The Implications of Number Sense on the Mastery of Addition and Subtraction Concepts

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Math in the Middle Institute Partnership
Action Research Project Report

in partial fulfillment of the MAT degree
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University of Nebraska-Lincoln
July 2011
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Abstract

In this action research study of my classroom of first grade mathematics, I investigated impact of developing number sense in first graders and the impact number sense has on mastering other concepts such as addition and subtraction. I discovered evidence that students could build quantities with manipulatives and that they can perform operations of addition and subtraction with or without manipulatives by applying their understanding of place value. Because of this research, I plan to start each new school year with a focus on number sense development to ensure that all students entering my classroom will have the solid foundation of numbers that will open the door to future computational success.
The problem of practice that is my focus for my action research project revolves around the impact of developing number sense in first graders and the impact number sense has on mastering other concepts such as addition and subtraction. As I returned to the classroom this year after two years in an administrative position, I came with firsthand knowledge of pre-kindergarten through sixth grade children and their struggles with mathematics. As an administrator, I worked with classroom teachers to implement curriculum measures that would help teachers address the diverse needs of their students. Often, because students do not have a good sense of what number names really mean, they struggle with operational concepts such as addition, subtraction, multiplication and division. Most first grade students can count, but not all first graders have a real sense that what they are naming have specific quantities and behave differently when working with mathematical applications.

I wondered as an administrator why some students did not understand numbers, but I felt passionate as a teacher to help my first grade students conceptually understand the meaning behind the symbols used for numbers. To me, teaching students about numbers is much like teaching a toddler to walk and talk; without such skills, a child’s future is faced with limitations. I wanted to send my first graders on to second grade really connected with numbers, so when they learn concepts that are more abstract they will be well equipped to work through new concepts. I wanted my students to understand number cardinality and number relationships.

There are many studies in the area of Reading and Language Arts that report that early literacy was essential to future reading and writing success and that students entering school from disadvantaged backgrounds begin significantly behind their peers. Ruby Payne was an author whom I looked to often in my classroom as she writes and researches a great deal on the impact of poverty on student achievement. My student population is highly disadvantaged
economically and many are English Language Learners. However, I did not want their current state of living to prevent them from a lifetime of success. As their teacher, I believe it to be vital that I make educational decisions that will positively affect who my students will grow to become. I believe that by knowing more about the factors that influence mathematical competencies in Number Operations, I could better prepare my students for their futures of learning and interacting with mathematics.

Through my journal reflections and the mandates of the scope and sequence embedded within my district’s pacing guide, I pursued research that helped me to understand the impact of number sense mastery. I wanted to know how number sense, mastery of place value, and number quantity influence the mastery of addition and subtraction concepts of first grade students? Within that question, I focused on the roles that manipulatives and multiple intelligence principles have on the development of number sense. I believed by understanding the impact that numbers sense mastery has on the developing mastery of addition and subtraction concepts that I could make appropriate changes to instruction for the benefit of my first graders and the benefit of future first graders.

At the start of my school year, there were immediate signs that my students did not arrive to first grade understanding that numbers are named based on their quantity. I had many students who could count to 20, but the same students had no concept of what 10 looked like and absolutely had no idea of quantity relationships beyond 10. How could I expect my students to understand concepts such as addition and subtraction or analyze data based on our new state and district standards if they had no sense of what number names mean?

The National Council of Teachers of Mathematics (NCTM’s) process standards (2000) address five key areas: problem solving, reasoning and proof, communication, connections and
representations. My students’ deficits of number quantity put them with deficits in all of these process standard areas. Being able to orally count was much like singing the ABC song: it was simple memorization without meaning. When educators expand a child’s understanding that those letter names work together to make sounds that build words, readers develop. I needed my students to develop reasoning proof, communication, and connections, so that they could see that those number names work together to build quantities that are then used to build connections that lead to problem-solving skills.

My ideal classroom is a world where my students are engaged in authentic mathematical conversations that involve problem solving where they use proof and reasoning to support their thinking. I believe I can achieve this if I can learn and do more to develop the sense of numbers that my students were missing. After attending a Kim Sutton workshop on number sense development on September 26, 2010, I heard her reiterate the intensity that educators need to address number sense as quantitative, comparative, and recordable. In the primary grades, teachers are to set a firm foundation of those three principals for number sense for all children in order to ensure the development of conceptual skills that the students need for future mathematics instruction. Students must have a sense of quantity, so that when addition or subtraction problems are introduced, students have means to make sense of the values derived under applications.

**Problem Statement**

Researching the influences of how number sense of place value and number quantity impact the mastering of addition and subtraction quantities was worth knowing about because, much like developing reading skills that are the foundation that all readers need, the principles behind number sense are equally the foundation that all mathematicians need. If students only
have surface exposures to number sense properties, they will have no foundation to build upon. Students limited in conceptual understand of number quantity have no basis for understanding when a calculation is right or wrong. The Common Core Standards (2010) and the NCTM’s Principles and Standards (2000) address the standard of number and operations as content that must be taught regardless of where a child attends school. In the past, many teachers including myself taught place value, but brushed past it quickly because it seemed too abstract for children to comprehend. It is obvious that this approach has left American students behind their peers of other nations as documented by many research studies. The development of instruction that really stems from an understanding of how place value and the development of a sense of numbers builds the foundation for all future mathematical learning is paramount.

As a researcher, understanding the impact of how number sense impacts the success of developing addition and subtraction concepts is one that I am eager to pursue. This research is worth knowing because of the implications in the Nation at Risk Report (1983). There are many research studies which show that United States’ schools are academically behind other countries in the area of mathematics. Many reports implicate conceptual understanding of mathematics as a reason for low achievement. The scope and sequence of educational classrooms in the United States looking to adopt the Common Core Standards will find that the first concept of mathematics at the first grade level to be taught will be number sense. This number sense strand is the Number and Operations content standard defined in the NCTM’s Principles and Standards book (2000). The current state of many schools produce students that demonstrate a mechanical understanding of place value where children can write numbers and perform calculations, but the same children lack the ability to explain the quantity relationships that exist in numbers. Previous research has shown that children incorrectly perform algorithmic operations in ways that they
would recognize as mistakes if they were more familiar with the quantities that numbers represent (Garlikov, 2000). Researching the impact of how addition and subtraction can be mastered when children have opportunity to fully conceptualize the quantitative principles of numbers is important. The skill for a student to know when an answer to a problem does not make sense comes from a conceptual understanding of place value.

**Literature Review**

Through the desire to understand what other researchers have investigated as they have pursued to answers to topics related to my problem of practice, I discovered a few common themes. Through the reading of research articles and educational journals, the following themes of number sense seemed to be most prevalent: number representation and number relationships through communication. Number representation is the ability to read and write numbers accurately and for a purpose. Number relationship is the ability to communicate and use what one knows to make comparisons of groups of things such as more and less, to use operational principles to identify a sets value, etc.

The term number sense comes with a variety of meanings and interpretations. Many articles and journals that I have read have a generalized common meaning, which denotes that the word “sense” means a personal ability to understand or know something. The idea of a “sense” of something is the ability to personally generalize about a situation. It is much like the term common sense: it is difficult to describe, but obvious in situations when a person is lacking it. For the purpose of this paper, the definition of “number sense” that I use comes from the web link [http://math.about.com/library/blglossarytopics.htm](http://math.about.com/library/blglossarytopics.htm), About.com: Mathematics’ glossary of terms defines number sense as an
Understanding of number including a grasp of counting, numeral recognition, a complex system of more-and-less relationships, part-whole relationships the role of special numbers such as five and ten, connections between numbers and real quantities and measures in the environment. Computational work dealing with fractions, decimals and integers. And….much, much more. (para. 11)

As I moved through literature reviews with this definition in mind, it was evident that each article of research focused on just a piece of the definition of number sense through the development of their study. Here is what I have found relevant to my action research from the works of others.

**Number Representation**

The process of developing number sense is a process in which children begin with counting orally, at times mimicking another person, chanting numbers. Early stages of counting can include the attempt to identify items in a collection of objects. According to the National Council of Teachers of Mathematics (NCTM), in the Multidigit Number Sense Framework by Jones, et al. (1996), the first number relationship children develop is a linear one. This particular article was an extension of research on a framework for assessing and describing children’s thinking. The extension was a case study involving 12 students: six from a first grade class and six from a second grade class in Illinois. The article reports that the students represented a broad multicultural and socioeconomic spectrum. This study was conducted over two years.

The purpose of Jones et al.’s (1996) research was to validate the five levels of thinking found in a previous framework study. It was said that there are markers in the developing number sense relationships in relation to counting, portioning, grouping and number relationships. Through interviews with the 12 children, each interview moved through a series of
questions until the student became unsuccessful. This study revealed the order in which number representation develops. Early development or level one skill set students lacked grouping skills. These students were successful with single digit counting of quantities. Students at the level five spectrums had what Jones et al. determined to be essential place value skills. These students had a grasp on counting, partitioning and grouping of numbers. They also had skills in number relationships. The early signs of number sense development in children beings with the ability to orally count. This number representation is essential to the mastery of future mathematical understandings.

It said that children begin with a fascination with numbers (Schwartz, 1995). Schwartz’s study was essentially observational data on a small group of preschool children during center activity time. Schwartz reports that early before formal mathematical instruction begin children use their number knowledge by counting aloud, or to count objects in a collection often without a one-to-one correspondence. Children naturally like to count things. Schwartz (1995) noted during one observational moment that other children joined in on the rhythmic pattern of counting initiated by one student’s activity. All of the children where chiming in with numbers regardless of accuracy. Each child had a limit of counting sequentially and some were content to continue to “count” using number names that they could recall. However, there was one 6-year-old who became discontent with the errors in counting and the group’s discourse that he started counting over again at one. The act of starting to count initiated others in the group to return to counting at one, but like the initial observation as reported in this study the children’s counting resulted in discourse again.

There are questions as to whether rote counting helps to establish number sense or it hinders development because it does not seek to answer the quantity question of how many.
Schwartz finds that the development of number representation begins when children begin to seek to answer the questions of quantity. The number representation develops as students gain an understanding in a set that some items belong while others do not and that counting serves the purpose to say how many are in a collection. Number representation is the ability to match a number name with an item in a set. The implications of Schwartz’s observations of this preschool classroom indicate the need to have a reflective practice of teaching.

Schwartz (1995) cautioned teachers to beware of how children are using center materials in the early development on number representation. At one point, Schwartz observed a child playing with most of the center blocks was self-talking his way through the notion of more and less, but the outward view of this activity appeared that the child was not being willing to share center materials. It is important to note that the early formal development of number representation begins before formal school, but early schooling must provide rich opportunities for children to experience the organization of sets and that they use accurate mathematical information every day. Educators that take a reflective observational role in the planning and development of instruction will foster the development of solid number representation in children.

As children progress in their development of numbers beyond orally counting, their system to represent quantity develops as well. Children begin to use pictures, diagrams and symbols to represent quantity relations that they are exploring. Harries and Suggate (2006) researched the link of computer programs on the developing understanding of mathematical concepts. Harries and Suggate sampled four schools in North East England with representation from both urban and rural schools. Each school had 18 students participate in individual interviews. Teachers from six classes were selected based on their knowledge of their classes
selected two students from a top performance, two from a middle performance and two from a lower performance ability group for a total of six students. The total population studied among all schools was 72 students. There were 24 students from first, second and third grades and 24 students in each ability group.

Harries and Suggate (2006) indicate that students were very willing to help and appeared to enjoy using the computer programs that were part of the study on student number sense achievement this research. The computer program provided students with a number representation and an explanation as the initial training occurred on how to use the program. Through the progression of the study, students were shown pictorial models of numbers and asked to identify the quantity. Students were then asked to explain how they knew quantity name or quantity size. The study looked at the accuracy of answers and commonality of errors.

The results of the Harries and Suggate (2006) study mimic in part the development of number sense that Schwartz (1995) found in preschoolers. The students in both studies relied on the physical model to make sense of quantity. Harries and Suggate’s data indicate that there is evidence that students’ ability to understand quantity develops as students grow. It was interesting that the visual model most often used, but that caused the most difficulties, was the number line. The structure of blocks and counters served a positive role in the development of number representation. The representations of quantity generated by the counters or base ten blocks provided a visual model for a student to concretely make sense of a number name. Harries and Suggate (2006) showed across ability level group the use of counters, arrow cards and figures groups of tens were the best model to use for the development of number representation, and that the linear number line provided the least positive support, but is the most common device used in schools. Harries and Suggate found that often students have success in reading
numbers well before they have an understanding of place value. The ability to count in tens and ones is associated with greater understanding of many number representations.

The implications of the Harries and Suggate study shows similar to the Schwartz (1995) findings that as children are developing their sense of numbers that the skills to orally read and write numbers does not equate to a solid understanding of number quantity. This sense of quantity is still developing and needs to be supported through physical models that a student can manipulate. According to Reubens and Crouch (2009), both of the Research Triangle Institute, children progress in similar ways with their development of number sense between the ages of 3 and 9. The early years are when children are developing the base of learning for all future mathematics. While Reubens and Crouch determined that lack of a strong base in number sense is difficult to overcome, they stressed the importance of understanding how number sense is developed as children grow. Reubens and Crouch highlighted the progression of development that all children go through regardless of their country of origin.

Educators need to understand the progression of development with number sense at all grade levels, but especially the early grade levels. It is noted through reviews of literature that all children bring similar background of numbers with them to formal schooling such as orally counting and understanding that numbers have names used to identify quantities (Rueben & Crouch, 2009). The notions of more and less that were shown by Schwartz (1995) developed, as children were able to make judgments about relationships of quantity. All researchers in this review agreed that the development of number representation was a sequential process. The process of number sense development must be carefully observed to provide appropriate interactions with students to ensure that number representation is developed successfully for all students. The number representation is the building block necessary for the future development
of number relationships. Students need to know number names associated with counting, but beyond the development of oral skills must come physical models to associate number names with quantities. Children who have the sense of what “five” means are then ready to progress to the development of relationships that occur within concepts such as addition and subtraction.

**Number Relationships through Communication**

Number Relationships appear to develop as children have developed the notation of quantity. Early development in children focuses primarily on the skills of counting and the desire to figure out how many items are in a collection. The next natural progression of number sense comes when children begin to compare quantities such as *if I have two and you have three who has more? How many do we have if we put ours together?* Children begin to explore relationships of more and less and even adding and subtracting principles long before educators put the vocabulary in place.

Jordan (2010, p. 82) said, “The development of number sense begins with precise representation of small numbers, whereas large quantities are initially captured through approximate representations,” Jordan went on to say that, “these primary abilities are the basis for developing secondary symbolic or verbal number competencies” (p. 82). It is this second stage of development that my action research is focused on. How does early number sense influence the development of the symbolic relationships that occur when working with number relationships?

Jordan (2009) indicated that children developed a “core” number sense battery for screening children in kindergarten and first grade. The longitudinal assessment over multiple time points showed three growth areas that were indicative of a child’s socioeconomic advantages and disadvantages. The study examined the predictive validity of the Number Sense
Brief screening measure. Children in this study were given the screening at the beginning of first grade and again at the end of first grade and the end of third grade. The assessment looked at overall mathematics achievement, written computation and applied problem solving.

Students in Jordan’s (2009) study came from the same public school in northern Delaware. There were 279 first graders assessed and 175 third graders. The population study participants in first grade were 55 percent male and 45 percent female with a similar balance of third grade participants. There was representation from minority and non-minority students at both grade levels. The largest discrepancy of balance was the ratio of low-income student to middle income student participates. This longitudinal study had roughly 70 percent middle-income participants in both first and third grade. The third grade participants selected were only participants if they had completed all parts of the first grade assessment battery. The method indicates that all participants were taught with the same curricular content and approach. Testing was completed by a specifically trained researcher and in a one-on-one testing procedure.

Students were assessed in counting knowledge and principles, number recognition, and comparability, story problems and number combinations. The Wechsler Abbreviated Scale of Intelligence as also given to the first-grader participants to assess oral vocabulary and spatial reasoning and a Digital Span test was given to assess short-term and working memory.

The results of Jordan’s study (2010) were the most interesting to me as I wrestle with my concern of developing number sense. The purpose of the study was to measure the long-term effects that early number sense has on future mathematical development. The testing measures had strong reliability, but the discussions that arose from the results are what I found most useful. The overall results of this longitudinal study were that early number sense is a powerful predictor of future mathematical knowledge. The article clearly stated that the underlying mathematical
difficulties came from early weaknesses in number competencies related to counting and number relationships which support the previous researchers’ statements in the number representation section. Jordan (2009) confirms the finding that number relationships are difficult to bridge when a child lacks early number representation sense. Jordan supports that children using the number representation relationships developed earlier have much better problem-solving skills.

Gill (1995) studied a second grade classroom in Newburgh, NY. The teacher, Mrs. Thompson, had prior training in Thinking Mathematics. Gill’s focus was on how students are thinking when working to solve a mathematical problem. The students had use of manipulatives to use in problem solving and noted that the teacher had worked very hard to develop a positive student climate where students felt comfortable to share with one another and encouraged to find many ways to solve a problem. The classroom instruction focused on number sense that of building numbers using standard and expanded forms. Students were asked to compose and decompose numbers in various ways demonstrating number relationships. Students in this class constructed, separated and partitioned groups of items in many ways while explaining their process. The particular study focused on how students would figure out how many three classes of students going on a field trip would represent. The particular problem involved regrouping and was noted that students at this point in the year would not have been exposed to this type of problem, but rather working on problems using worksheets and a traditional algorithm.

Gill (2009) looked at the conversation students had through the solution of this problem. Gill expressed that students were pushed to express themselves in language that made sense to them as they worked through solving the field trip problem. Gill noted that dialog was often difficult for the teacher because it did not follow traditional methods. Mrs. Thompson noted that she needed time to develop the process of bridging student thinking. She could really see how
the students were working through the problem and earlier on would have supplied language for students, but learned that by bridging student thinking with restatements of students’ words that students developed the principles she was instructing.

Gill (2009) pointed out that the language development of numbers and symbols in turn developed the concepts and understanding of number relationships. Gill stated when children “understand concepts, and can describe and justify what they write, they can deal with formal mathematical symbolism with ease and understanding” (p. 359). The progression of number sense involves building language as the early development of number sense (such as orally counting) and then moving to physically counting a collection. The ability to describe the mathematical relationship that Gill observed in her study of Mrs. Thompson’s second grade class is the further development needed in the journey to full number sense understanding.

The communication of how numbers are related is the other essence of number sense development. The NCTM Principles and Standards for School Mathematics (2000) made a similar statement in their publication. The communication piece of selecting symbols to represent ideas in mathematics is evidence of real understanding. Dilek (2009) conducted a research experiment with 150 fourth graders that were divided into two experimental and two control groups. This study looked at the effects of the cooperative learning method supported by Multiple Intelligence Theory on student achievement using a comparison of a pre- and post-test method. The focus of this research article was the integration of cooperative learning groups that were supported by multiple intelligence principles as proposed by Howard Gardner. The study was conducted over a nine-week instructional period in a state school setting in the country of Turkey where students were mostly middle-socioeconomic class. The researcher herself instructed one of the control groups and experimental groups. The control groups were taught
using traditional teaching methods where the experimental groups received instruction using cooperative learning and multiple intelligence principles.

The analysis of Dilek’s (2009) research study showed significant overall treatment effects regarding student achievement, but did not support positive effect on student retention of learned material. This article is relevant to the theme of number representation through communication because it highlighted the importance of seeking to represent understanding using a variety of methods. There is some discussion regarding the validity of Multiple Intelligence theory in the classroom, but the research within this article showed that when students are provided other methods to communicate understanding they have greater success. Researchers would agree that children could often show what they know mathematically using manipulatives before they have the official language to explain what they know.

Zhou, Peverly and Lin (2005) studied 160 first-grade students, 80 students from America and 80 students from China. The U.S. students were randomly selected from two middle class schools in New York. In China, students were randomly selected from two average elementary schools in Beijing. Zhou et al. investigated if there were cross-cultural differences in the development of mathematical knowledge in four domains. The study found that Chinese students and U.S. students are more equipped in the area of numbers and operations than other mathematical areas. Students were individually assessed during the month of October using a broad battery of task assessments. The results of the assessment were then analyzed for any cross-cultural differences. Zhou et al. suggested that the differences prevalent in China might be due to the family support provided for informal mathematical development as documented through parent interviews. As seen in Schwartz (1995), the early development of number sense happens before formal education begins Zhou et al. supplied additional evidence that the early
foundation is key to the development of number sense relationships. The implications of this particular study imply the need to emphasize problem solving through communication. Early in students’ education, students must be asked to use mathematical reasoning as they are developing their ideas about numbers.

Summary

The development of number sense within the guidelines of the Common Core Standards is complicated. Defining number sense and then applying the principles within the meaning of number sense takes direct focus on the part of educators and parents of young children. It is not enough to say that a child can count to 100. Every child must have a personal mathematical vocabulary that includes number names, but must also have personal understanding of how the names of numbers identify quantity. The development of number sense is a gradual process that teachers must reflectively develop in students. Through careful observations and instructional practices, students need to be encouraged to use language that supports the development of number relationships. The research is abundant in how to assess the development of number sense and that most children regardless of the country they are born in develop and understanding of numbers in similar ways.

Purpose Statement

The purpose of my action research project was to understand how number sense mastery of place value and the mastery that number names represent quantity influence the mastery of addition and subtraction concepts by first grade students. I sought to answer the following questions in my research:
1. How do number sense, mastery of place value, and conceptual understanding of cardinality influence the mastery of addition and subtraction concepts by first grade students?

2. What happens to my teaching when I focus on emphasizing building number sense, mastery of place value, and conceptual understanding of cardinality in my first grade class?

When considering my question I also had to consider the variables that would impact the outcome of my inquiry.

- First graders’ confidences of numbers 0 to 100 when counting is a measure of number sense ability. The quality of oral reasoning when counting in a sequence such as by 2, 5 and 10, the ability to count forward from a designated starting point and the ability to count backward with a designated starting point is a measure of number sense.

- The extent of reasoning and proof of the cardinality of number and the relationship that numbers have in respect to counting and number name quantity.

- The quality of students reasoning and proof in orally defending the validity of an answer of an addition or subtraction problem.

- The efficiency of applying the concept of cardinality within applied concepts of addition and subtraction.

**Method**

My research began with deep reflection on my instruction and on the needs of my students as I formulated the purpose of my research. Through journaling, student interviews, classroom observations and work samples I sought to answer the essential questions of my action research project. During the day of parent teacher conferences in March of 2011, I explained to my students the concept of my research project as best I could with 6- and 7-year-olds. Then
again, I explained in detail to parents and students that attended conferences what I was researching. I explained that I would need their permission to include their child’s work in my analysis of my research study.

It was normal practice for me to video my students and record samples of student work by photographs or by scanning of student work that I used to develop their student electronic portfolios documenting their first-grade year of learning. This collection of works I have always assembled to share with parents during conferences or other academic meetings. Based on my normal course of practice, I continued to gather all student work that I felt necessary to document the journey of how the mastery of number sense affects the mastery of addition and subtraction concepts.

I used this habit of practice to help me develop my journal reflections and to capture student work. During our daily mathematical warm-ups, the students and I focused on number sense through songs, discussions and problem solving. As students chattered and shared about their thinking of mathematics, I captured photos and sound bites that I would use later for journaling.

The organization of my work utilized the methods of scanning and importing photos to word documents or in the case of my interviews; I utilized slideshow software to record student data. The first set of interview questions is found in Appendix A. The first interviews were conducted on February 1, 2011. Student work was collected and scanned throughout the research timeframe from February 1, 2011 to April 15, 2011. Appendix B was the small group Interview Number Two used with student work to document or demonstrate number sense mastery. Appendix C documents the topics I utilized for journaling during the period of research.
The analyzing of data occurred through my reflections of the day’s events, which included conversations that occurred through math instruction. I looked at the achievement progress of students through the evaluation of formative and summative grades collected for report cards. I found the scanning of documents most helpful in organizing my data. It provided me the opportunity to group students by level of mastery. The photographs of students engaged in activities with notes helped to capture the essence of my students thinking about how numbers grow or shrink when adding or subtracting.

I found the method of collecting data of first graders was a bit of a challenge due to their limited ability to explain their thinking. Many of my students are newcomers in our country and as a result, their status as English Language Learners limited rich conversations that perhaps older students with stronger language skills would provide. The use of photographs helped to capture the raw material of learning activities.

**Findings**

A typical day in a first grade begins at one o’clock back at the calendar corner where one of my students begins our classroom routine of setting up the calendar zone. At the calendar zone the lead student will identify the day of the week and the full date. It would be said like this, “Today is Friday, May 6, 2011.” The leader would then lead the class in identifying what tomorrow is called and what yesterday was called. Next, we look at the pattern that is displayed on our monthly calendar display. Then the student leader leads a discussion that relates the number of days we have been in school with our place value blocks and how we would display that value using our money system. Once this portion of calendar is complete we using our math concepts music to review our multiples of 2, 3, 5, and or 10’s. The student leader during song
time walks our number line as we all chant our multiples. After song time, I then do a quick introduction as to what our big lesson idea for the day is.

My students then engage in a brain break activity, which includes music and dance at this point, because they are 6- to 7-year-olds and can only focus on a task for about 15 minutes at a time. After the brain break we return to our calendar work area where we engage in a warm-up brain stretch activity such as patterning extensions, math fact review, guess my digit, greater than, less than, and or name another name for a given number. During this warm-up time I am careful to create connections to the four strands of our newly adopted content standards of Number Sense, Algebra, Data Analysis, and Geometry. This is often the moment of the day that a student may ask a loaded mathematical question that takes our brain stretch activity into a full exploration and completely delays our big idea of the day’s lesson.

If we are not moved by powerful question or a need to explore an idea further, we move to our big idea lesson. Again, my students are given a mini brain break as we move from the calendar corner to the front of our room where the lesson will be presented using the classroom Promethean board. The use of this technology lends itself to modeling concepts using virtual manipulatives. Our district has just adopted new state standards and our current textbook provides practice for students, but does not fully supply the depth needed for mastery of concepts so instructional materials are pulled from a variety of resources. I rely heavily upon the tools within Active Inspire software when presenting instructions in a whole group format. This technology provides opportunities for students to interact with ideas throughout the lesson. If a lesson is a new concept I will first model for students what to do and then we will practice a concept together. We go from virtual manipulatives to real manipulatives for concrete practice at this moment. A concrete model that students visually and kinesthetically connect with the lesson
idea during guided activities further develops internal thinking and connections with the new learning. The whole group lesson time typically takes about 15 to 20 minutes to present and discuss.

After whole group instruction, all students then have a period of independent work time with the exception of a small flexible group of students that are pulled to our worktable for further small group instruction of a concept previously taught or to receive guided practice of the new concept. The small group work time consists of students who need specific practice at a specific moment in time. As students finish their independent practice portion of the lesson they move to a center exploration. Center exploration consists of options such as a computerized fact practice station, explore with manipulatives station, read a math story station, or a building value station, which is typically a dice or card type game. Students rotate throughout the week, so that they each spend some time at each station weekly.

The last ten minutes of the math hour is a lesson wrap-up: students may be asked to share something they discovered today, liked today, or are still confused about. There are days I ask students to write their thoughts down or we share orally a few ideas of the day. On occasion, I will share observations I made or give them a think-about topic.

I have found that number sense, mastery of place value, and conceptual understanding of cardinality influences the mastery of addition and subtraction concepts by first grade students significantly. The evidence of understanding cardinality has shown itself in the discussions that I wrote about in my Journal 2 entry. After reading, Donald Crew’s *10 Black Dots* story my students became very interested in how many dots were actually in the book from 1 to 10. I observed the exploration of two very different groups during that week as they sought to answer their own question of “How many.”
Group One physically counted the dots and kept track as they went with a tally mark system while Group Two began to see pairs of numbers that summed to 10. There was strong evidence that having cardinality of numbers prompted solving that problem with addition versus simply counting using a one to one correspondence. When Group 2 shared their process with the class, Group One (whose members had more one-to-one correspondence skills) were amazed at how the other group used their addition skills. Sarah\(^1\) from Group One asked for Group Two to show them how they worked to solve the problem again. Sarah, a child who I have observed is just on the edge of breaking through the cardinality barrier, said, “Wait show me again how you knew to pair up a low and high number to make 10.” My observation of this exchange of knowledge reveals a child carefully laying out with manipulatives how she paired a low number with a high number to make 10. The “a ha” moment for Sarah was, “Of course that makes sense!” She said to the group, “Can I try to find the next pair before you show us?”

With encouragement from her peers, she went back to the front of the book for the page containing three dots, which was the next pair to be partnered in Group Two’s explanation. “Okay,” Sarah said, “I know I need a number to add to the three that will make 10. Now let me see.” She carefully took out three counters and stared for a minute and said, “Can I count forward until I get to 10 with a different color and then I will know what partner page to look for.” Group Two erupted in cheers. Emily exclaimed, “She sees the numbers now like we see them, doesn’t she Mrs. Lee?”

The observation of this groups interactions, the journaling of this event and student dialogs is the evidentiary support I offer to document that number sense, mastery of place value, and conceptual understanding of cardinality influences, the mastery of addition and subtraction concepts by first grade students significantly.

\(^1\) All names are pseudonyms.
I assert stronger number sense abilities of Group Two put them at an advantage to apply addition as a skill to solve questions such as the one we explored when finding the answer to how many dots were in the book, Ten Black Dots by Crew.

It is evident that my students all had the desire to know how many dots were in the story, but the skills to attain and answer the question were vastly different. The desire to make sense of how many is a number sense skill that Schwartz (1995) indicated this desire is the early stages of number representation. I have had most of my students all year, so I know they have all received many opportunities to explore numbers and to build the one-to-one correspondence of numbers that students need to have cardinality. Observing their desire to make sense of how many total dots were used to make the book is evidence that cardinality of numbers and understanding of place values builds mastery of addition and subtraction concepts.

Consider a comment from Ed, one student from Group Two, when he said, “Look, Mrs. Lee if I take the one dot from the first page and put it with the nine dots on the page with nine I know that $1 + 9 = 10$ those two pages together give me 10 dots. Did you know there are pairs like that all over this book? I think my group can find the total quick if we use our pairs.” Ed’s comment is evidence that cardinality of numbers builds on the mastery of addition.

The story of my teaching was still under development as I looked back over my journal. There was evidence that most of my students have shown growth in the area of number sense. Daily, students are more explicit in their descriptions of how they can identify quantity. My students eagerly look for patterns during our warm-up time activities. During our group activity time, they eagerly share ideas and work to help each other accomplish daily mathematical tasks. I know that my teaching has changed as I am seeking out opportunities for my students to be reflective.
One way that my teaching has changed is the way that I have let the use of manipulatives take the place of a traditional worksheet. This activity required partners to roll a dice and place the value of the roll in either the ones, tens or hundreds place with the desired outcome to create a number larger than their opponents. In prior years, I would have given a greater than or less than worksheet and they would have placed the appropriate symbol of comparison. This work would have been done independently. As a result of this change in student work I was able to observe and document this encounter, which provides evidence of how my students are developing number sense. Observe the encounter with Samuel and Joshua as they explained their strategy with the greater than or less than game. I stopped by to ask how they were doing on building their numbers. Samuel said, “I am so going to with this game!”

“Really how do you know, Samuel?” I asked. He responded, “Well, the person with the biggest number will win the round. Every time I roll the dice if I get a number like 6, 5, or 4, then I put it in the hundreds place. Because the more hundreds I have the bigger number I will make. I always put my small numbers like 1, 2, or 3 in the ones place because they won’t help me get a big number. Look at my paper; see my strategy is working!” This evidence supports the development of number sense.

Emily, Sarah, and Elizabeth, when working another number building activity
using dice and place value cubes, shared their knowledge of how they knew their place value cubes matched their value on their dice. In journal 5, I shared how my students were working on demonstrating the quantity 100. Students were rolling their dice and keeping track of their totals until they reached 100. In my Journal 5 I said, “Listening to them build their numbers and make sense of the difference between tens and ones was very insightful.” This is still the case as students that are able to share with one another their reasoning of how they build numbers with place value blocks.

Emily’s explanation to her group, “Look, our green cube is our tens cube so we have to have two sticks because each still equals 10 ones. If you have two tens that is the same as twenty ones cubes. It is just easier to see. Emily scratching her head says, “Can you say that one more time?”

Elizabeth takes charge, “See how we have the two sticks and if we count the little cubes we would count to twenty; two sticks is the same as twenty.” Then, Emily jumps up and screams, “Mrs. Lee, Sarah can do it; she knows and I was her teacher!” There is no stronger evidence of knowledge then when a person can teach someone else what she knows.

During our warm-up time for the reflection of Journal 6 my students and I spent the week working to make predictions as to how many 2 addend facts the numbers 0 to 10 have. Student’s previous thinking and experimenting with numbers and fact families were amazing as shared above. During the exploration of how many paired facts a sum would have, I had a few theories emerge that the number of facts depends on whether a number is odd or even. Our group spent
time clarifying what we meant when we said it depended on odd and even and we agreed as a group the working meaning of that statement before groups set out to explore. As our experimentations and discussions moved on through the week as we worked to solve the question of how many addend pairs equal the sum there seemed to emerge a pattern. Students were making conjectures such as “If I want to know how many addend pairs I would have for a sum value of 5 I believe there will be six and here is why I think that.” “There seems to be one more then the value of the digit for how many two addend facts there were” said Jared.

The sophistication that my students worked through this exploration still amazes me weeks later. I can guarantee that in years past, I would not have stopped “instruction” for this type of chatter, my mind would have said, “we have too much ground to cover.” I see now that the minutes we take with explorations that lead to these sorts of discoveries make covering ground so much more efficient. One has to see it in action before they can believe in its action.

The initial development of understanding number quantity and value is making a difference for most of my students. They are all currently at different levels of mastery; however, when it comes to the mastery of addition and subtraction concepts, which is what my main exploration of research was, I wondered: How does the mastery of number sense affect the mastery of addition and subtraction concepts? Currently, I have three students who still do not understand that they have 10 fingers every time they count them. When they work with other manipulatives they constantly have to recount and recount to verify quantity. As a teacher, I do not understand how I have missed making the
The other 12 students have moved along in their desire to understand quantity and to make sense of numbers, which is reflected in their math formative assessment quarterly grades. The 12 students understand when finding a sum the answer will be greater than the two addends and when finding the difference the answer will be smaller than the minuend.

Looking at student individual interviews and groups discussions that have occurred, there is evidence that my students are mastering addition concepts at a faster rate than they are mastering subtraction concepts. I have witnessed the success of addition concepts through the collection of student work and student discussions. Students understand that the sum of addends will be greater than the addends themselves. Through journal entries, when students strategize methods to figure out how many fact pairs exist for the number 0 to 10 and how many dots were in the book from 0 to 10 they applied their knowledge of addition concepts to reach solutions. Ed and Emily during interviews could articulate fully the strategies they used to figure out an addition problem such as 5+4. A quote from Ed: “I can get my math done quickly when I know I have one more or less then the double facts I know.” Emily shared during a discussion something similar. It went like this, “Math is so simple when you see the double facts and their friends.” I said, “Friends? What do you mean?” “Well you see if you have 5+6, it is
really $5+5$ and 1 friend that makes 11.” Both students used the double fact concept and a plus or minus one method to quickly determine the answer. They could articulate in their interview how they solved that particular problem and could carefully model with blocks what they did in their head. This articulation and modeling of a solution is clear evidence of the mastery of addition concepts.

Ed and Emily’s work on a later assignment serves as additional evidence of their mastery of addition concepts. The two aforementioned children are not isolated in their evidence of mastery, but stand out as exceptional examples of how the development of number sense has made mastering addition so much more concrete for them.

I have claimed that I had evidence that my students were in fact developing number sense and supported that claim using whole group class discussions and activities document through photographs. This photograph of Samuel and Jared building with place value blocks is evidence that they are building a sense of number quantity and that they could identify that a value of a digit in the tens place looks different then a digit in the one place. They were working to roll the dice until they built the quantity to 100. This photo shows Samuel and Jared building the number 46.

Samuel provided to Jared a very articulate explanation when he said, “Jared look, our orange cube says four and we know that is the tens place digit, so we need four of our sticks because those equal ten.” He went on to justify in the same manner why they needed six cubes. As the boys continued on their assignment their conversations continued to use their knowledge of place
value to build two digit numbers with their manipulatives I observed. Their explanation of how they knew their answer was correct and was very articulate.

During Interview 1, which took place February 1, 2011, students were not as articulate in their explanation of the meaning of the four in 46. Manipulatives were not available to use in their explanation of the answer. I was seeking to identify if students really understood what each digit represents in a two-digit number as they had been so articulate in earlier activities with manipulatives present. During interviews of my students, 90 percent of them could write the number 52, but only 45 percent of those students could articulate that the 5 in the tens place meant the value 50. This leads me to believe that they are still reliant on the manipulatives to support their understanding of number quantity. I believe that using manipulatives to build number sense has become concrete for students. They use the place value blocks in a one-to-one relationship with counting numbers. They are familiar with the meaning of manipulatives and how they represent numbers, which has helped with mastering the concepts of addition and subtraction. Students understand that addition is joining property while subtraction is a reduction property as
Number Sense 30

observed in interviews and discussions.

When collecting this body of work from my class, a distinction arose between the students who have mastery of number quantity (who had success on this assignment) and those who did not. Students who were able to articulate an understanding of the one-to-one relationship of quantity did very well on this assignment. Jared was so excited when we were working on our modeled part of this lesson when he squealed, “Our work today is so easy! I know what I am going use my count back strategy we have been working on.”

The students who are still unsure that every time they count their fingers that they have 10 did not have the same level of success with this assignment. This photo indicates a lack of understanding of subtraction. When I stopped by to visit with Gordon about his strategy on this assignment he said, “I am using my crayons to count the big numbers and then I take some away.”

“That is a good strategy,” I tell him, “show me how it works.”

“Okay, I am going to do the 12 take away nine one right now. First I take 12 crayons from my box,” Gordon said. Observing him counting out his crayons took several minutes. He recounted his pile about three times. The first time he started to count over because one crayon rolled off his desk, the second time someone asked him a question and he appeared to lose his count and the third time he made it to 12. “Okay, I have my 12, now I take away nine. I get three left. See I can do it!”

Both sets of student work indicate an understanding of the subtraction concept, but the strategies to solve the problem reveal a different level of number sense mastery.
During Interview 1 when students were asked, “What is the sum of 15 + 6? How would you solve a problem like that?” Here are some of the response I received that provide evidence to how students are using their number sense to complete sums:

“I would tell my brain, (15+5) and then add 1 more,” said Ed.

“I saw our class bulletin board where we have our count by five numbers marked in yellow. I found 15 and went to the next yellow dot to get 20 plus one more gives me 21,” said Sarah.

These two students are using similar number sense strategies making use of counting by five although one is still relying on a visual model used previously during instruction.

Emily used a completely different number sense strategy when she said, “When I see (15 + 6) I think about the (6+5) and the extra one goes in the tens place to get the sum of 21.”

Gordon said, “I can do 15 + 6 by counting forward by ones. Listen 15…16, 17, 18, 19, 20 and 21. See I used six fingers so I know I counted forward the right amount.”

The sense of numbers that these students described when solving the above question has provided evidence that a sense of numbers helps to build mastery of addition and subtraction concepts.

There is evidence through journaling and discussions now at the end of March that my students can build quantities with manipulatives and that they can perform operations of addition and subtraction with or without manipulatives demonstrating understanding of place value. Students are able to articulate why a problem like 42 + 12 = 54. I observed explanations such as this from Ed, “If you have four tens and put it with one more then you have a total of five tens. Then you have two ones place and another two ones to make a total of four ones. Five tens is expanded to 50 and four is just four we can decompose our number to standard form to get 54.”
Sarah modeled for the class at the activboard 66 – 32, “First I set up my problem vertically like Mrs. Lee says for us to do. I line up the ones digits and the tens digit before I start my work. Then it is just like solving two little math problems watch. 6 – 2 is 4 in the ones place. 6 Tens minus 3 tens is 3 tens. So my answer of 3 tens and 6 ones is 36.” It was amazing to observe these first graders using high-level mathematical vocabulary in their explanation as to how they solved a problem.

Further evidence that first graders are utilizing their mastery of number sense came from Journal entry 6 by observation of a classroom activity, which was started, by a student’s observation of patterning. As a whole group, we have worked on writing the facts to ten during our calendar time. We started to write all of the ways we could add using two addends to equal a given sum such as 10. As we worked our way through all the values 10 to 0, the students who have been searching for patterns everywhere began to indicate that they were seeing a pattern with how many addends and fact they could make.

My students asked if we could organize our pattern in a T-chart to see if that would help us to find out what the pattern was.

The chart looked a bit like this chart on the right. We were at the sum of six when my students thought the pattern was if the sum was odd there was an even number of addend pairs and if the sum was even there would be an odd number of addend pairs. It appeared that that was a true finding, but I asked them to look further at the pattern. Could we predict how many addend pairs a sum would have? How do we know we have found all the addend pairs?
The buzz of conversation that erupted amongst this group of 6-year-olds was astounding. I knew by asking the questions above meant that I would not get my regularly scheduled lesson taught that afternoon. The students would need time to explore and investigate. I just had to wait for the buzz to slow down as groups gathered their thoughts. I just stood back to listen to the group chattering. Some groups were stuck on the odd even pattern and were not sure if we found all the sets. Other groups were trying to figure out how they could tell if they found all the addend pairs. It was about 10 minutes later when a group said, “Mrs. Lee, I think we need our counters and number lines out to really know if we found all the sets.” In unison the class agreed that would make a difference and could they please investigate? What teacher could say no to that request? Groups gathered the supplies they felt that they needed and set to work to figure out a way to know if they found all the facts and if they could predict how many facts, other sums would have. Here is how the chart of sums grew as my students worked through their investigation.

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<th>Sum</th>
<th># of Facts with turn around facts</th>
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<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

All students were actively engaged in building the models of addends used to find the sums of a given quantity. The groups of students that show evidence that their number sense skills are developing were content to contribute to the building of the fact sets. They were not able to articulate much about patterns they were seeing or how they might be used to predict how many facts another sum would have, but I was delighted that they could build successfully the addend pairs for a given sum.
The groups that have shown previously that they had strong number sense were getting so excited about the developments before them. They began to see that the even sums had the extra double fact that odd number sums did not. They were able to articulate the findings that they saw had an increase of one more fact because they could use all of the previous numbers to make the new sum. Consider the sums of six there are seven addend pairs [(0+6), (6+0), (1+5), (5+1), (2+4), (4+2) and (3+3)]. The efforts of their work helped to build the understanding that there was always one more addend pair then the sum itself. Even number sums had the extra double fact that odd sums did not.

I am amazed at the confidence my students exhibit when they describe the patterns they see or how they utilize what they know about numbers to develop new thinking. I can see evidence of strong number sense in most of my students. It is this number sense that helps them understand the principles of addition answering the question of how many or how much.

The analysis of my collected data continued to support that students must have mastery of number sense if they are going to advance to mastery of addition and subtraction concepts.

Consider the evidence from Journal 9 observations, student work collected, and group discussions where my students completed a modified five-minute math assessment that our district uses with students in grades two through six. We took the assessment utilizing a one hundred-question test that the students are given five minutes to complete.

Before setting the students to work, we discussed as a whole group what strategies students might use to complete their work.

Nicole said I am starting with my zero plus facts those are easy: “Zero plus a number is always that number.”
Samuel said, “I am going to use my double facts. I know those answers are even sums and they are my favorite!”

Ed said, “I am doing all my nine plus numbers because you turn them to quick tens by taking one from the addend then the sum is just $10 + \text{the number easy}$!”

Esther said in a small voice, “I am going to look for the one plus numbers because I know I can count forward one to get my answer.”

I stood in amazement as students grabbed their pencils and set to work on the go command. Every student had a strategy they planned to use to complete the mission. As I walked the room, I witnessed students using their strategies that they had described. I also observed that students were working with great accuracy. They had the sense that their answers were correct and kept moving through the assessment.

I had two students that under pressure to work just wrote numbers down regardless of the problem. These two students are two that I have pulled separately to work building number recognition throughout the year. They have not yet subitized values to 10, which was limiting their ability to complete assignments such as this one. When I stopped to check in after this quick assessment I asked Esther if she used her number plus one strategy she mentioned earlier. She said, “I tried and then I just let my brain tell me a number to write.”

I have attached her work to support the claim that
when a student has not yet subitized their numbers, mastering addition and subtraction concepts presents a struggle. I find this to support my research question that the foundation of number sense greatly influences the ability to master addition and subtraction concepts.

What happens to my teaching when I focus on emphasizing building number sense, mastery of place value, and conceptual understanding of cardinality in my first grade class. The biggest change in my instruction happens in my delivery of content. If I had not kept a journal that documented how my lesson plans changed based on a student question of inquiry I would not believe it myself. As I combed back over my journal entries I was struck by what my students initiated on behalf of their desire to know. I wrote in entry 2 during the early weeks of my study that I stopped my planned instruction due to a student’s desire to know how many dots were in the book we just read as our opener. In journal 2, I observed the dialog of Sarah from Group One asking Group Two to show her how they worked to solve the problem again. My teaching changed. Sarah, a child who I have observed was just on the edge of breaking through the cardinality barrier, said, “Wait show me again how you knew to pair up a low and high number to make ten.” Group two was very articulate in how they approached their solution. Journal 2 indicates, “we explored the number of two fact sums we had when looking at the total dots in the story, given sums to 10. I know that without the reflective practices of journaling I would not have put aside the daily lesson to pursue the answer to a student’s question. I would have forged ahead with what the lesson plan said. Stopping to have real mathematical conversations with
students helped me to know truly as an educator what my students were really thinking mathematically. This awareness helped me to move instruction along at a pace that connected directly with the needs of my students. I would have missed what my students were thinking and how they were building math concepts as they grew. The few days that we spent exploring the number of dots in the book opened my eyes, ears and heart, to be able to see feel and hear what my students were really thinking about numbers. Previously, I would have delivered my lesson, my students would have practiced the lesson I delivered and we would have moved to the next step the proceeding day. Student inquiry has helped me to be a better teacher. I feel allowing the explorations helped my students personally connect with their own learning and desires to know, which in turned brought number sense mastery for most of my students. One of my students’ favorite math centers is “guess my number.” It is a set of cards numbered from 1 to 50 with number clues on the backside. One student secretly writes a number from one to 20 down and proceeds to answer questions about the number for her
center mates. The clues on the back of the cards help provide clues to the secret numbers.

Values are ruled out as students utilize their number sense. Having math centers as part of daily math instruction is new to my teaching as a result of this project. I found the engagement in this type of center helped me to listen to my students’ thinking. Sarah said on this day, “I have a plan to trick my buddies I am going to pick a number that is a multiple of 3 and is an odd number. They will never guess my number.” Sarah was so proud of her number facts and that she thought she had a number no one would guess. I was so proud that she knew two number properties about the number she was selecting.

During our addition with regrouping lesson, we began our exploration with rolling dice and using place value blocks to build our quantities. I asked, “Can anyone show us what happens to our blocks when we have more than a sum of 9 in a place value place?” Ed said, “we already know that we cannot have a digit larger than nine in a place value place, so we are going to have to do trading with our blocks, right?”

“Good, Ed. Let us work with what we know to see if we can find the sum of two rolls of our double dice,” I said. Students eagerly set to work with dice, cubes and paper for recording what they saw happening. During the wrap-up part of this lesson students shared out a few of their problems
and solutions and it was there that we put in the vocabulary of regrouping to the work that they had just completed. The value that I had seen through this research helped me to know that my students would see the process and find value in what they were doing before they were able to grasp with words the concept I would have told them how to do if my teaching had not changed.

Ed was so excited about the work I was asking him to do he quickly filled his paper with accurate and detailed solutions. One cannot overlook the pride on this young man’s face.

“I am a real mathematician,” exclaimed Ed. I said, “I feel that I am a real teacher of mathematics.”

**Conclusion**

The purpose of Jones, et al.’s (1996) research was to validate the five levels of thinking found in a previous framework study. It was said that there are markers in the developing number sense relationships in relation to counting, portioning, grouping and number relationships. Through Jones et al.’s interviews with 12 children, each interview moved through a series of questions until the student became unsuccessful. This study revealed the order in which number representation develops. Early development students lacked grouping skills. These students were successful with single digit counting of quantities. Students at the level five spectrums had what Jones et al. determined to be essential place value skills. These students had
a grasp on counting, partitioning and grouping of numbers. They also had skills in number relationships. The early signs of number sense development in children begins with the ability to orally count. This number representation is essential to the mastery of future mathematical understandings.

The finding of Gill and Thompson (1995) stated when children, “understand concepts, and can describe and justify what they write, they can deal with formal mathematical symbolism with ease and understanding” (p. 359). I found with my own sampling of data for that to be exactly right. My three students that to this day struggle to realize that every time they count their fingers there are still 10 just do not have the knowledge to process concepts such as addition and subtraction. They can move through an assignment using manipulatives like blocks or number lines, but they are unable to articulate why the attained the answer that they did. I have shown evidence that students’ understanding of quantity provided the necessary foundation of numbers that provide them confidence to solve addition and subtraction problems. They understand the principles behind the operational task and they have a sense of number relationships, which serves in knowing if a solution is reasonable.

Jordan (2009) confirms the finding that number relationships are difficult to bridge when a child lacks early number representation sense. Jordan supports that children using the number representation relationships developed earlier have much better problem-solving skills. This also was in agreement with the findings of my research.

Implications

This research says to me that I must work to develop lessons that foster number sense very early on in the year. The foundation of understanding cardinality and number relationship of quantity make on future operational skill development is paramount to all future mathematical
learning. Students leaving first grade without solid number sense skills are essentially set up for lifelong struggles with mathematics. This awareness is something I must address in my classroom, but I also must share with others educators teaching similar grade levels.

The traditional teaching methods of standing at the board and delivering instruction to a whole group will not provide all students with what they need to truly develop number sense. I witnessed that difference in the growth of my students at they explored their ideas about what they saw happening in their mathematics. The explorations make the connections that students need to acquire number sense. I feel that it will be important for me to share my finding with primary grade teachers.
References


Appendix A

Individual Interview Session 1

Interviewer: I am going to count down to a number and then I want you to continue to count. Let’s begin... 60, 59, 58, 57, 56 ...

Listen and record student responses (stop after receiving 5 values)

Interviewer: Now I am going to count using a pattern of multiples and I would like you to continue counting when I stop. Let’s begin. 0, 5, 10, 15, 20, 25 ....

Listen and record student responses (stop after receiving 5 values)

Interviewer: Can you tell me the number name that represents how many bears I have on the tray?

Listen and record student responses

Interviewer: Can you tell me the number name that represents how many bears I have on this tray?

Listen and record student responses

Interviewer: Which of the two trays has more bears on it? How do you know?

Listen and record student responses

Interviewer: Can you tell me what comes before and after 23? How about 49? And 12?

Listen and record student responses

Interviewer: Can you show me how you would find the sum of the following problems? (Orally ask the problem to see for number name recognition and operation procedure)

8 + 6 ? How do you know your answer is a good one?
How about $5 + 0$?

And now $7 + 2$?

**Listen and record student responses**

**Interviewer:** What is the answer to $7$ minus $5$? How do you know? Is there more then one way to find the answer? Can you show me?
Appendix B

Interview 2 Question Small Group

Using small groups have students work to solve questions from the worksheet and explain their methods. Provide variety of manipulatives for students to use as they work through problems.

Record student thinking and discussions

Prompt for strategies on what students are thinking as they work through to a solution.

Inquire as to another way a problem can be solved.

Ten to Twenty

Name: ____________________________

\[
\begin{array}{ll}
12 - \square &= 11 & 16 - 4 &= \square \\
19 - 3 &= \square & \square + 6 &= 19 \\
18 - 1 &= \square & 16 - \square &= 14 \\
12 + \square &= 14 & 19 - \square &= 12 \\
11 + 6 &= \square & \square + 7 &= 17 \\
18 - \square &= 11 & 19 - \square &= 15 \\
\square + 2 &= 18 & 16 - 2 &= \square \\
\square + 5 &= 17 & 18 - 4 &= \square \\
\end{array}
\]
Appendix C

Teacher Journal Topics

1. How does each of the two incidents I wrote about relate to my research question(s)?

2. What changes have I seen in my students this week?

3. Are they utilizing the same addition and subtraction algorithms or does the strategy vary with the problem presented?

4. What surprised me this week, related to students thinking about number patterns?

5. What went really well this week, related to how my students are working through addition and subtraction problems?

6. What did I learn this week that will inform my teaching and/or journaling next week?

7. Tensions I felt this week between my roles as teacher & researcher.

8. What evidence is there that students are using their sense of numbers to build understanding of addition concepts? Are they using the same strategies to solve subtraction problems?

9. How are students using place value understanding to solve problems?