Using Physical and Virtual Manipulatives with Eighth Grade Geometry Students

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Abstract

In this action research study of my classroom of eighth grade geometry, I investigated the impact of the use of physical and virtual manipulatives on students’ relational understanding of geometry concepts. I discovered that my students enjoyed and actually looked forward to manipulative-based lessons; they were more engaged in their learning when they worked with manipulatives, whether concrete objects or virtual tools; and manipulative-based activities positively affected student performance. Students reported that they found manipulative-based activities to be helpful in developing understanding of geometry concepts, but when interviewed, four out of five students stated that they preferred hands-on learning to computer applets. As a result of this research, I plan to continue to use manipulative-based lessons in my geometry class. Due to the short duration of this study, I was only able to use manipulatives to teach a limited number of geometry concepts, so I would like to conduct further research on the use of manipulatives with a wider variety of topics.
In today's political climate of high-stakes testing, I, like many other teachers, find myself faced with ever increasing pressure to improve student test scores while making sure that all students meet minimum proficiency levels. Nebraska’s move from district level assessment to state testing through this year’s implementation of NeSA-M (Nebraska State Accountability-Math) has further intensified demands on educators. Teachers are pressed to cover all tested standards by the date of the state test, which according to work by Hunt et al. (2009), has caused many instructors to set aside time devoted to hands-on activities and decrease the amount of time spent on non-tested standards (pp. 69-70). Furthermore, as the math specialist and a geometry teacher at a fifth through eighth grade middle school magnet center, I have found that one of the major challenges confronting teachers in my building is the issue of students’ retention of previously learned concepts. Teachers are often forced to re-teach prerequisite material that students have forgotten before they are able to teach the standards that will be tested at their particular grade level. The issue then, becomes one of teaching in a manner that facilitates students’ long-term retention of math concepts in order to decrease the instructional load on teachers at subsequent grade levels and to promote students’ future success in mathematics.

Why is it that students don’t retain learning from one school year to the next? Skemp (1978) reasons that there are two types of mathematical understanding: instrumental and relational. He defines instrumental understanding as “rules without reason” and relational understanding as “knowing both what to do and why” (p. 9). Skemp makes a case in favor of teaching for relational understanding by arguing that students who learn the reasons behind mathematical procedures are more likely to remember what they have learned by relating the method to the problem, and he maintains that these students are also more likely to adapt the method to new problems (p. 12-13). As an eighth grade geometry teacher, I feel passionate about
providing the kind of lessons that will support my students in gaining a relational understanding of the concepts I teach so that they will continue to excel in math classes at the high school level and beyond. I feel confident that by providing a sound foundation in math courses at the middle school level, I can increase the likelihood that my students will be able to successfully apply mathematics learning to real life situations throughout their lifetimes.

This is my first year teaching eighth grade geometry. (The bulk of my teaching experience has been in the fifth grade.) My previous class assignments at the middle school level have been eighth grade Algebra 3-4 (two years) and Pre-algebra (one year). This year’s eighth grade geometry class meets in the school’s math lab, which houses 30 Mac computers and a host of physical manipulatives. The class consists of 24 students (13 girls and 11 boys). All 24 pupils are considered to be high ability math students (by virtue of their placement in eighth grade geometry), and therefore, have access to programs and services provided by the school’s gifted and talented program. Many are also part of the dual language program and receive both science and language arts instruction in Spanish. Most of my students are from Spanish-speaking homes, and 16 of them are also former English Language Learners. Although all of my students have exited the ELL program, some are still developing the scope of Academic English required to easily navigate advanced math classes.

As a fifth grade teacher, I frequently used physical manipulatives to teach math concepts, but since moving to the middle school, I have not used them nearly as often because of the time involved and the students’ penchant for using these “tools” for purposes other than which they were intended. I have tried to introduce the use of manipulatives in a couple of geometry lessons this year, but the activities did not go well because students either did not use the tools appropriately, did not take the activity seriously, or did not make the intended connection
between the physical representation and the mathematical concept. However, I still feel that manipulatives (whether physical or virtual) are a valuable tool, and I would like to be able to effectively incorporate their use in a variety of lessons as a means of promoting the development of students’ relational understanding of geometry concepts.

**Problem Statement**

In working with my students, it has become increasingly apparent that a majority of them have little more than instrumental understanding of many foundational geometry concepts. I want to find the most advantageous means of helping my students to achieve true relational understandings. NCTM’s overarching principle of learning maintains “students must learn with understanding, actively building new knowledge from experience and prior knowledge” (National Council of Teachers of Mathematics, 2000, p. 20). Additionally, NCTM’s process standard of representation states that

> Instructional programs … should enable all students to: create and use representations to organize, record and communicate mathematical ideas; select, apply, and translate among mathematical representations to solve problems; (and) use representations to model and interpret physical, social, and mathematical phenomena. (p. 67)

After reading several research articles on the use of physical and virtual manipulatives to teach math concepts, I wanted to explore the connection between the use of such representations and the development of relational understanding of geometry concepts in eighth grade students.

NCTM’s Principles and Standards for School Mathematics emphasize the importance of conceptual understanding in conjunction with “factual knowledge” and “procedural proficiency” to students’ competency in mathematics. They also stress the need for students at all levels “to work at developing their understandings of the complex ideas captured in conventional
representations” (NCTM, p. 68). However, from reading related research, I am reminded that the simple inclusion of manipulatives in math lessons will not automatically result in relational understanding. Teachers need more than knowledge about the technologies they choose to incorporate into their lessons. Rather, they need extensive skill in using these technologies to teach mathematical concepts to their students.

There is a somewhat limited body of research available on the use of manipulatives in the teaching of mathematics, and the majority of the most recent studies conducted on the topic have dealt solely with the use of virtual manipulatives. Furthermore, very few of these inquiries apply strictly to the use of manipulatives with middle school students. Knowing about this particular Problem of Practice will serve to inform me in my own practice with students by increasing my knowledge base and assisting me in making informed adaptations to teaching in my classroom. This research will help me to develop lessons that promote the development of students’ relational understanding and will assist me in determining how my particular group of students learns.

As a math specialist, I also feel that I have the ability to affect school-wide change in the teaching of mathematics. In addition to learning about how this Problem of Practice applies to my own teaching and learning situation, I can share what I learn with other math teachers in my building - some of whom will have very similar groups of students. It will also give math teams building-wide the ability to build on what has been learned and to then apply that knowledge to other grade levels and math courses. Ultimately, teachers throughout the district might be interested in the findings of this particular case study, especially teachers in classes with high numbers of non-English speaking students. Since NCTM has advocated the importance of the development of students’ relational understanding of math concepts as well as students’ ability to
translate among mathematical representations, this Problem of Practice is relevant to teachers of all levels of mathematics.

**Literature Review**

Identifying instructional strategies that promote students’ understanding of concepts and the retention of skills has become a fundamental concern in today’s educational climate of high stakes testing and accountability. Educators across the country are seeking out effective research-based practices in an effort to improve student performance. In a 1989 report on curriculum and evaluation standards, NCTM (1989) advocated the increased use of physical manipulatives during mathematics instruction. Additionally, in a subsequent publication of guidelines and standards, NCTM included *representation* as one of five recommended process standards for mathematics instruction of students in pre-kindergarten through grade 12 (NCTM, 2000). In 1989, NCTM defined *manipulatives* as “materials and tools such as spinners, tiles, cubes, geoboards, pattern blocks, compasses, rulers, protractors, scales, scissors, graph paper, grid and dot paper,” which can be used to illustrate and clarify abstract mathematical concepts (NCTM, 1989 p. 6). This increased focus on the use of technology in the classroom has paved the way for the development of virtual manipulatives. A virtual manipulative can be defined as “an interactive, Web-based visual representation of a dynamic object that presents opportunities for constructing mathematical knowledge” (Moyer, Bolyard & Spikell, 2002, p. 373).

NCTM’s emphasis on the use of representation to promote mathematical understanding has called for educators to reflect on the use of manipulatives in mathematics instruction. As a result, various studies have been conducted to address the following questions: 1) What are the effects of physical manipulatives on student achievement and understanding? 2) Do physical
What Are the Effects of Physical Manipulatives on Student Achievement and Understanding?

The belief that concrete objects aid children in the development of abstract concepts has been widely accepted in academic communities (McNeil & Jarvin, 2007), and a number of studies have corroborated this idea by showing that the use of physical manipulatives does positively affect student performance (Cass, Cates, & Smith, 2003; Martin & Schwartz, 2005; Raphael & Wahlstrom, 1989). Using data collected through educators’ participation in the Second International Mathematics Study (1981-1982), Raphael and Wahlstrom found that student achievement improved when experienced teachers employed physical manipulatives in geometry instruction. Records also revealed a relationship between a higher frequency in the use of physical aids and increased topic coverage in math courses (Raphael & Wahlstrom, 1989).

Physical manipulatives have also been shown to have a favorable effect on the development of students’ problem-solving skills and conceptual understanding. Martin and Schwartz (2005) studied fourth- and fifth-grade students to determine how interaction with physical manipulatives contributed to the development of fraction concepts. Researchers found that the act of using physical manipulatives enabled students to find solutions for problems they were otherwise unable to solve either mentally or with the aid of pictorial models. Video-taped interviews confirmed that students more frequently gave correct interpretations and tried a greater number of strategies when solving problems with physical materials. Researchers used a variety of manipulatives in the study and noted that when students used interfaces that required the adaptation and interpretation of concepts, they were more likely to transfer learning to new situations (Martin & Schwartz, 2005). Similarly, a comparative case study of 12 pairs of middle-
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class eighth graders, found that students were able to use a physical manipulative called a “winch” to generate and solve simultaneous linear functions for which they received no direct instruction. Students used the winch to construct their own understanding by simply drawing on previous knowledge of physical and numerical patterns (Izsak, 2004).

Retention of skills and knowledge can be tied to academic achievement and has also been a focus of recent study. Cass, Cates, and Smith (2003) extended previous findings in a study of three secondary students with learning disabilities in mathematics. Using a multiple baseline design, researchers found that students acquired and retained problem-solving skills when the concepts of perimeter and area were taught using physical manipulatives.

However, not all research validates the use of manipulatives in math instruction. McNeil, Uttal, Jarvin, and Sternberg (2009) investigated whether highly realistic physical manipulatives were more helpful to fourth- through sixth-grade students in problem solving than simple, unadorned manipulatives or no manipulatives at all. During the first experiment, students in the experimental condition were provided with highly realistic bills and coins and asked to solve a series of word problems involving money. In contrast, the control group was asked to complete the same problems without the use of concrete objects. Data from the study showed that students from the experimental group solved fewer problems correctly than did students in the control group. Results seemed to suggest that physical manipulatives were counterproductive to students’ problem-solving performance (McNeil, Uttal, Jarvin, & Sternberg, 2009).

In a second experimental investigation of the same study, researchers divided 85 fifth-grade students into three conditions. The first experimental condition provided students with highly realistic coins and bills and asked them to solve word problems involving money. The second experimental condition asked students to solve the same word problems using plain,
black-on-white coins and bills labeled simply by denomination. The control group was asked to solve the same problems without the use of concrete objects. Results showed that students who used the realistic manipulatives were least likely to solve problems correctly; however, the errors made by students using realistic manipulatives were less likely to be conceptual errors than those made by students in either of the other conditions. These data suggest that different types of manipulatives may offer distinct advantages to student learning.

Interestingly, data from the majority of studies looked at here indicate that the use of manipulatives does have a positive effect on student learning. However, it is worth noting that researchers in each case related the use of manipulatives to positive effects on slightly different aspects of student learning. For example, Raphael and Wahlstrom (1989) found that the use of manipulatives during math instruction increased student achievement, while Martin and Schwartz reported improved student problem-solving skills. Izsak linked manipulative use to students’ development of relational understanding, whereas Cass, Cates, and Smith (2003) associated manipulative use to long-term retention of skills. Although McNeil, Uttal, Jarvin and Sternberg (2009) found that some manipulatives actually had a negative effect on student problem-solving ability, they also pointed out that the type of manipulative employed during instruction appeared to be a relevant factor with respect to student learning.

Do Physical and Virtual Manipulatives Have an Equal Effect on Learning?

Over the past several years, technology has become more readily available to classroom educators, and there has been a resultant increase in the number of virtual manipulative applets available to support math instruction. A question of fundamental importance in the use of virtual manipulatives to teach math concepts is the issue of effectiveness. Do virtual manipulatives and concrete manipulatives have an equal effect on student achievement and conceptual
understanding? Suh, Moyer, and Heo (2005) examined field notes, student interviews, and classroom videotapes to explore ways in which virtual manipulatives assisted fifth graders of varying math ability in developing a connection between conceptual and procedural understanding of fraction concepts. Results indicated that the speed of the applets, in conjunction with computer generated feedback, encouraged student experimentation and discovery learning. Student interviews suggested that working with virtual manipulatives supported representational fluency because it helped students to make connections between visual depictions and symbolic models.

Additional support of virtual manipulatives was found by Steen, Brooks, and Lyon (2006), who examined the influence of computer manipulatives on the academic achievement of 31 first graders in a quasi-experimental study. Findings revealed that students in the treatment group, who used only virtual manipulatives for math practice, showed a statistically significant improvement from pretest to posttest during a unit of geometry instruction. However, researchers observed that the treatment group had lower pre-test scores, and the difference between the posttest scores of the two groups was not significant. It was noted that both groups performed extremely well on the post-test with all students demonstrating mastery of second grade geometry concepts. The data suggest that virtual manipulatives are as effective as their more traditional counterparts. Even so, field notes indicated that the teacher who used virtual manipulatives felt there were definite advantages to using technology such as elimination of set-up time, immediacy of feedback, increased motivation, time-on-task, ease in checking, and support of in-depth exploration (Steen, Brooks, & Lyon, 2006).

In addition to promoting mathematical understanding, virtual manipulatives have been found to have a positive impact on student attitudes. In an action research project involving 19
third graders, Reimer and Moyer (2005) explored the effects of virtual manipulatives on conceptual and procedural knowledge as well as students’ attitudes during a fraction unit. After a two-week unit of study covering basic fraction concepts, the majority of students’ scores on a posttest of conceptual knowledge showed improvement over the pre-test scores. A paired t-test showed no significant difference in students procedural knowledge from pre-test to post-test, but it was noted that one of the limitations of the data was that pre-test scores for the entire group were extremely high, which suggested that students may have begun the project with a sound understanding of procedures in working with fractions while lacking an equally strong conceptual understanding. Interview responses indicated that students enjoyed working with virtual manipulatives and that they found immediate feedback from the applets to be particularly helpful (Reimer & Moyer, 2005).

While virtual manipulatives are often computerized versions of familiar physical manipulatives, there is some evidence that the virtual tools offer additional benefits over the same tool in its concrete form. The ease with which students are able to maneuver virtual manipulatives has been shown to support student exploration and the development of problem-solving strategies. Yuan, Lee, and Wang (2010) used polyominoes in a quasi-experimental study to examine the differences in eighth grade students’ problem-solving ability, problem-solving strategies, and attitudes when working with virtual manipulatives as compared to a control group, which used only physical manipulatives. Researchers found no significant difference in problem-solving ability between the two groups; however, students who used virtual manipulatives tended to exhibit more exploration, which led to a greater range of problem-solving strategies. The majority of students who used virtual manipulatives looked forward to using them to study math topics in the future (Yuan, Lee, & Wang, 2010).
Data from the research compiled here supports the notion that both virtual and physical manipulatives can have a positive effect on student understanding. However, none of the researchers found an advantage to using virtual manipulatives over physical representations with respect to effect on student achievement. In spite of this, several of the studies cited examples of unique benefits available only to students working with virtual manipulatives. Among the benefits mentioned were speed of applets and immediacy of feedback (Suh, Moyer, & Heo, 2005; Reimer & Moyer, 2005), reduction of time spent on set-up and clean-up, and increased student motivation (Steen, Brooks, & Lyon, 2006; Yuan, Lee, & Wang, 2010).

Do Manipulatives Ensure Student Learning?

While various reports have shown that the use of both physical and virtual manipulatives can be beneficial to student learning, there are also a number of studies, confirming that the use of manipulatives alone will not ensure student learning. The essential factor in successful teaching with manipulatives is the classroom teacher. Moyer and Jones (2004) conducted a comparative case study of 10 middle-grade teachers participating in a summer mathematics institute. Their findings suggested that teacher demonstration of correct use of manipulative materials and careful structuring of lessons to allow for multiple representations before replacing concrete representations with abstract symbols alone, supports students’ understanding of abstract symbols (Moyer & Jones, 2004).

Teachers need to be able to decide which learning objectives will be supported by the use of manipulatives, as well as which manipulative will best support students in making connections between informal representations and symbolic representations (McNeil et al., 2009). Puchner, Taylor, O'Donnell, and Fick (2008) conducted a comparative case study suggesting that student
understanding and achievement is strongly dependent on careful teacher planning and structuring of learning activities. Qualitative data analysis was conducted on data collected from 23 elementary teachers enrolled in a professional development institute focusing on pedagogical strategies compatible with NCTM principles and standards. While the majority of teachers said they believed manipulatives were advantageous to student learning, most of the participants still experienced difficulty in implementing the use of manipulatives in math lessons after completing the program. In many teachers’ follow-up lessons, manipulatives appeared to be used with no specific purpose, and manipulative use had become an end in itself. Researchers proposed that successful reform of teaching practices would require substantially more time than made available through the institute, as well as ongoing support and professional development opportunities (Puchner, Taylor, O’Donnell, & Fick, 2008). Similar results were found in a qualitative study of 20 eighth-grade algebra teachers who participated in a summer in-service project focused on technology knowledge, pedagogical knowledge, and content knowledge. Findings pointed to the importance of ongoing professional development to the successful implementation of virtual manipulatives in mathematics instruction (Richardson, 2009).

Data from this collection of research suggests that classroom teachers’ familiarity and skill in working with manipulatives is, perhaps, the greatest determinate of whether manipulative use will positively affect student learning. Moyer and Jones (2004) along with Puchner et al. (2008) found that effective teachers planned structured lessons and modeled correct use of manipulative materials. Both Puchner et al. (2008) and Richardson (2009) proposed that teachers who lack the necessary skills and background knowledge required to properly implement manipulative use in the classroom require ongoing training and support programs to develop these skills.
Literature Review Conclusion

The driving question behind research on the use of manipulatives is whether these instructional tools will increase student achievement and conceptual understanding. However, recent studies linking the effects of manipulatives to students’ achievement and the development of conceptual knowledge has been somewhat inconclusive. Some current studies support the NCTM contention that physical materials have a positive effect on student understanding (Izsak, 2004; Martin & Schwartz, 2005), while other research has found evidence to the contrary (McNeil et al., 2009). Findings from several contemporary studies have shown that virtual tools not only helped students to learn mathematics concepts as effectively as physical manipulatives, but also sometimes, afforded supplementary benefits not offered by their more traditional counterparts (Reimer & Moyer, 2005; Steen, Brooks, & Lyon, 2006; Suh, Moyer, & Heo, 2005; Yuan, Lee, & Wang, 2010). Results from numerous research projects have yet to give a conclusive answer. Evidence suggests that the use of manipulatives does not in and of itself guarantee success (McNeil & Jarvin, 2010), but instead, is highly dependent on teacher knowledge and skill in the use of manipulative materials.

A major limitation of the current body of research is that only a small number of studies have focused on the use of manipulatives in teaching the higher-level concepts of secondary mathematics courses. One study looked at students in a regular eighth grade math class and an additional study looked at eighth grade algebra students; however, the current research project expands upon these findings by examining the use of manipulatives with advanced placement eighth grade geometry students. Another aspect of this study that makes it unique is that the research population is composed primarily of students who are fluent in both Spanish and
English and for whom English is a second language. The current study also differs from much of the other research on the topic in that it is a teacher as researcher action research project.

**Purpose Statement**

The purpose of my project was to investigate the impact of use of physical and virtual manipulatives on the development of relational understanding of geometry concepts in eighth grade students. The variables I examined were: the quality of students’ written reasoning in explaining geometry concepts, the quality of students’ verbal reasoning in explaining geometry concepts, students’ ability to correctly transfer learning to similar problems, the quality of student work handed in, students’ homework completion rates, amount of instructional time spent in direct instruction, and amount of instructional time spent in inquiry-based instruction in seeking to answer the research questions:

- What will happen to students’ relational understanding of geometry concepts when lessons include the use of physical or virtual manipulatives?
- What will happen to the level of student performance in geometry when I include the use of physical and virtual manipulatives in class?
- How will the increased use of manipulatives impact my teaching style?

**Method**

My study was conducted in an eighth-grade geometry class at an urban middle school magnet center located in the Midwest. The school serves more than 1,000 students in grades five through eight. The student population is approximately 67 percent Hispanic, 23 percent Caucasian, and 10 percent African American, with 75 percent of all students receiving free or reduced lunch. A total of 24 students participated in the data-gathering process, but based on completed consent forms, interviews and work samples are included from only 22 of those
students. My class of 13 girls and 11 boys met for 90 minutes every day in a classroom that was equipped with 30 computers. The curriculum used for instruction of this course included a self-paced computer component in addition to a consumable student text and workbook.

Data collected as a part of my normal teaching practice included students’ class work and homework samples, pre- and post-tests, and student journals. Additional sources of information used to ensure triangulation of the data were student interviews and a personal teacher journal.

Homework was assigned on average four days per week. All graded homework assignments and class work was evaluated using a grading rubric (see Appendix A). After sharing grades and comments with students, all work directly related to manipulative based activities that had been taught in class were collected and kept for data analysis.

Academic growth was measured by means of a pre-test and post-test, which were identical in form. The test was taken from two different chapters of the geometry curriculum and covered measurement of both two- and three-dimensional figures. The test included items requiring a wide range of skills, from simple identification of formulas to problems requiring the application of multiple formulas (see Appendix B).

All students were required to maintain a writing journal in which they were asked to write evaluations of the learning activities presented during the week. Students were encouraged to include their thoughts and opinions, as well as any additional information that was relevant to their learning. Writing journals were handed in weekly, but entries were not graded.

Five students were interviewed on two separate occasions during the course of the study, once near the beginning and once near the end (see Appendix C for interview questions). Interviews were conducted during the regular class period and each lasted approximately 15
minutes each. Interviews were recorded on a laptop computer; all students were asked the same interview questions, and questions were posed in the same order to all students.

**Findings**

The district curriculum for this class encourages the incorporation of direct instruction and student collaboration with independent computer-based learning. My students are typically introduced to four daily lessons from the text each week and spend the fifth day working independently on the computer portion of the curriculum and reviewing skills.

Students in this class worked from a set of two consumable textbooks. Both books were fairly large so the assignment books were kept in the classroom where homework and skill pages were torn out as needed. Upon entering the classroom, students picked up a graphing calculator and their assignment book. Students then turned in the current homework by placing it in the assignment basket, and retrieved their checked work at the same time. Thirty desktop computers were situated along the back and side walls of the room, and groups of two to four students were assigned to work tables arranged in the center of the room.

Several manipulative-based activities were incorporated into lessons during the second semester, but repeated experiences were necessary before students were familiar with the appropriate routine and procedure for use of these materials. Therefore, my research focused on the activities presented in the context of a unit on measurement. A typical 90-minute class period during this study began with a 10-minute warm-up problem to review and reinforce previously taught concepts. These problems generally posed a question that led students into the ensuing lesson.

The next 45 to 50 minutes were spent on the daily lesson, which began with an introduction of new vocabulary along with a review of key terms. New terms were discussed,
illustrated, and then added to a classroom word-wall. After highlighting the vocabulary, a new problem was posed followed by discussion and preliminary work requiring students to build on prior knowledge. Work that required the use of a formula was preceded by group work to derive the formula. The manipulative-based activities that are the focus of this project are activities meant to assist students in deriving and understanding measurement formulas.

Manipulative-based activities were incorporated into the lesson approximately one to two times per week over the 12-week course of the project. Content objectives were used to dictate the types of activities that were chosen. Activities that included the use of concrete objects generally required from 20 to 30 minutes, while virtual-manipulative activities were usually accomplished within 10 to 20 minutes. When the lesson required the use of physical manipulatives, all of the materials for the activity were set out prior to the beginning of the class period, but the manipulatives were not placed on student tables until after the activity had been introduced.

Students completed most of the activities with a partner or in a small group, however, one of the physical-manipulative activities and three of the virtual-manipulative activities were conducted as independent activities. Students evaluated each manipulative-based activity by means of a student writing journal. In their journal entries, students were asked to describe the activity, explain what they had learned, and make a personal assessment of the value of the activity to their own learning.

Following the daily lesson, homework was previewed and assigned. After directions were given, students had the opportunity to ask questions and get clarification if needed. Students were allowed approximately five minutes of class time to begin homework.
The final 30 minutes of the class period was devoted to teaching and reviewing eighth grade standards in preparation for the state math test. This was accomplished through a combination of direct instruction and computer-based, instructional resources and practice activities.

**Impact on the Development of Relational Understanding**

My first research question addressed the impact of the use of manipulatives on my students’ relational understanding of geometry concepts. During the measurement unit of study, which was the focus of this project, learning objectives included using formulas to calculate perimeter/circumference and area of two-dimensional shapes as well as volume and surface area of three-dimensional figures. Student interviews, student journals, and a personal teacher journal gave a good picture of the changes that occurred in my students’ relational understanding of these formulas throughout the course of the study. As my students progressed through this instructional unit on measurement, two things became apparent:

- Students began using the manipulative-based activities that had been integrated into lessons as assistive tools in recalling the formulas.
- Students’ explanations of the steps in their own problem solving procedures became more complete.

Student interviews were the first piece of evidence that provided support for these claims. Initial interviews were conducted in early February prior to introduction and instruction of the measurement unit. Second interviews were held in mid-April, after all of the lessons for the unit had been taught. Questions were asked in the same order, and problem-solving questions were the same on both dates. In the problem-solving portion of the interview, all problems were directly related to the manipulative-based activities taught during the instructional unit on measurement, problems were presented to students one at a time, and students were asked to
explain their reasoning aloud while solving each problem. Four of the five students interviewed indicated during the initial interview that they did not remember ever having used physical manipulatives or hands-on learning activities in any previous math class. The remaining student recalled making vocabulary posters and using graph paper in a seventh grade class.

At the outset of my project, all of my students demonstrated, through a pre-test, class discussions, student interviews, and student journals, that they were able to identify and use formulas to correctly calculate perimeter and area of a rectangle. About half could also correctly recite and use a formula to compute the area of a triangle. However, when asked to explain the area formula for a triangle, students’ comments revealed that there was little understanding of the logic behind the formula. None of my students were familiar with area formulas for a parallelogram, trapezoid, or regular polygon.

The following excerpt is from the initial interview with Yesenia¹, a student of average ability, who was struggling to keep a C average in geometry. Yesenia was asked to find the area of a given parallelogram (see Figure 1).

Question: Can you find the area of this parallelogram?

Response: It's like a rectangle, so that's length times width. So I count the squares for length and width and get 17 here and 9 for the side. Then, that's 17 times 9...153. It's 153 meters squared.

Figure 1

¹ All names are pseudonyms.
Yesenia’s interview response suggests that she may be aware that a relationship exists between the formula for area of a parallelogram and the formula for area of a rectangle; however, her reasoning isn’t complete, and she isn’t able to recognize that the two polygons in her drawing do not have the same area.

Rafael, a second student in my class, had demonstrated high math ability, but managed to maintain only a C average due to poor test scores and a number of related factors. He was inattentive during instruction, rarely participated in class, and seldom turned in completed homework assignments. Raphael gave the following initial interview response to a question that required him to calculate the area of a trapezoid:

**Question:** Can you find the area of this trapezoid?

**Response:** Uhm...I don't remember how to do that, Miss.

Figure 2

Raphael’s response reiterated what I had heard from over half of the students in my class during a whole group discussion: namely, students had once been taught the formulas, but had forgotten the previously learned information.

In response to data collected from student interviews and class discussions, I incorporated two different manipulative-based activities into a series of lessons on calculating the area of polygons. The first activity was a computer applet. Students logged into the following website (http://illuminations.nctm.org/ActivityDetail.aspx?ID=21) and explored the concept by finding area for a variety of triangles, rectangles, and parallelograms.

The second activity was a hands-on activity in which students used grid paper to draw triangles, rectangles, parallelograms, and trapezoids to derive area formulas for the various
polygons. Since my students were already comfortable with using the formula for area of a rectangle, I used this activity to relate area formulas for triangles, parallelograms, trapezoids, and regular polygons back to the area formula for a rectangle. For example, after students drew right triangles on grid paper, they were lead to discover that the area was equal to one half of the related rectangle. They then wrote a formula for area of a triangle based on the already familiar formula for area of a rectangle. Next, students were lead through a similar activity to discover that a parallelogram drawn on a grid can be rearranged into a rectangle by cutting, sliding, and pasting a triangular portion of the parallelogram. After making this visual alteration, students were able to write a formula for area of a parallelogram by relating the activity back to their prior knowledge (area of a rectangle). Like activities were conducted for trapezoids, and regular polygons.

The following is an excerpt is from Yesenia's follow-up interview in February, after she had completed both the virtual- and physical manipulative-based activities:

**Question:** Can you find the area of this parallelogram?

**Response:** First, I can draw a line here, and then move the triangle over here. So, now I have a rectangle. Then I can just do length times width.

**Question:** Do you remember the mathematical term for the line you drew here?

**Response:** Uhm... It's an altitude or maybe height, and length is the base.

**Question:** Can you write a formula for the area of a parallelogram? (Student demonstrates by writing.)
The differences in Yesenia’s responses from the first to the second interview indicate that she was using the previously described grid-paper activity to help her remember the formula for area of a parallelogram as she solved the problem.

Likewise, Raphael’s second interview attempt to calculate the area of a trapezoid gave evidence of the student making a connection between the in-class manipulative-based lessons and his chosen problem-solving strategy.

Question: Can you find the area of this trapezoid?

Response: Well, I could make a copy of the trapezoid and flip it so it becomes a parallelogram. And if I draw in the altitude, and then slide this triangular piece over to this side, like this, it becomes a rectangle. Then I can use the formula for a rectangle and divide it by two because the area of the original trapezoid is half the area of the whole rectangle. But the formula is written so that the length is the sum of base 1 and base 2, and the width is the height.

The second piece of evidence to support my claims that students began using the manipulative-based activities as assistive tools in recalling formulas and that students’ explanations of their problem solving procedures became more complete as the unit progressed was my teaching journal. In mid-April, I introduced an activity in which students divided a circle into 16 congruent parts, cut the circle into wedges, and rearranged the pieces to form a figure that

Figure 4
was similar to a parallelogram. Students then derived a formula for the area of a circle that was based on prior knowledge about the area of a parallelogram (see Figure 5).

After the activity, I made the following notation about Ian, a bright but typically unmotivated student, in my personal journal:

During a whole-group discussion this week, Ian described the manipulative-based activity we had done in class as he explained his method of remembering the formula for area of a circle. He said, "If I cut the circle into lots of wedges (Ian draws a circle on the board and divides it into wedges), and rearrange the wedges with some of the teeth up and some of the teeth down, the new shape will make...or almost make, a parallelogram (redraws the wedges to form a shape similar to a parallelogram). The width of the parallelogram is one radius (makes motion to relate the similar parts of the two drawings), and the length of the parallelogram is half of the circumference of the circle...'cause this part goes half-way around and this part goes the other half-way (indicates the
bases of the parallelogram and traces the two halves of the circle with his finger. So the formula is $1/2(2\pi r)$, which is equal to $\pi r^2$ (writes both formulas on board). (April 8, 2011)

From his explanation, it was apparent that Ian was drawing on the in-class activity to aid him in recalling the formula for area of a circle and explaining the logic behind it.

The third piece of evidence for these claims is the students’ journals. On April 21, the objective of the lesson was to derive a formula to calculate the volume of a pyramid. I introduced a manipulative-based activity using relational geo-solids and rice to assist students in deriving the formula for volume of a pyramid. Students were given relational geo-solids in the form of a cube and a rectangular pyramid with equal bases. They filled the pyramid with rice and poured the rice into the cube, counting the number of times it took to fill the cube. They then built on prior knowledge about volume of a cube to write a formula for volume of a pyramid. After completing the activity, Ariana wrote the following entry in her student journal:

Today, the activity we did was really helpful to make me understand this lesson...It took 3 times to fill up the whole prism/cube. We also made an observation that the base and height of both figures was the same, but the prism had a greater volume. It was really easy to do, but still made me understand this lesson better...The volume of the prism is 3 times greater than the volume of the pyramid. $V_{\text{prism}} = \ell \, \text{wh} = Bh$

$$V_{\text{pyramid}} = \frac{1}{3} \, Bh$$

None of my students could accurately calculate the area of a pyramid prior to completing this lesson; however, after the manipulative activity, many students like Ariana were not only able to compute volume, but also were able to give a coherent explanation of the logic behind the formula.
Manipulatives in Geometry

On April 28, Liseth, one of my strongest students, wrote a second journal entry that suggested students were developing relational understanding of measurement formulas. My lesson had included a manipulative-based activity using oranges to derive the surface area of a sphere. This was accomplished by having student pairs find the circumference of an orange, calculate the radius of the corresponding great circle, and then calculate the number of times the orange peel would cover the great circle (see Figure 6).

After completing the activity and deriving the formula, Liseth wrote:

After we put all the peels in the circle, we noticed that it covered four circles. We noticed that mostly everyone also got four circles covered. So after we ate the orange, we came up with a formula. The formula was $S = 4\pi r^2$. When you look at the formula, you can see you got the four circles from the number of circles the peels covered, and the $\pi r^2$ comes from finding the area of a circle. I think this activity will help me remember the formula because since using oranges was part of it, it will remind me that it took up four circles, and then I just have to remember the area of a circle. And so the formula will stay in my mind.
It is clear from these journal entries that my students found the manipulative-based activities both memorable and useful in helping them to solve problems.

Data gathered from student interviews, student writing-journals, and my personal teaching journal clearly show that students, who could not adequately explain geometric measurement problems prior to this instructional unit, were able to do so after lessons that incorporated manipulative-based activities had been taught. After instruction of this unit, students demonstrated greater understanding of the logic behind measurement formulas in both oral and written explanations of their problem-solving strategies. Student explanations made frequent references to manipulative-based activities that had been presented within the context of the unit, and the activities themselves became a reference point for students as they solved geometric measurement problems.

**Impact of Manipulative Use on Level of Student Performance**

My second research question looked at the impact of the use of physical and virtual manipulatives on the level of student performance. Over the course of the study, it became evident that students’ ability to solve measurement problems improved after completing manipulative-based lessons.

The first piece of evidence to support this claim is a comparison of students’ pre- to post-test scores (see Figure 7). The pre-test and post-test were identical. The test assessed students’ ability to calculate perimeter and area of two-dimensional polygons and composite figures, compute volume and surface area of three-dimensional figures and composite solids, and apply knowledge of the associated vocabulary. The pre-test was given prior to introduction of the measurement unit, and the post-test was the administered after all of the manipulative-based lessons for the unit had been completed. Twenty-two students completed both the pre-test and
Students’ scores from pre-test to post-test improved by an average of 42.68 percent per student, suggesting that the activities presented throughout the unit of instruction comprised an effective method of learning for my students.

A detailed comparison of student work on individual problems from pre- to post-test also provided evidence for the assertion that student performance increased after the incorporation of manipulative-based activities in geometry lessons. For example, on the pre-test, when a given problem required students to calculate the area of a cylinder, Liseth responded by drawing a representation of a cylinder and labeling the given dimensions of radius and height. Her work indicated that she understood the relationship between the radius and diameter of a circle, but that she did not have an accurate strategy for calculating the volume of a cylinder (see Figure 8).
However, after completing manipulative-based instructional activities, Liseth correctly answered the problem by applying the appropriate formula in which she first calculated the surface area of the two bases of the cylinder and then added the product of its circumference and height (see Figure 9).

In both pre- and post-test examples, Liseth began her work by sketching the problem situation; however, her mathematical thinking and problem-solving strategy clearly became more complete and accurate over the course of the instructional unit.
Alejandro, a student who showed little interest in geometry prior to the incorporation of manipulative-based lessons, also demonstrated improved problem solving ability from pre- to post-test. On the pre-test, Alejandro was unable to accurately determine the area of a trapezoid. He simply multiplied the four given side lengths, demonstrating no understanding of an area formula (see Figure 10).

When presented with the same problem on the post-test, Alejandro chose to calculate the area of the trapezoid as a composite figure, using the formulas for area of a rectangle and area of a triangle rather than the formula for area of a trapezoid. Although Alejandro’s method of problem solving did not verify an understanding of the formula for area of a trapezoid, he was able to accurately determine the area of the trapezoid, and his work demonstrated improved problem solving ability from pre- to post-test (see Figure 11).
Students’ daily work also indicated that students’ proficiency in using measurement formulas improved over the course of the project. A rubric was used to assess daily homework during the measurement unit of study. The rubric assessed student work for content knowledge, written explanation of work, and completeness and accuracy of diagrams. Over the course of instruction, 12 student assignments were evaluated using the grading rubric. Taken as a whole, 64 percent of graded daily work demonstrated basic understanding of geometry measurement concepts and processes, 31 percent demonstrated proficiency, and 5 percent exhibited advanced knowledge and skills. A possible explanation for this low level of proficiency on daily assignments is that all daily work was assigned directly after the related lesson. Students may have continued to build understanding of basic geometry measurement formulas in subsequent lessons, which required basic formulas as prerequisite knowledge.

Despite the fact that less than half of my students demonstrated proficiency in their daily work, given students’ low level of understanding as shown by the pre-test, it is reasonable to say that students’ ability to solve geometric measurement problems improved over the course of the study. Students were able to solve problems in their daily work which they had been unable to
solve prior to instruction of the measurement unit. Specific comparisons of examples of students’
problem solving from pre- to post-test, as well as a comparison of pre- to post-test scores
confirm that the students’ ability to apply measurement formulas and solve problems was
positively affected.

**Impact of Increased Use of Manipulatives on Teaching Style**

My final research question dealt with how the increased use of manipulatives would
impact my teaching style. Prior to introducing the measurement unit on which this project is
focused, I introduced my students to the use of physical and virtual manipulatives through
various learning activities that supported a wide range of geometry concepts other than
measurement. When I first began incorporating manipulative-based activities into my lessons,
my students were extremely talkative, frequently off-task, and generally made poor use of class
time. I was frustrated, and after one particular activity in which students used pattern blocks to
explore tessellations, I wrote in my personal journal:

> My students’ comments indicated that they liked the idea of using physical
> manipulatives to learn about tessellations, but their behavior was frequently off-task.
> Instead of looking for regular polygons that tessellate as I directed, the majority of the
groups spent most of their time stacking the pattern blocks and combining different
shapes and colors of blocks to create pictures. Despite a good deal of redirection
throughout the activity, when asked to share their findings with the whole group, three of
the five groups had very little or nothing to share. I can see that I need to convey my
expectations and establish a routine that promotes learning when students are involved in
lessons that involve physical manipulatives. (Feb. 10, 2011)
After incorporating several manipulative-based activities into my lessons using both virtual- and physical manipulatives, I found that I needed to set specific expectations before sending students to the computer or handing out physical manipulatives. More students demonstrated on-task behavior when they were given a specific problem or set of questions to answer during the activity. I also found that setting a limit on the amount of time students had to complete the activity helped my students to stay focused on the task at hand. On April 29, I wrote:

My students were excited and immediately engaged as soon as they saw that materials had been set out for a hands-on learning activity. All of my students quickly sat down at their tables, and most listened carefully as I introduced the lesson and gave instructions for the activity. Four of the six groups completed the activity without teacher redirection, while one group needed to have the directions for the activity repeated and a second group required a reminder about staying on-task. After I had repeated the directions, all students fell to work. I was occasionally called over to share in a group’s discoveries, verify their conjectures, or to clarify an understanding, but students directed the majority of their questions to their peers. At that point, I realized we had successfully established an efficient routine where students were engaged in their learning and meeting content objectives.

From my journal entries, it is clear that by responding to the instructional needs of my students, I was able to enhance my ability to teach the concepts my students needed to learn. I also found that the students themselves could be equally effective teachers when given opportunities to share what they had learned with their peers. When I incorporated manipulative-based activities into my lessons, I found that I spent less time
in direct instruction and more time guiding my students in the development of their own relational understandings.

**Conclusions**

My first attempts at incorporating manipulative-based activities into lessons were hampered by students’ unfamiliarity with hands-on learning and their associated, off-task behavior. This limited picture of the project substantiates the findings of McNeil, Uttal, Jarvin, and Sternberg (2009), who maintain that manipulatives are counterproductive to students’ problem-solving performance. However, after repeated experiences of working with both virtual tools and concrete objects, my students were able to develop a relational understanding of abstract concepts as suggested by McNeil and Jarvin (2007). Students were then able to apply formulas with logic and a more complete understanding, which positively affected their performance. These findings corroborate those of Cass, Cates, and Smith (2003); Martin and Schwartz (2005); and Raphael and Wahlstrom (1989).

Unlike the students studied by Reimer and Moyer (2005), my students did show a significant improvement in their procedural knowledge from pre-test to post-test. However, at the beginning of the unit of study, my students had only a rudimentary knowledge of geometric measurement formulas, giving them ample room for improvement and offering a possible explanation for the difference in results between the two studies.

According to Suh, Moyer, and Heo (2005), students reported that virtual manipulatives were helpful in making connections between visual and symbolic models. All of the students I interviewed also indicated that they found virtual manipulatives to be useful. However, four of the five students stated that they preferred hands-on tasks to computer applets. One factor in this preference might be explained by the computer component in our curriculum, which results in
students spending approximately 20 percent of their class time on computers. Hands-on learning activities with concrete materials may offer my students an attractive alternative to their regular learning routine.

Finally, findings by Moyer and Jones (2004) highlighted the significance of teachers choosing appropriate concepts for manipulative support, carefully demonstrating correct use of manipulative materials, and adequate pedagogical and content knowledge. In my work, the role of the classroom teacher became remarkably clear. Without the establishment of appropriate expectations and guidelines, the incorporation of manipulative-based activities during classroom instruction will be futile.

**Implications**

This action research project has shown me the value of incorporating manipulative-based activities into geometry lessons. My students looked forward to these lessons and were eager to participate in the activities. While using manipulatives, my students were genuinely engaged in building relational understanding of geometry concepts. Next year, I plan to continue to use both hands-on learning and computer applets as a primary teaching tool during instruction of the measurement unit. In addition, I want to promote the development of relational understanding over a wider range of mathematical concepts by increasing the integration of manipulative-based activities into other instructional units as well. I intend to offer more opportunities for peer tutoring as a means of strengthening students’ ability to explain their own math reasoning and problem-solving strategies.

In my role as a middle school math specialist, I share the responsibility of supporting and guiding math teachers in their delivery of mathematics instruction to students throughout the building. I want to share what I have learned with my colleagues and encourage them to consider
how the increased use of manipulative-based activities might be used to develop relational understanding of math concepts for all of our students and the impact this would have on their instruction and learning in classrooms throughout our building.
References


## Appendix A- Student Grading Rubric

### Geometry Grading Rubric

<table>
<thead>
<tr>
<th>Content Knowledge</th>
<th>Does Not Demonstrate Understanding 1 point</th>
<th>Partial Understanding 2 points</th>
<th>Understanding of Basic Concepts &amp; Processes 3 points</th>
<th>Proficient 4 points</th>
<th>Advanced 5 points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student does not attempt</td>
<td>Incorrect procedure/does not show steps to solve the problem</td>
<td>Minor errors in procedure/Shows partial steps to solve problems</td>
<td>Accurately solves basic problems and shows steps with no major errors</td>
<td>Correctly applies skills to all</td>
<td>In depth inferences and applications</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Written Explanation of Work</th>
<th>Does Not Demonstrate Understanding 1 point</th>
<th>Partial Understanding 2 points</th>
<th>Understanding of Basic Concepts &amp; Processes 3 points</th>
<th>Proficient 4 points</th>
<th>Advanced 5 points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student does not attempt</td>
<td>No understanding or skill demonstrated</td>
<td>Demonstrates partial understanding of simpler details and processes</td>
<td>Explains all steps with some degree of difficulty</td>
<td>Explains all steps of problem or sketch</td>
<td>Explains work with full knowledge and initiates new ideas</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Accuracy and Completeness of Diagrams</th>
<th>Does Not Demonstrate Understanding 1 point</th>
<th>Partial Understanding 2 points</th>
<th>Understanding of Basic Concepts &amp; Processes 3 points</th>
<th>Proficient 4 points</th>
<th>Advanced 5 points</th>
</tr>
</thead>
<tbody>
<tr>
<td>Student does not attempt</td>
<td>Diagrams do not represent the problem situation</td>
<td>Portions of the diagram are inaccurate or missing important information</td>
<td>Basic information is included and there are no major errors in the diagram</td>
<td>Diagrams are accurate and include all important information</td>
<td>Diagrams are complete, accurate, and help to explain the problem solving process</td>
</tr>
</tbody>
</table>
Appendix B- Pre-Test/Post-Test

Pre-Test

Name ________________________________ Date __________________

Write the area formula for each shape.

1. triangle

2. rectangle

3. parallelogram

4. regular polygon

5. The area of a rectangular field is 6800 square meters. If the width of the field is 80 meters, what is the perimeter of the field? Draw a diagram and show all your work.

6. Calculate the area of the trapezoid shown.

<table>
<thead>
<tr>
<th>12 cm</th>
<th>10 cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>6 cm</td>
<td>20 cm</td>
</tr>
</tbody>
</table>
7. The area of the triangle shown is 38.48 square centimeters. What is the value of \( n \)?

8. A side length of the square is 14 inches. The diameter of the circle is 8 inches. Calculate the area of the shaded portion. Use 3.14 for \( \pi \).
9. The figure shown consists of a rectangle and two congruent circles. If the area of the rectangle is 1250 square feet, what is the radius of one of the circles?
Pre-Test

1. How many vertices, edges, and faces does the prism have?

2. Without creating a model, identify the polyhedron that is represented by each net. Then, identify the base(s) of each polyhedron on the net.

The names of three-dimensional objects are shown. Match each object with the correct formula.

3. Rectangular Prism
   a. \( S = 4\pi r^2 \)

4. Cylinder
   b. \( V = \pi r^2 h \)

5. Cone
   c. \( S = \pi r h \)

6. Sphere
   d. \( V = \frac{1}{3} \pi r^3 \)
7. Your aunt’s rectangular driveway is 55 feet long, 15 feet wide, and 0.5 feet thick. Pre-mixed cement is sold by the cubic yard. Determine the number of cubic yards of cement that your aunt used to construct the driveway.

8. A basketball with a diameter of 9.5 inches is packaged in a cubic box measuring 9.5 inches on each edge. Determine the volume of the empty space in the box.

9. Your cousin works for a company that makes aluminum cans. A large soup can has a radius of 8 centimeters and a height of 20 centimeters. How much aluminum is used to make the can?
Appendix C- Student Interview Questions

Student interviews were focused on a subset of these questions. (Questions pertaining to past experiences with math manipulatives were not repeated in the second interview.)

1. Do you remember using math manipulatives in any math lessons before this year's geometry class? If so, do you remember what manipulatives you used and what math concept you were learning about when you used them? Explain.

2. Do you think using math manipulatives in the past helped you to understand the math concepts you were learning about? In what way(s)?

3. Do you enjoy this year's math lessons that include the use of physical or virtual manipulatives? Why or why not?

4. Does the use of math manipulatives in geometry class help you to better understand the math concepts and ideas that are being taught? Explain.

5. Do you have a preference for using either physical or virtual manipulatives? If yes, which do you prefer and why?

6. When you complete homework, test, and quiz questions, does it help you to think about the lessons you learned using manipulatives? Explain.

7. Has your attitude about working with manipulatives changed during your 8th grade year? Explain.

8. I would like you to work on this problem, saying aloud whatever it is you are thinking as you work through the problem. I especially want to hear you talk about how you decide what to do to solve the problem. If you choose to, you may use any of the manipulatives we have used in class to help you solve the problem.

9. Is there anything you want to know from me?

10. Is there anything else that you think it is important for me to know about you or your understanding of math concepts?