

**Precise Mathematical Language: Exploring the Relationship Between Student Vocabulary  
Understanding and Student Achievement**

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### **Abstract**

In this action research study of my classroom of fifth grade mathematics, I investigate the relationship between student understanding of precise mathematics vocabulary and student achievement in mathematics. Specifically, I focused on students' understanding of written mathematics problems and on their ability to use precise mathematical language in their written solutions of critical thinking problems. I discovered that students are resistant to change; they prefer to do what comes naturally to them. Since they have not been previously taught to use precise mathematical language in their communication about math, they have great difficulty in adapting to this new requirement. However, with teaching modeling and ample opportunities to use the language of mathematics, students' understanding and use of specific mathematical vocabulary is increased.

## Introduction

I have taught fifth grade mathematics for the last seven years in a Pre-K through fifth grade Catholic elementary school. Our curriculum, *Saxon Mathematics*, is the only major math series available today that incrementally distributes instruction and practice and assessment rather than presenting the instruction of related concepts in a chapter-based approach in a short time period. In other words, Saxon does not divide its texts into chapters or units such as “time and money” or “measurement.” Saxon’s methodology, often called “distributed practice,” is intended to ensure that students retain essential skills while moving ahead to other required knowledge.

The pedagogy of *Saxon Mathematics* seems straightforward. Each increment is designed to build on earlier increments, leading students to a deeper understanding of mathematical concepts. The authors believe that by carefully distributing related increments throughout the grade level, they can ensure that students have the opportunity to master each skill before moving on to the next one and that students can achieve automaticity of basic skills (Harcourt, 2005).

I have consistently noticed that my students struggle with the precise language of mathematics while proceeding through *Saxon*, both in using it to explain their thinking and ideas, and in solving written problems which contain the precise language. Often, they ask questions about specific words, such as “What does ‘factor’ mean?” or, “Does ‘sum’ mean add?”

When I first learned that I would have to do an action research project, I was unsure about how to even start thinking about an area. After the first Math in the Middle course, “Math as a Second Language,” with Dr. Ken Gross, I was intrigued with the idea of having my students write about math in order to explain their thinking. The Saxon curriculum that I am required to

use includes a supplemental booklet entitled, “Writing About Math.” It contains a sampling of problems from various textbook lessons for which students are to provide written solutions. No one in the school had ever used this booklet. I saw the “Math as a Second Language” course as an opportunity to use some of the problems from that booklet to have my students explain their thinking. My initial desire was simply to have my students do what I had been required to do in Dr. Gross’s course—to explain the “steps” of a problem solution in words. I wanted to be able to understand my students’ thinking and the reasoning behind it. At the same time, I believed that by having to take the time to select appropriate words to describe the steps they used, they might discover errors in their own thinking and reevaluate their solutions. They might even begin to realize, after hearing or reading others’ solutions and use of language, that there can be more than one solution to a problem.

As I continued to use the “Writing About Math” problems to evaluate student understanding, as well as students’ abilities to communicate, I was concerned that most of my students did not use specific mathematical language in their explanations. This was also true of the verbal explanations they gave during the math lesson itself. Instead of saying, for instance, “You change the second fraction to its reciprocal and then multiply the fractions” (dividing fractions), they often said, “You flip the second fraction upside-down and multiply.” When referring to fractions at any time, they almost always said “the bottom number” and “the top number” rather than using the terms denominator and numerator.

With this in mind, I decided to do my research in the area of communication, specifically focusing on mathematical vocabulary. Since I noticed that my students struggled simply to remember the terms and what they mean and with using the language appropriately to explain their thinking, I decided to start at what I considered the beginning—defining terms. I assumed

that my students had some useable knowledge of mathematical concepts that are addressed in fifth grade. However, I had the working hypothesis that students need to have a greater sense of the definitions of the terms and how to connect them to what they are doing in isolated problems. For example, it seems that students should first know the definition of *factor* in order to “List the factors of 18.” They need to know that a *sum* is the answer to an addition problem, so that when they are asked to “find the sum of 5 and 8,” they will add and not multiply. They need to be able to keep straight other terms such as *addend*, *factor*, *product*, *difference*, *quotient*, *divisor*, and *dividend* according to which operation they apply to. Furthermore, it seems most important that students to be able to make the connections between the terms and the concepts. After becoming proficient in making these connections, they can effectively use the precise mathematical language in their written explanations.

In deciding on this area of focus, I considered the four criteria for selecting an area of focus, according to *Action Research: A Guide for the Teacher Researcher* (2007). The area I chose involves teaching in my own area of practice, and it is something I would definitely like to improve if I am able to do so. The last criterion is that it be something I am passionate about. While I am passionate about my area of focus, I do question whether specific instruction in mathematics vocabulary will make a difference.

My area of focus, as I indicated before, relates to the National Council of Teachers of Mathematics (NCTM) standard of communication. The NCTM asserts that communication is much more effective when the parties communicating speak a common language (2000). Students who can speak in mathematical terms, with precision, can presumably communicate effectively with the teacher and with each other. Effective communication can be achieved between students if each knows and understands the terms being used. Ideally, they should to be

able to communicate their thinking to anyone. According to the NCTM, students move from using familiar, everyday language to explain their thinking in the lower grades to the formal mathematical language in later grades (2000). Certainly, anyone with teaching experience at various grade levels has observed this among students. I desire for my students to have such a command of mathematical language that language is not a barrier to their learning at higher levels of math.

### **Problem Statement**

As I have observed my students' difficulty with using the precise language of mathematics in both verbal and written explanations, I have questioned what modifications I might be able to make in my classroom in order to remedy the situation. With my observations in mind, I decided to focus my research on the NCTM standard of communication, specifically focusing on vocabulary (2000). As I studied the research that has been done in this area in the past, I noted that most of the recent research in the area of communication in mathematics has been focused on aspects other than specific vocabulary. Quite often, others have looked at how aspects such as the readability of the text, how students' reading abilities affect understanding in math or how "talking" about math in the classroom facilitates understanding. However, within each of these themes, there is the underlying notion that mathematics is a language all its own. Knowledge of this unique language, then, determines, at least in part, the effectiveness of classroom discourse and the comprehension of students as they read the written text.

My interest in the relationship between student vocabulary understanding and student achievement grew when I began to understand what researchers such as Kotsopoulos and Manoucherhri had already learned about the subject. I realized that modifications in my current

instruction and isolated, planned instruction in precise mathematical vocabulary could greatly improve students' understanding of the vocabulary specific to math. I assumed that the effect of this new understanding would be, at least for some students, improved math performance. If so, I would be able to help my students to not only increase their understanding of math and improve their scores, but also to gain confidence in their abilities to do the work of math.

My choice of this topic is undoubtedly worthwhile research for my practice, since *Saxon Mathematics 65* is rich in mathematics vocabulary. The broader scope—the immediate community of teachers within my school system and the larger community of educators—will benefit as well from this research. My school system uses the Saxon Mathematics curriculum K-5 and Accelerated Math software for the 6-12 curriculum. Both of these programs rely heavily on the use of precise mathematical language in their textbooks and assessments.

### **Literature Review**

I extensively researched the work of scholars and researchers who were interested in the connection between students' understanding of precise mathematical vocabulary and their achievement in mathematics. I found sixteen related articles, and for discussion purposes here, I have organized them into four themes: text readability, classroom discourse, prior reading ability, and specific vocabulary instruction. Much of the previous research that I found is quite dated; still, it is worthwhile to discuss in this paper because the researchers were seeking answers to some of the same questions that I have.

#### Text Readability

Prior to beginning my research of these articles, I had not thought about how the readability of the text impacts student understanding. A “tunnel vision” sort of mentality is

responsible for my ignorance. In my 11 years of teaching, I have taught only from Saxon Mathematics which is, for the most part, a highly scripted text. However, while the students follow along, I have complete flexibility in restating directives or information in ways that I know will be easier for my students to understand. My students are not left “on their own” to make sense of the text, but I am sure that in some classrooms using Saxon, that may be the case.

Noonan (1990) was a schools inspector in England who extensively investigated the readability problems presented by math textbooks. He wrote that children, while able to decode and read the mathematics textbook with fluency, may have great difficulty in learning new math concepts by just reading the written text. This ability develops slowly, because of the nature of mathematical language, so that even by age 16, many students cannot understand new material using the text alone. New topics and concepts should always be introduced and established in classroom discussion.

Similarly, though not as specifically, in 1972, Aiken found differing opinions about the effect of textbook readability on students’ understanding and ability to solve math problems. Aiken reviewed others’ research on the relationship between verbal factors and mathematics learning. He noted that using at least some of the common readability formulas led the researchers he studied to a conclusion that the vocabulary of math textbooks is often at a higher readability level than that of the students’ abilities.

However, Dale and Chall (1948) created a readability formula for determining the level of written text. The formula computes a raw score, called the Reading Grade Score (RGS), which rates the text on a grade level based on the average sentence length and the number of unfamiliar words, using a predetermined list of 3,000 words commonly known by 4th grade students. The formula for the Reading Grade Score is:  $RGS = (0.1579 \times DS) + (0.0496 \times ASL) + 3.6365$ ,

where DS is the Dale Score, or the percent of words not on the common list, and ASL is the average sentence length (number of words divided by the number of sentences). Then the RGS is matched to its corresponding grade level. The idea behind this formula is that readers typically find it easier to read, process and recall a passage if the words are familiar. Using it, educators discovered that, among California public schools mathematics textbooks, a wide range of readabilities exist in books intended for the same grade level.

The work of these researchers investigating the effects of text readability on student understanding reveals that the vocabulary instruction may result in increased student achievement. Language of the text itself might be a stumbling block for some students' understanding, as teachers guide students through lessons in a book. However, this research gives credence to my working hypothesis that teaching specific vocabulary in advance of the lesson can lead to better understanding and ultimately improve student achievement. Students can be able to make sense of the language of the text because of the preliminary instruction.

### Classroom Discourse

Students who can speak in mathematical terms, with precision, can communicate effectively with the teacher. Effective communication will be achieved between students only if each knows and understands the terms being used. They need to be able to communicate their thinking to anyone. Burton and Morgan (2000) analyzed 53 published research papers on the topic of mathematics communication and the ways in which language is used in the mathematics classroom by teachers and students. They found that the language used, both in and out of the classroom, influences the perceptions that students form about math and shapes their mathematical interest.

Students develop interesting perceptions about math, as noted by Kotsopoulos (2007). After observing that many students perceive the language used in mathematics as a sort of “foreign language,” she investigated the problem by taping other teachers’ classroom discussions and interviewing students about their difficulties. Her investigation revealed that teachers, including herself, engaged in talking 80% of the instructional time, leaving students just 20% of the time to engage in mathematical conversation. Some students did not talk at all. The concern was that students could not become proficient in math if they were unable to participate in mathematical discussions. Kotsopoulos also learned that students experience interference most often when words are borrowed from their everyday language and inserted into the math discussion in a different context. They then have to relearn familiar word or assimilate them into the mathematical context, and they do this most effectively through classroom discussions.

Along these same lines, Manouchehri (2007) used her “cereal box problem” to support her assertion that math instruction should provide opportunities for students to make meaning of math through classroom discourse. During just the second week of school, in the context of a problem-solving unit, she gave her students several problems. They worked on the problems in small groups or individually and then presented their solutions to the class. During the presentations, the students were asked to ask questions and make comments. Manouchehri’s focus was not so much on teaching specific vocabulary, but rather on using the language of math that her students used to explain and defend their ideas—that which made sense to them. She concluded that these experiences help students to construct meaning, make connections, and enrich their learning.

Some of the work of Kotsopoulos and Manouchehri was particularly helpful to me during my research. Kotsopoulos reported that students often have difficulty understanding how some

words from their everyday language make sense in the context of mathematics. Students can relearn these words in a mathematical context through class discussions. While Manouchehri did not focus on teaching specific vocabulary, she did conclude that using the language of mathematics to explain their ideas and solutions helps students to construct meaning.

### Prior Reading Ability

It is easy to understand how a student's ability to read might aid or interfere with his ability to comprehend written math problems. In his review of earlier research on language factors related to mathematics teaching and learning, Aiken (1972) cited Martin's (1964) work looking at the relationships between problem solving in math and reading ability (as indicated in results on the Iowa Test of Basic Skills). Aiken formed this conclusion: The correlation between reading comprehension and problem solving abilities, when not taking into account computational abilities, is higher at both the fourth and fifth grade levels than is the correlation between computational ability and problem solving ability, when reading comprehension is not taken into account.

Much later, Montis (2000) conducted a year-long qualitative case study of a 12 year old girl who had experienced many difficulties in reading and deficits in mathematical ability. She tracked the girl's experiences during her math class and tutoring sessions and discovered the important role that language processes play in the understanding of math. The extra help the girl was given to improve her reading comprehension resulted in higher scores in math.

### Specific Vocabulary Instruction

In 1933 Monroe and Engelhart conducted an experiment in fifth grade classrooms in Decatur, IL, to learn the effectiveness of a program involving the teaching of how to solve word problems in mathematics. Students in the experimental groups were asked to define the math

terms used in the problems, restating them in their own words and rewording the problem itself without changing the context of the problem. Later, Dresher (1934) sought to determine the effects of (extensive) and specific vocabulary instruction on learners of junior-high mathematics. His contention was that, as a result of this specific instruction, the students would have a larger vocabulary. He wondered if they would then have a better understanding and ability to solve word problems. All four tests that he used revealed little increase in knowledge of specific vocabulary under usual teaching conditions. However, there was a significant gain in knowledge of vocabulary and in ability to solve problems when specific vocabulary was taught first. Focusing his study on a higher level of education, Ogilvy (1949) found that among 60 college freshmen males, 40 of whom were pre-science majors and 20 of whom were pre-arts majors, the percent that missed each question about the definition of a specific mathematical term ranged from 20%-55%, depending on the term in question.

Almost two decades later, Vanderlinde (1964) experimented with nine control classes and nine experimental classes in fifth grade, comparing IQ and achievement test scores in vocabulary, reading comprehension, math concepts, and math problem solving. The experimental classes studied a list of math vocabulary terms for up to 24 weeks, and the tests were readministered; like the results of Dresher's (1934) testing, Vanderlinde's results indicated greater gains by the experimental classes in both math concepts and problem solving.

It seems, from my investigation of the research done in this area, that educators have been interested in the effects of vocabulary to a greater extent in the past than they are now. This interest even led some to compare students across decades. Olander and Ehmer (1971) reported on a 1968 study by Buswell and John, which compared students of 1930 and of 1968 to discover which group of students had a better understanding of math vocabulary. The researchers

concluded that younger children of the day (1968) knew the vocabulary better than did their counterparts of 1930, but that advantage disappeared as the children grew older.

Aiken (1972) concluded from his research that, because knowledge of vocabulary is crucial, vocabulary training should be a goal of math instruction. He found that the understanding of specific mathematical vocabulary had improved among elementary school student over the previous 40-year period, but that problems with difficult vocabulary continue to interfere with effective problem solving. He cited both Dresher (1934) and Johnson (1944) who found that problem-solving ability improved following instruction in mathematical vocabulary.

More recently, Jackson and Phillips (1983) conducted a study among 111 seventh grade students to determine if achievement in a unit on ratio and proportion was improved by vocabulary-oriented instruction. They found, at the end of their experiment, that the mean score for students in the experimental groups was significantly higher for both verbal and computational problems. Somewhat surprisingly, the experimental groups outscored the control groups on computational items, despite having less time for computation practice each day. The researchers advised that further research should be done to verify that their results can be generalized to other similar student groups. However, they concluded that concentrating on the definitions of a few specific terms for only a few minutes daily can result in increased achievement.

Some researchers have studied the effects of specific vocabulary instruction in the content areas, and their conclusions can be applied to the teaching of math. Bauman, Edwards, Boland, Olejnik, and Kame'enui (2003) compared the effects of teaching vocabulary using morphemic and contextual analysis with the effects of teaching vocabulary using the curricular materials provided in the textbook. Based on their study of the 157 fifth grade social studies

students, they concluded that specific vocabulary should be taught using the textbook and, for some words, using morphemic analysis. The notion that some of the terms taught in social studies classrooms are specific to the content and would not be considered “everyday” language fits well with the ideas of Hersh (1997), an emeritus professor of mathematics at the University of New Mexico who studies mathematics as a part of the human culture. He cited several examples of specific mathematical vocabulary in his assertion that math is a language all its own, which he calls “math lingo.” He contended that teachers must teach their students *all* of the interpretations of a particular math term to help them understand that, in the context of math class, the words they hear or read in the text are technical terms, not plain English.

Rubenstein (2007), a teacher of preservice and master’s degree students at the University of Michigan, studies mathematics communication and curriculum development. She cited the NCTM goal of enabling students to use language to express mathematics precisely. Her article focused on one part of communication—acquiring mathematical language and being able to use it fluently—while she explained a variety of strategies to foster vocabulary development. Among these, the practice of developing concepts before introducing new terms, was particularly relevant to my project, because it gives purpose to the idea of enabling students to make connections between the new vocabulary and the concepts they already know.

My research looks similar to that of some of the researchers that I have described in this vocabulary section. At the same time, it is very different from other researchers’ work. Like Monroe and Englehart (1933), I conducted my research with fifth grade students, evaluating their ability to define or demonstrate their understanding of mathematical terms. I did not expect them to rewrite problems in their own words, however. Vanderlinde (1964) also studied fifth grade students, finding that students who were given specific instruction in selected vocabulary terms

did better than the control groups on both math computation and problem solving items. I, too, gave instruction to fifth graders on certain vocabulary terms and then assessed achievement to see if the instruction made a difference. Dresher (1934) conducted his work with middle-schoolers, but our objectives were the same. Like him, I attempted to increase student achievement by teaching mathematical vocabulary in isolation.

Jackson and Phillips (1983) also found that vocabulary instruction significantly improved student achievement. Their research, however, was limited to one unit on ratio and proportion, and it was done at the seventh grade level. My research within my fifth grade class was extended for several weeks and across many concepts. Bauman, Edwards, Boland, Olejnik, and Kame'enui (2003) compared two methods of teaching social studies vocabulary to fifth grade students; my work was done at the same grade level, but it took place in the math classroom, and I did not evaluate the difference in effects of two methods of instruction.

Finally, other researchers I mentioned, such as Ogilvy (1949), Aiken (1972), Hersh (1997), and Rubenstein (2007), all concluded that knowledge of the language of math is a critical factor in students' understanding in math and that this knowledge can be achieved through specific instruction in mathematical vocabulary. This sort of specific instruction was the focus of my action research.

### Conclusion

Effective communication can be achieved between students and teachers only when each knows and understands the terms being used. My desire for my students to have such a command of mathematical language that language is not a barrier to their learning at higher levels of math was the reason I chose vocabulary as my area of focus.

Most of the recent research in the area of communication in mathematics has been focused aspects other than specific vocabulary. My action research is different from several of the researchers I have discussed here, because I did not account for prior reading ability and text readability. I did note, to some extent, how vocabulary affects discourse in my classroom. The focus of my research, however, is most similar to those researchers that I outlined in the section above, entitled “Specific Vocabulary Instruction.”

I was excited that several of the studies that I read about were done with fifth grade students, since that is the grade that I currently teach. Monroe and Engelhart (1933) and Vanderlinde (1964) conducted their studies in fifth grade classrooms and had similar results: math achievement increased when the specific language of math was taught prior to and with the relevant concepts. However, their research is quite dated, so I hoped that, while looking at the same factors and effects that they did, I would be able draw similar conclusions. I hoped to learn if giving specific instruction in the language of math would improve students’ understanding of concepts and, more importantly, enable them to communicate more clearly about math.

### **Purpose Statement**

The purpose of my project was to investigate the relationship between specific, isolated instruction in mathematics vocabulary and students’ understanding of written math problems. I examined three features in seeking to answer my research questions: students’ accuracy/precise use of vocabulary terms in written solutions (“Writing About Math” activities), students’ beliefs about the importance and benefits of using precise mathematical vocabulary, and, to a lesser extent, the number of correct assessment answers on problems containing precise mathematical vocabulary.

I was interested in learning if giving specific instruction in the language of mathematics would improve students' understanding of concepts and, more importantly, enable them to communicate more clearly about math. I formulated the following research questions:

1. What will happen to students' use of precise mathematical vocabulary in written solutions after they receive specific, isolated instruction in mathematics vocabulary?
2. What are students' views on the use of precise mathematical vocabulary in written solutions and on assessments?
3. What does my teaching look like when I require the use of precise mathematical vocabulary in students' written solutions?

### **Method**

I decided that I would collect data in four forms throughout my project: teacher journal, written samples, student interviews, and assessments. In my journal, I documented my thoughts, ideas, concerns, and general, overall problem-solving process throughout my project. I also recorded my ideas in designing problems for students to explain. These problems, which I call "WAMs" (Writing About Math), were initially assigned three times each week and were scored using a rubric.

I began by analyzing the "Writing About Math" (WAM) problems that I had assigned beginning the second week in January. I noticed that they all obviously lacked one important requirement—the students were not using mathematical language in their solution descriptions. This would not be such a problem if it was true of one or a few of the students, but as a class, my students were not yet using precise mathematical language. I decided to take a "time-out" from

assigning the problems in order to model for them how to write explanations using precise language.

I purposefully sampled the work of six of my students. For the purpose of this research, I divided my class into three groups based on ability—high, medium, and low. I gave this list to my principal and asked her to choose two students (whose IRB permission forms had been turned in) from each of the groups. I had her choose since I could not know who had turned forms in at that time.

Soon after selecting my six students, I was ready to begin collect my third form of data, which was through student interviews. I decided to modify my original interview protocol by selecting the questions, typing them in an easy-to-follow questionnaire (See Appendix), and giving them to my six focus students to complete at the end of the day. For the second set of interviews, I conducted a one-on-one interview with each of the six focus group students, and for the final interview, I was again forced to give my students written questionnaires.

The last form of data that I began to collect was student assessments. These were the Saxon assessments given after every fifth lesson in the program. Because of Saxon's methodology of distributed practice, each assessment typically has 20-23 problems that test various skills and concepts. I quickly realized that, again, because of the nature of distributed practice, each of the assessments may have only three to seven problems containing mathematical vocabulary. I would have liked to continue my analysis of the assessments to discover if there was a relationship between my students' mathematical vocabulary understanding and their achievement on Saxon assessments, but I decided that I would gain little useful information from the scores. I discontinued my quantitative analysis of assessments at

that point. Instead, I made observations about how my students answered specific assessment questions, and I recorded these observations in my journal.

### **Findings**

Throughout my action research project, I collected and analyzed data both from my whole class and from my focus group of six students. While making these analyses, I attempted to learn whether or not my data provided answers to each of my research questions. In this way, I was able to make assertions based on what I found to be true at each step along the way. In this section, which I have organized according to each research question, I explain my findings of my action research project.

#### **Students' Use of Precise Mathematical Vocabulary in Written Solutions**

Students resist using precise mathematical language in their solutions because it is not natural to them; therefore, specific vocabulary instruction and repetition in using the vocabulary during instruction is necessary to make using the language more natural for the students. They take the shortest route in explaining their thinking, and they make assumptions that the reader will understand what they mean without mathematical language. In a sense, they write how they talk.

One of the first and simplest WAM questions I assigned is:

**Mitchell bought a half-dozen eggs. On the way home he dropped them, and four eggs broke. So Mitchell went back to the store and bought another half-dozen eggs. How many eggs does Mitchell have now?**

Students' typical responses were as simple as mathematical symbols and digits, e.g.,  $6-4=2$ ,  $2+6=8$  eggs. Others were more explanatory, such as:

*First you take six eggs, subtract four eggs and get two eggs. Then you add six more eggs and you get eight eggs.*

Neither of these solutions sufficiently explained the students' thinking or helped the reader to understand the reasoning in the solution. After I took the "time-out" from assigning written solutions and completed several models, I received more sophisticated solutions:

***How many different ways can you write a number sentence with a sum of 13 using two numbers from 1 through 9?***

*First I have to find all the numbers that when added have a sum of 13. Then I'm going to add I start with 9. So  $13-9=4$ . Then  $13-8=5$ . Next  $13-7=6$ . I keep going in this pattern and see that the numbers can be reversed and still equal 13. So  $13-6=7$ . Then  $13-5=8$  and  $13-4=9$ . But  $13-3=10$  and we can only use the numbers 1-9, so there are 6 ways to write a number sentence with a sum of 13 using the numbers 1-9 if you can reverse the same numbers (3 if you don't reverse numbers).*

I would have liked my students to use the term "addend" instead of number and to have recognized the use of fact families in this problem. However, this example does indicate an understanding of the term "sum" and that the operation used in the problem is addition. It is a good explanation of his thinking and it is easily understood.

Precisely because using the mathematical language is so unnatural to them, students would rather use the words that come naturally to them. Again, they resist using the mathematical vocabulary. One of the questions asked in my first interview (which actually turned into a questionnaire) was:

*Would you rather use words that come naturally to you when explaining your solutions, or would you rather use precise math language?*

All six of the students interviewed stated that they prefer to use their own words in written explanations of their solutions. The reasons they gave for this preference were similar, yet in each of them, there were undertones of their individual personalities.

Scott and Emily are two students who have “average” ability but do not always achieve their potential, mainly because they don’t put forth a great deal of effort at times. They both stated that they would rather use their own words “because it is easier.” Even their responses to this question did not require much effort.

I consider Grace and Annikka “thinkers”. Grace commented that she would rather use her own words to explain math because even though she understands the words to do regular problems, she can’t explain her solutions as well with them. Grace, then, has learned to recognize the mathematical vocabulary used in the context of a problem, but she still has difficulty using the language to explain her work. Annikka is more of a reflective thinker than are her peers. She stated that she would rather use words that come naturally to her because that is what she learned to do when she was younger. It makes sense that she considers herself in sort

of a transition stage, in which she is now getting used to using the precise language of mathematics.

Kinsey and Trent are above-average students, both in terms of their academic achievement and in their critical thinking skills. Trent has little difficulty with mathematics, although he sometimes becomes careless in his computation. Sometimes he simply does not focus on what the problem is asking. Interestingly, even when answering this interview question, he seemed to have been focused on a less-important aspect of the vocabulary. He stated that he would rather use words that come naturally because they are easier *to say* than most mathematical words. Kinsey, on the other hand, is very conscientious and tends to be a worrier. She worries that she will make mistakes or not understand a problem correctly, so she asks a lot of questions, even though she could be a much more independent worker. Her answer to this interview question reflects a tendency to worry. She said that she would rather use words that come naturally to her in her writing because then she doesn't have to worry about what she is saying or if she is using the right word.

Further evidence of students' preferences toward using the language that comes to them naturally is seen in their responses to another of my first interview questions:

***Does using precise mathematical language help you to understand, or does it make mathematical understanding difficult?***

Five of the six students indicated that the mathematical language sometimes confused them because they "don't remember what the words mean." This was not a surprising answer, given that using such language was a new experience for these students. Even though they had

seen and heard the words in the past, they had not been required to use them in their explanations. Only Grace indicated that precise mathematical language helps her to understand. She said: “Precise mathematical language helps me understand because I see it often in my math book and I’ve gotten used to what they mean.”

By the end of March, students were beginning to use precise mathematical vocabulary in their written solutions as a more habitual means of communicating with others. Most of them demonstrated an understanding of the need for a common language to communicate their ideas. In late March, I noted that students seemed to recognize the need to use a common language to explain their solutions. I wrote in my journal that I was seeing more precise language, and therefore, more detail in my students’ written explanations:

I spent most of Sunday finally catching up on assessing my students’ Writing About Math (WAM) solutions. I am so excited about the improvement I’ve seen in their explanations and the extent to which they are using precise mathematical language. The majority of them seem to understand how important it is to explain their thinking. It is clear that **they are trying** to use precise math language, and that now, using some of the vocabulary that I’ve taught is automatic for them (words like sum, difference, product). For some, though, it still seems sort of “forced,” like it’s very unnatural for them to write this way, because it’s not how they would explain it in spoken words. I look forward to seeing if they continue to improve over the next few weeks.

Again in April, I observed that some of my students were using the mathematical vocabulary quite readily, while others were still very resistant to it. I recorded this in my journal:

I've noticed a definite subset of students in my class who have "adopted" the mathematical language more readily than others. The majority of my students, however, continue to use the word answer (for example) when talking about the answer to any operation (i.e. "The answer to  $11 \times 4$  is 44," rather than the product of 11 and 4 is 44). These students continue to use the words that come to them naturally first.

Additional support for this observation is gained from student responses to the following WAM question. I asked this question because I wanted to see if my students would connect the mathematical operations that I had no doubt they would use—addition and subtraction—to the vocabulary words used to describe the answers they found in each step. I wanted to see if they would use the word *sum* in their first step of adding the numbers of students who got on the bus and the word *difference* when they described how they subtracted the number of students who got on the bus from the number of students that they bus could carry. The work submitted by my students varied, but the majority used the vocabulary that I was looking for to some degree. Evidence of this improvement can be seen in four representative solutions below:

***The bus could carry 80 students. The 32 students from Room 8 and the 29 students from Room 12 got on the bus. How many more students can the bus carry?***

*I know the bus can carry 80 students. I know that 32 students from Room 8 and 29 from 12 got on the bus. First, I have to find the sum of 32 and 29. Nine plus two is 11, carry the one to the ten's place. Three plus two is five. Then you have to add the carried one.*

*That is six. So the answer is 61. Now I have to find the difference of 80 and 61. You then subtract one from ten. That's 9. Seven minus 6 is one. The answer is 19 students. So the bus can carry 19 more students.*

*First I find what I need to do. I see I first have to add the students together. Next, after I have a sum, I subtract it from the capacity of the bus to get the answer.*

*I know the bus can hold 80 students. First we add the students in room 8 to the students in room 12 and get 61 students. We have 61 students, and we know we subtract 61 from 80 and the difference is 19. Nineteen more students can ride the bus.”*

*First I find the sum of all the students in room 8 and room 12. Then I subtract the students already on the bus from all the students the bus can fit. So the bus can fit 19 more students.*

In the solutions given above, students used terms such as *sum*, *difference*, *minus*, *capacity*, and one even explained how to regroup and included the concept of place value. I was especially pleased with the responses of those students who used precise terms to explain both operational steps of the problem.

In the second interview I conducted with my 6 focus students, one of my questions was:

***Do you think using precise mathematical vocabulary helps you to communicate your ideas more clearly so that others can understand your work?***

Five out of the six students answered “yes” to this question. I am almost certain that Emily, the one student who didn’t answer affirmatively, didn’t really understand the question. She simply answered, “No,” and when I asked her to explain why she didn’t think so, she shrugged her shoulders and said she didn’t know why, but she just didn’t think so. Emily often does not comprehend oral questions, and I did not want to “lead” her to a certain answer (that I perhaps desired) by questioning her further.

The other five students agreed that using precise mathematical vocabulary does help them to communicate their ideas to others. In fact, four of the five gave responses that indicated their recognition of the notion that a common language facilitates communication. Trent stated that precise mathematical vocabulary probably does not help if the other person does not know the words, but if they do know them, it helps communication. Grace and Scott answered that if the other person knows the words you are using, they’ll understand your explanation, but if you use your own words, they might not understand because of the words you choose. Kinsey added that if the other person doesn’t understand the words you use, you might try to explain it by using different words that they might understand. Annikka was unable to explain why she believed that precise mathematical language helps to communicate ideas to others. She simply said that it does.

### **Student Views on Precise Mathematical Vocabulary Used on Saxon Assessments**

Early in my project, I determined that students need repetition and constant use of mathematical language in order to feel confident in their understanding and in their ability to

solve assessment problems which use the language. One of the questions I asked in my first interview was:

***When you read a word problem that contains mathematical vocabulary, do you think you know the meanings of those words?***

Grace, Scott, Kinsey, and Annikka said sometimes most of the time they do know the words, but sometimes they are confused by them. Trent and Emily, however, said they usually don't remember what the "special" words mean.

A second question asked in the first interview, concerning the assessments, was:

***When you don't understand something, what makes it difficult to understand?***

Two students, Trent and Scott, gave answers specific to a type of problem they have difficulty with, such as long division, so their answers were unrelated to mathematical vocabulary. The other four students indicated that they have difficulty understanding what the question is asking or what it means because of the way it is asked.

By late March, I observed that many students in my class had made progress in their understanding of specific mathematical vocabulary words that I had taught in isolated lessons. I wrote the following entry in my journal:

Overall, I've seen improvement in students' ability to correctly answer assessment questions containing some of the vocabulary from the isolated lessons. The vast majority

can now correctly give the product of two numbers, because they know that **product** means they must multiply the factors given. Prior to vocabulary instruction, few students answered this type of question correctly. Instead, many would add and some would subtract. I haven't done enough detailed analysis to provide concrete data on scores to reflect how much improvement has occurred to this point.

Despite the improvement that I had observed, I recognized that some students still felt confused when faced with word problems containing precise math vocabulary; they did not recognize that instruction in the use of precise mathematical vocabulary in their written solutions had actually helped them to comprehend word problems on assessments.

During the second interview I conducted with my focus students, I asked the following question concerning assessments:

*Does using the precise mathematical language help you to understand, or does it make understanding more difficult? Why?*

My students were split on their opinions about this question. Three students said precise language on assessment questions made them easier to understand. Trent said, "It makes it easier because I know most of the words." Similarly, Kinsey said, "It's easier, I think, because I know some of the words better now." Annikka agreed. She said that the precise language helps her to understand, but she couldn't explain why that was true. The three students who said precise language makes the assessment questions more difficult indicated that the language continues to confuse them.

Even though I found it necessary to not use student assessment scores in my analysis of student vocabulary understanding, I did make some noteworthy observations of the group's success on specific assessment problems. On early assessments which tested on the concept of perimeter, I had several questions from students such as, "What does perimeter mean?" or "What do I do here?" or even "I don't get it." Half the class usually answered incorrectly. After reteaching the definition and concept of perimeter and then teaching area, I observed that students still confused the two, often multiplying two sides to find perimeter. Again, I retaught both concepts.

On Saxon Assessment #15, which was given at the beginning of the semester in early January, question 8b asked them to find the perimeter of a given square with length 5 and width 3. Fourteen of sixteen students correctly answered the question.

On April 3, I administered Assessment #24 to my class (16 students). Question 10 asked them to find the perimeter of a given square, which was pictured (a  $\frac{1}{2}$  in. square). Again, 14 students answered correctly. Of the two students who responded incorrectly, one showed that she (correctly) multiplied  $\frac{1}{2} \times 4 = \frac{4}{2}$ , but then she incorrectly simplified the improper fraction to equal  $2 \frac{2}{2} = 3$ . The other student found that the perimeter was 2, but she labeled it as an area ( $2 \text{ in}^2$ ).

The next assessment again contained a perimeter question. Question 18a asked for the perimeter of a 3 in. x 2 in. rectangle. Part b of the same question asked for the area of the same rectangle. One student was absent, so only 15 students took this test. Fourteen of the fifteen answered the perimeter question correctly. The incorrect student confused perimeter with area. She answered perimeter for area and vice versa.

However, of the fourteen students who answered the perimeter question correctly,

12 correctly found the area (part b) to be 6. However, six of those students correctly labeled with square inches, and six forgot that the units needed to be squared. Of the two incorrect students, one multiplied ( $l \times w$ ) and then doubled the product. The other student simply found the perimeter again (probably just a careless error—not paying attention to what she was doing).

In addition to the evidence found in the student assessments, my working hypothesis that students need repetition and constant practice in the use of precise mathematical language in order to gain understanding and confidence in using it is partly verified by my observation that I recorded in my teacher journal.

On February 22, I recorded what had happened in my class that day, before giving assessment #19. I had decided to take a few extra minutes to review concepts which would require my students to recall several vocabulary terms: pint, quart, gallons, obtuse, acute, parallel, greatest common factor, perimeter, reduce, milliliters, liters, and the names of several geometric shapes. I did this because the assessment seemed to be heavier on vocabulary than normal. As the review progressed, however, I was very pleased that there was very little confusion among the students for words that they might easily confuse, such as obtuse and acute. In fact, while I normally would not allow the “talking out” and shouting out answers, I just let it go. All but a couple of students were very participative in the review (and able to provide correct answers to my questions). On the two or three occasions that a student answered incorrectly, several other students promptly jumped in and corrected the wrong answers.

### **Teaching Behaviors When the Use of Precise Mathematical Vocabulary is Expected**

Once I started focusing my instruction on precise mathematical vocabulary, my teaching became increasingly student-driven. I made more critical observations of student behaviors in

terms of using the vocabulary, and I adapted instruction based on what I observed. These behaviors are evidenced mainly by anecdotal evidence recorded in my journal. I wrote that I began by analyzing the “Writing About Math” (WAM) problems that I had assigned beginning the second week in January. I noticed that they all lacked one important requirement—the students were not using mathematical language in their solution descriptions. This would not be such a problem if it was true of one or a few of the students, but as a class, my students were not following the instructions to use precise language. I decided to take a “time-out” from assigning the problems in order to model for them how to write explanations using precise language. After that time-out, I began to see an improvement in some of the students’ writing about math.

On March 14, I recorded in my journal that in a vocabulary lesson that week, I focused on the difference between the definitions of area and perimeter. Many of my students commonly substituted one for the other. If they were asked to calculate the perimeter of a rectangle, for example, many would have multiplied the length and the width. If asked to find the area of a rectangle, they might add the sides instead. During that lesson, I wanted to give the students a concrete, tactile experience with area and perimeter. For the first part of the lesson, each partner group was given a 6” x 4” rectangle (piece of construction paper), a 24” piece of yarn, and a ruler. We again defined perimeter as the sum of the measures of the sides of a polygon. I instructed the groups to use the yarn to “trace” the outer border of their rectangles. I wanted them to “see” what we mean when we say perimeter is the sum of the length of the sides. Next, I instructed them to measure each side of the rectangle using the ruler and then to find the sum of those sides. Then, to make it even clearer that we are finding the sum of the sides, I had them measure the length of the yarn that they had traced the rectangle with earlier. Taking this measurement helped them to understand that perimeter includes “the outside edge” only.

For the second part of the lesson, I passed out 1 inch squares (a handful of more than 20) to each group and told them to start arranging them on their rectangles in equal rows. When done, they were to count the number of squares that formed the longest side, the length, and the number of squares that formed the shorter side, the width. They had already learned how to find area (multiply  $l \times w$ ), but we reminded everyone of this. Then they were instructed to count—one by one—the squares they had used to see if they got the same number (area). The discussion that followed included many comments by students that this activity made it much easier to understand that perimeter is “around the shape” and area is “inside the shape” (not exactly in mathematical language, but in their words).

As my action research continued during the semester, I continued to be very aware of the teacher behaviors that I needed to exhibit in order to emphasize the usefulness of precise mathematical vocabulary. During the course of instruction, when I called on a student to explain a solution or answer a question, I asked them to restate what they said using mathematical language if they did not do so the first time. An example was when I asked Tori to explain how to divide two fractions. Tori explained that we “leave the first one alone, change the division sign to a multiplication sign, and then flip the second fraction upside down.” I responded by asking her how we could rephrase the third step using mathematical language. Tori then (and all students, almost without fail) made the correction in her explanation without hesitation. It seemed they knew how to explain in mathematical terms, but they need to be “reminded” to use the desired language.

### **Conclusions**

Students resist using the language of mathematics in their explanations because it is not natural to them; it is not what was expected of them from the beginning of their education.

However, the work of noted researchers such as Kotsopoulos, Manoucherhri, and Aiken provided significant evidence to support the notion that children can be taught the language of math. Similarly, my findings from my action research project lead me to conclude that continued vocabulary instruction can lead to increased student understanding and enhanced communication between students or students and teachers. However, the desired understanding cannot be achieved immediately. Only with the necessary components of teacher modeling, clarifying questions, and written practice will help students become more comfortable to more naturally use precise math vocabulary in their written solutions.

### **Implications**

Throughout my action research project, I made a qualitative analysis of my students' achievement in terms of learning precise mathematical language. My observations have given me cause to look eagerly toward the upcoming school year, during which I plan to implement my findings. I will have the advantage of a semester's worth of observations on which to structure my plan for vocabulary instruction. This instruction will begin at the start of the semester, when I will begin teaching pertinent mathematical vocabulary before even introducing the Saxon curriculum. I will continue to require written explanations of problem solutions using precise mathematical vocabulary. At this point, I have no definitive plan for how I can assess student achievement using the Saxon assessments, but I hope to develop a method of collecting data that will inform my practice.

My principal and I have already met to discuss what I learned from my research. We have discussed the implications that it has for my classroom and perhaps for our entire elementary school. With her support, I will share my research with my colleagues during a

staff in-service prior to the opening of the school year. I am confident that they will embrace the idea of teaching specific mathematical vocabulary prior to teaching the Saxon lessons. With their cooperation, we can instill this behavior in our students from the start of their educational journey.

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## Appendix

## Research Questionnaire

1. In general, do you understand math? Explain your yes or no.
  
2. When you don't understand something in math, what makes it difficult to understand?
  
3. Does using the precise mathematical language help you to understand, or does it make understanding more difficult? Why?
  
4. Would you rather use words that come naturally to you when explaining your solutions, or would you rather use precise mathematical language? Why?
  
5. When you read a word problem that contains mathematical vocabulary, do you think you know the meanings of those words?
  
6. Why is it important to know the meanings of vocabulary words you see in math problems?