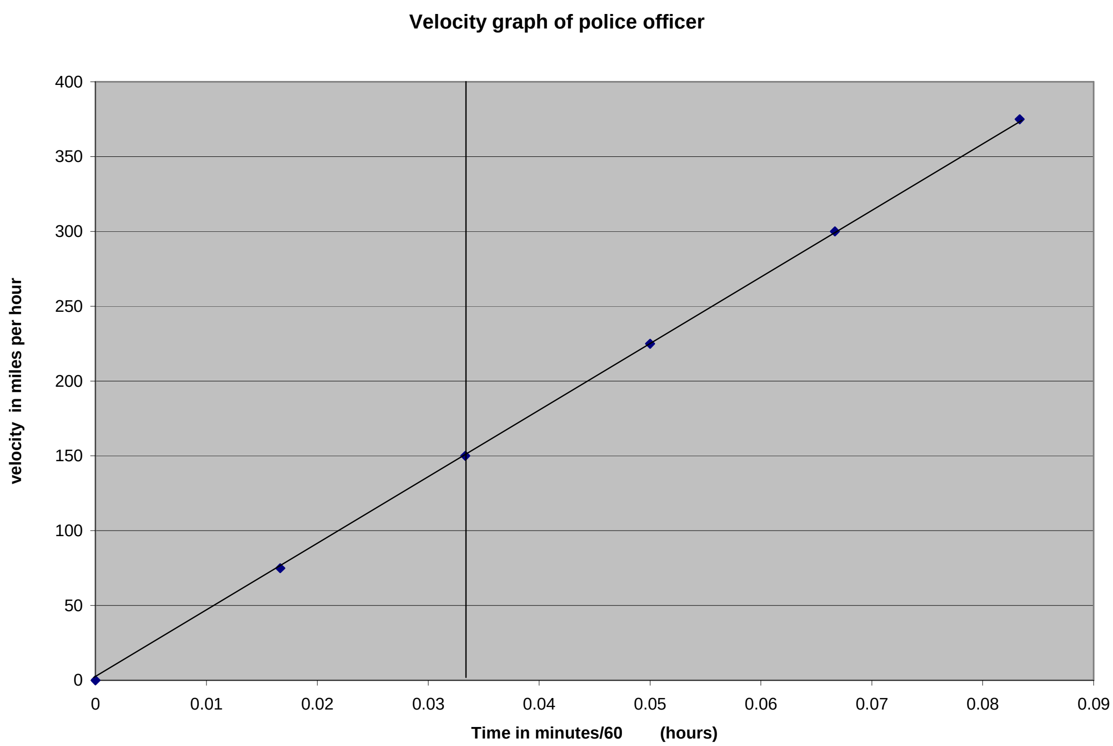
Let's look at a problem where we solve for a velocity. A motorist travels at 30 mph. A police officer starts 1.5 miles behind the motorist, and overtakes the motorist in 2 minutes. How fast is the officer traveling if she travels at constant velocity throughout the chase?

I will use a table which I frequently use when teaching.

	Velocity	×	Time	=	Distance
Motorist	30 mph		$\frac{1}{30}$ hour		1 mile
Police	?		$\frac{1}{30}$ hour		2.5 miles

The entries in the chart are explained as follows: We know the velocity of the motorist is 30 mph and is constant. The time that the drivers are traveling is 2 minutes because that is when the officer overtakes the driver, however we need to be consistent with units of measure so I converted 2 minutes to $\frac{1}{30}$ of an hour. The distance for the motorist can be found using *velocity* × *time* = *distance*, so 30 mph × $\frac{1}{30}$ hour = 1 mile. We did not know the officer's velocity (that is what we are looking for) but we know the time and we can figure the distance. The officer started 1.5 miles behind the motorist plus she needed to drive the 1 mile distance that the motorist had driven before she caught up with the motorist. So the police officer had a distance of 2.5 miles. Now that we know the distance and the time we can use division to calculate the velocity: 2.5miles $\div \frac{1}{30}$ hour, or 2.5 miles $\times \frac{30}{1}$ hour = 75 miles per hour. So the officer was driving at a constant rate of 75 miles per hour to overtake the motorist.

Let's look at another problem from a different angle. A motorist travels at 30 mph. A police officer starts 1.5 miles behind but overtakes the motorist in 2 minutes. Suppose the officer starts at zero velocity and then accelerates at a constant rate until she overtakes the motorist. How fast would she be traveling at that point? I will be looking at this from a calculus viewpoint.



The graph above is looking at the velocity of the police officer. We know that the officer is increasing at a steady rate with velocity starting at zero. We can use calculus to look at this graph, because when using calculus the area under a velocity graph is the distance that the object moves. The only information we need to note about the graph is that it shows the officer was initially stationary, and then increased her velocity at a steady rate. This is indicated by a straight, diagonal line with positive slope which passes through the origin. When we look at the shape that is formed by the function and the vertical line in the graph above, we see a triangle. We also know that the two cars were at the same place after two minutes (time), which I converted to $\frac{2}{60}$ = .033 (repeating) of an hour. I can figure the area under the function using this triangle and the formula *Distance* = *Area* = .5 (*base* × *height*). I know that the distance was 2.5 miles and the base was 1/30 hour. So using this we can find the height of the triangle, or the velocity:

$$2.5 = .5(.033 \times \text{height})$$

$$5 = .033 \times \text{height}.$$

So the height of the triangle is 150. Therefore we can say the police officer had a velocity of 150 mph when he overtook the motorist.

Now, although we did not use this information to determine the height of the triangle (the officer's velocity), I actually configured the graph above by finding the slope of the line through the two points (0, 0) and (.033, 150). Using the slope formula, I found the slope to be 4500. So I could write the function for the diagonal line as $f(x) = 4500x + 0$ (since the velocity started at zero). After drawing the graph I realized I had a triangle whose area determined the distance traveled, and that I could use this distance to find the velocity of the officer.

What is really interesting about these two problems is that they can be solved by an Algebraic method using the *distance* = *velocity* × *time* formula or by Calculus in using a function to create a line or boundary and figuring the area below to find the distance.

Gravitational Acceleration

Gravitational acceleration was originally discovered by Isaac Newton. Newton's law of gravity describes the force that pulls everything to the center of the earth. The gravitational force between two objects depends on their masses. With the earth being so large in comparison to the familiar objects which surround us, we can see one example of gravity in action; when one object has a mass that is really large. Isaac Newton was the first scientist to describe gravity using mathematics. The larger the objects' masses and the closer they are together, the larger the pull of gravity between them.

If air resistance was nonexistent, all objects would fall to the earth at the same rate. In other words if there was no resistance or air friction, a feather would fall at the same rate as a book. According to Newton's second law, it is possible for us to calculate the acceleration of any object when that is acceleration is caused by gravity; "we must take the gravitational force on that object and divide by the inertial mass" (Kobes, Kunstatter, 1999). Newton noted that an object's mass does not change from place to place, but an object's weight does change depending on the distance the object is from the center of the earth. Also, the greater the mass of the object, the greater the pull of gravity there is on the object.

The website entitled *Acceleration of Gravity* (cited below) states:

> "The acceleration of an object due to the earth's gravitation is a = $\frac{GM}{r^2}$, where G is Newton's gravitational constant (G = (6.67428 $\pm$ .0010) x $10^{-11} \text{m}^3 \text{kg}^{-1} \text{s}^{-2}$), M

is the mass of the earth which is 5.9742 x 10^{24}, and r is the distance to the center of the earth which varies from 6,356.750km to 6,378.135km. This computes to an acceleration of 9.8 meters per second per second. Inasmuch as the radius of the earth is very large compared to laboratory dimensions, an object near the surface of the earth, r is approximately constant and equal to the radius of the earth. Thus an object near the surface of the earth will experience a nearly constant acceleration."

Gravitational acceleration is also called free fall. When an object is dropped it will experience a constant acceleration due to gravity. As this object falls downward, the magnitude of the velocity increases at a constant rate (i.e. acceleration) in a downward direction. I had the opportunity this spring to go on a field trip to the museum in Aurora, Nebraska where we used a strobe light to observe falling drops of water. The lights were turned off and as the water was dripping it was illuminated by a strobe light. The strobe light was adjusted so that we would see each drop of water as it fell into the bowl. I remember the instructor there telling us that free-falling objects on earth fall at a rate approximately 10 meters per second per second. That was a neat experience for me.

Escape Velocity

If you would throw a baseball hard enough would it travel into outer space? Well, first of all it would need to escape the earth's gravitational force. One would have to throw the ball with sufficient velocity so that it overcomes the earth's gravitational pull. This initial velocity is called escape velocity. The website www.physllink.com provides information about the energy that an object must have to escape the gravitational pull. The equation $\frac{1}{2} mv^2 = GMm/R$, where m is the mass of the object, M is the mass of the earth, G is the gravitational pull of the earth, R is the radius of the earth, and v is the escape velocity, simplifies to $v = \sqrt{(2 \times GM) \div R}$. This formula does not include the mass, m, of the object because the mass of a baseball is negligible when compared to the mass of the earth. This formula indicates the escape velocity of the earth is approximately 11,100 meters per second, 40,200 kilometers per hour, 25,000 miles per hour or 7 miles per second.

If the object thrown does not reach this escape velocity, it will do one of two things. It will either return to the earth's surface or enter into orbit around the earth. In order for a baseball to escape this gravitational pull we would need to throw it at a velocity of 7 miles per second.

What is interesting about all this is that gravity extends to all objects in space in all directions and no matter how far you go, theoretically you can never fully escape the pull of the earth.

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References

Acceleration of Gravity. Downloaded from http://teacher.nsrl.rochester.edu/phy_labs/Measure_g/Measure_g.html on June 27, 2007.

Kobes, Randy, Kunstatter Gabor. "Gravitational Acceleration". Physics 1501 – Modern Technology. September 9, 1999. <http://theory.uwinnipeg.ca/mod_tech/node55.html>.