Teaching Teachers Mathematics

The Importance and Components of Quality Professional Development

Jim Lewis

University of Nebraska-Lincoln
What do you notice?

This was located in the upper left hand corner of USA Today.
The realities of scale
The Current U.S. Education System

Many K-12 students do not learn challenging mathematics

Those students enter college unprepared for college mathematics; the ones who do understand mathematics often do not enter teaching

Teachers are left to their own devices in terms of continuing their mathematical education

Those same prospective teachers are apprenticed in classrooms of teachers who themselves know little mathematics

Future elementary teachers take very few mathematics courses; those taken by secondary teachers do not prepare them to develop mathematical knowledge for teaching
The result

– Many teachers do not know the mathematics that they will now be required to teach

– Many teachers do not have an opportunity to learn that mathematics in preservice preparation or **professional development** in any methodical, sustained, planful way

Suzanne Wilson, former chair, Teacher Education at Michigan State, presentation at the 2011 CBMS Forum on Teaching Teachers in the Era of the Common Core
My view

Investing in high quality teachers is the single most important thing we can do to improve K-12 mathematics learning.
The Challenge We Face as We Teach Teachers

• What Mathematics do Teachers “Need to Know” and How Should They “Come to Know” Mathematics?

  – What does it mean to offer challenging courses and curricula for math teachers?

  – How do we help teachers translate the mathematics they come to know into classroom practice that leads to improved student learning?
Can you compute 49 times 25?

Of course:

\[
\begin{array}{c}
49 \\
\times 25 \\
\hline
245 \\
+ 980 \\
\hline
1,225
\end{array}
\]

Why might a fourth grader think the answer is 1,485?*

\[
\begin{array}{c}
49 \\
\times 25 \\
\hline
405 \\
+ 1080 \\
\hline
1,485
\end{array}
\]

* On May 4, 2010, Deborah Ball, Dean of the University of Michigan School of Education used this question as part of her testimony before the U.S. House of Representatives Education and Labor Committee.

\[5 \times (4+4) = 40\]

or

\[5 \times 4 + 4\]
What is so difficult about the preparation of mathematics teachers?

• Our universities do not adequately prepare mathematics teachers for their mathematical needs in the school classroom. Most teachers cannot bridge the gap between what we teach them in the undergraduate curriculum and what they teach in schools.

• We have not done nearly enough to help teachers understand the essential characteristics of mathematics: its precision, the ubiquity of logical reasoning, and its coherence as a discipline.

H. Wu, Professor, University of California, Berkeley
What is so difficult ....?

• The mathematics taught should be connected as directly as possible to the classroom. This is more important, the more abstract and powerful the principles are. Teachers cannot be expected to make the links on their own.

• Get teachers to believe, that mathematics is something you think about - that validity comes from inner conviction that things make sense, that mathematical situations can be reasoned about on the basis of a few basic principles.

• The goal is to have teachers develop flexibility in their thinking, to be able to reason about elementary mathematics.

Roger Howe, Yale University
Mathematical Education of Teachers II Themes

• There is intellectual substance in school mathematics

• Proficiency with school mathematics is necessary but not sufficient mathematical knowledge for a teacher

• The mathematical knowledge needed for teaching differs from that of other professions

• Mathematical knowledge for teaching can and should grow throughout a teacher’s career.

Available at: http://www.cbmsweb.org/
MET2 Recommendation 1

Prospective teachers need mathematics courses that develop a solid understanding of the mathematics they will teach.

- The mathematical knowledge needed by teachers is substantial yet quite different from that required in other mathematical professions.

- Prospective teachers need to understand the fundamental principles that underlie school mathematics.

- Coursework for prospective teachers should examine the mathematics they will teach in depth, from a teacher's perspective.
MET2 Recommendation 2

Coursework that allows time to engage in reasoning, explaining, and making sense of the math that prospective teachers will teach. Well-started beginning teachers need

- Elementary teachers — at least **12** hours on fundamental ideas of elementary mathematics.

- Middle grades (5-8) teachers — at least **24** hours of mathematics that includes 15 hours on fundamental ideas of school mathematics appropriate for ML teachers.

- High School teachers — the *equivalent of a major* that includes three courses with a primary focus on high school mathematics from an advanced viewpoint.
MET2 Recommendation 3

Throughout their careers, teachers need opportunities for **continued professional growth** in their mathematical knowledge.

- Satisfying the minimum requirements for initial certification to teach mathematics does not ensure that even outstanding future teachers have the knowledge of mathematics, of teaching, and of students that is possessed by successful experienced teachers.
- The need for professional development takes on increased importance due to the wide adoption of the CCSS.
- A reasonable goal for initial certification at the secondary level is to create beginning teachers who are able to teach competently a portion of the high school curriculum.
MET2 Recommendation 4

- All courses and professional development experiences for mathematics teachers should develop the habits of mind of a mathematical thinker and problem-solver, such as reasoning and explaining, modeling, seeing structure, and generalizing.

- Courses should also use the flexible, interactive styles of teaching that will enable teachers to develop these habits of mind in their students.

- To help their students achieve the CCSS Standards for Mathematical Practice, teachers must not only understand the practices of the discipline but also how these practices can occur in school mathematics and be acquired by students.
MET2 Recommendation 5

Teacher education must be recognized as an important part of mathematics/statistics departments’ missions and should be undertaken in collaboration with mathematics educators.

More mathematics/statistics faculty need to become deeply involved in professional development for teachers and become involved with local schools and districts. National and regional efforts are needed to help prepare mathematics/statistics faculty to contribute effectively to teacher education.

Mathematics/statistics departments must provide graduate level courses designed to meet the professional needs of PreK-12 mathematics teachers.
MET2 Recommendation 6

• Mathematics education, including the mathematical education of teachers, can be greatly strengthened by the growth of a mathematics education community that includes mathematicians as one of many constituencies committed to working together to improve mathematics instruction at all levels and to raise professional standards for teaching.
Operations and Algebraic Thinking (Kindergarten – Grade 5)

The different types of problems solved by addition, subtraction, multiplication, and division, and meanings of the operations illustrated by these problem types.

Teaching-learning paths for single-digit addition and associated subtraction and single-digit multiplication and associated division, including the use of properties of operations (i.e., the field axioms).

Recognizing the foundations of algebra in elementary mathematics, including understanding the equal sign as meaning “the same amount as” rather than a “calculate the answer” symbol.

- Recognize that commutativity for multiplication is not obvious and use arrays to explain why multiplication is commutative.
Middle Grades Teachers
Essential Grades 6 – 8 Ideas for Teachers

• **Ratio and Proportional Relationships** (Grades 6–7)
• Illustrative activities:

• **2.** Compare and contrast different ways to find values in proportional relationships and in inversely proportional relationships. For example, explain why linear interpolation can be used with proportional relationships but not with inversely proportional relationships.
High School Teachers
Introduction

The need for opportunities to engage in mathematical practices and develop mathematical habits of mind:

“teachers need opportunities for the full range of mathematical experience themselves: struggling with hard problems, discovering their own solutions, reasoning mathematically, modeling with mathematics, and developing mathematical habits of mind.”
High School Teachers

Introduction

Outline:

1. **Essentials in the mathematical preparation of high school teachers.**

2. **Important additional mathematics content that can be learned in undergraduate electives or in professional development programs for practicing teachers.**

3. **Essential mathematical experiences for practicing teachers.**
High School Teachers
Important Additional Mathematics

• “It is impossible to learn all the mathematics one will use in any mathematical profession, including teaching, in four years of college. Therefore teachers will need opportunities to learn further topics throughout their careers.”
“...teachers need experiences that renew and strengthen their interest in and love for mathematics, help them represent mathematics as a living discipline to their students by exemplifying mathematical practices, figure out how to pose tasks to students that highlight the essential ideas under consideration, to listen to and understand students’ ideas, and to respond to those ideas and point out flaws in students’ arguments.”
NSF Supported Professional Development

• Math in the Middle Institute
  – A master’s program for middle level (5-8) teachers
• Primarily Math
  – An 18-hour certificate program for K-3 teachers
• Nebraska Algebra
  – A 9-hour program for Algebra 1 teachers
• New Teacher Network
  – A 24-hour PD and mentoring program for new teachers
• Robert Noyce NSF Master Teaching Fellowships
  – A program for extraordinary master teachers
• Robert Noyce NSF Teaching Fellowships
  – A postbac master’s and certification program
Math in the Middle Instructional Model

**SUMMER**

- Offer 1 and 2 week classes.
- Class meets from 8:00 a.m. - 5:00 p.m.
- 35 teachers – 5 instructors in class at one time.
- Substantial homework each night.
- End-of-Course problem set
  - Purpose – long term retention of knowledge gained.

**ACADEMIC YEAR**

- Two-day (8:00 – 5:00) on-campus class session.
- Course completed as an online, distance education course using Blackboard and Adobe Connect.
  - Major problem sets
  - End-of-Course problem set
  - Substantial support available for teachers
Goal: Mathematical Knowledge for Teaching

• Teachers need specialized content knowledge:
  – Deep understanding of content
  – Representations and connections
  – Understand student thinking
  – Assess student learning
  – Make curricular decisions

• This type of knowledge is not typically gained through most pre-service mathematics programs (i.e., Ball, Thames & Phelps, 2008; NCTM, 2000)
A common theme: Courses for math teachers should emphasize solving problems to develop teachers’ Mathematical Habits of Mind

**Goals:** Give teachers experiences to develop their:

- Strategies for solving problems
- Flexibility in thinking
- An appreciation for the importance of precise mathematical definitions and careful reasoning
- Ability to explain solutions to others
- Persistence and self-efficacy
Our Definition:
Teachers who possess a rich set of mathematical habits of mind

1. Understands which tools are appropriate when solving a problem.
2. Is flexible in their thinking.
3. Uses precise mathematical definitions.
4. Understands there exist (therefore encourages) multiple paths to a solution.
5. Is able to make connections between what they know and the problem.
6. Knows what information in the problem is crucial to its being solved.
7. Is able to develop strategies to solve a problem.
8. Is able to explain solutions to others.
9. Knows the effectiveness of algorithms within the context of the problem.
10. Is persistent in their pursuit of a solution.
11. Displays self-efficacy while doing problems.
The Triangle Game

(Paul Sally, U. Chicago) Consider an equilateral triangle with points located at each vertex and at each midpoint of a side. The problem uses the set of numbers \{1, 2, 3, 4, 5, 6\}. Find a way to put one of the numbers on each point so that the sum of the numbers along any side is equal to the sum of the numbers along each of the two other sides. (Call this an Equal Side Sum Solution.)

– Is it possible to have two different Equal Side Sum Solutions?
– Which Equal Side Sum Solutions are possible?
– How can you generalize this game?
New math courses for middle level and high school teachers

- Experimentation, Conjecture and Reasoning
- Number Theory and Cryptology for teachers
- Using Math to Understand our World
- Math in the City for Teachers
- Algebra for Algebra Teachers
- Geometry for Geometry Teachers
- Functions for Precalculus Teachers
- High School Mathematics (I and II) from an Advanced Viewpoint
- Statistics for High School Teachers
Algebra for Algebra Teachers content

• The arithmetic and algebra of the integers, especially the (Euclidean) division algorithm, the Fundamental Theorem of Arithmetic and important school mathematics applications related to these results.
• The integers modulo \( n \) as a tool to broaden and deepen our knowledge of the integers.
• Defining polynomials, roots, polynomial functions, and polynomial rings;
• Special attention paid to linear and quadratic polynomials/functions in connection to their importance in school algebra (slope/rate of change, graphs, quadratic formula, etc);
• Comparing properties of \( k[x] \), \( k \) a field with \( \mathbb{Z} \), the integers;
• In \( k[x] \), Division algorithm, Euclidean algorithm and applications, unique factorization and applications;
• Irreducibility in \( \mathbb{Q}[x] \), \( \mathbb{R}[x] \) and \( \mathbb{C}[x] \) (Irreducibility tests); (brief overview of properties of \( \mathbb{C} \)).
Mathematical Knowledge for Teaching Algebra

Teachers need knowledge of mathematics that enables them to address a wide range of mathematical ideas and questions. Here are some questions that school algebra teachers might be asked.

1) My teacher from last year told me that I whatever I do to one side of an equation, I must do the same thing to the other side to keep the equality true. What am I doing wrong when I add 1 to the numerator of both fractions in the equality 1/2 = 2/4 and get 2/2 = 3/4?

2) My father (who is very smart) was helping me with my homework last night and he said the book is wrong. He said that $\sqrt{4}=2$ and $\sqrt{4}=-2$, because $2^2=4$ and $(-2)^2=4$, but the book says that $\sqrt{4}\neq-2$. He wants to know why we are using a book that has mistakes.

3) Why does the book say that a polynomial $a_nx^n + a_{n-1}x^{n-1} + \ldots + a_1x + a_0 = 0$ if and only if each $a_i = 0$, and then later says that $2x^2 + 5x + 3 = 0$?

4) I don’t understand why $(-3)(-5)=15$. Can you please explain it to me?
Side Sum Solutions for Hexagons

Side Sum 17:   3, 8, 6, 4, 7, 9, 1, 11, 5, 10, 2, 12
Side Sum 18:   None
Side Sum 19:   6, 2, 11, 5, 3, 9, 7, 4, 8, 10, 1, 12
                And   4, 10, 5, 8, 6, 2, 11, 1, 7, 9, 3, 12
                And   5, 11, 3, 9, 7, 4, 8, 10, 1, 6, 12, 2
                And   3, 9, 7, 11, 1, 10, 8, 6, 5, 2, 12, 4
Side Sum 20:   7, 11, 2, 8, 10, 4, 6, 9, 5, 3, 12, 1
                And   9, 3, 8, 5, 7, 11, 2, 12, 6, 4, 10, 1
                And   8, 2, 10, 4, 6, 9, 5, 3, 12, 7, 1, 11
                And   10, 4, 6, 2, 12, 3, 5, 7, 8, 11, 1, 9
Side Sum 21:   None
Side Sum 22:   10, 5, 7, 9, 6, 4, 12, 2, 8, 3, 11, 1
Patterns with Minimums & Maximums

<table>
<thead>
<tr>
<th>Polygon</th>
<th>Minimum Side Sum</th>
<th>To find the next Minimum</th>
<th>Maximum Side Sum</th>
<th>To find the next Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangle</td>
<td>9</td>
<td>+3</td>
<td>12</td>
<td>+3</td>
</tr>
<tr>
<td>Square</td>
<td>12</td>
<td>+2</td>
<td>15</td>
<td>+4</td>
</tr>
<tr>
<td>Pentagon</td>
<td>14</td>
<td>+3</td>
<td>19</td>
<td>+3</td>
</tr>
<tr>
<td>Hexagon</td>
<td>17</td>
<td>+2</td>
<td>22</td>
<td>+4</td>
</tr>
<tr>
<td>Heptagon</td>
<td>19</td>
<td>+3</td>
<td>26</td>
<td>+3</td>
</tr>
<tr>
<td>Octagon</td>
<td>22</td>
<td></td>
<td>29</td>
<td></td>
</tr>
</tbody>
</table>
A Solution for an n-sided polygon, n odd

• General solution for an n-gon where 
  \( n = 2k + 1, \) n odd
• For a Heptagon Solution, \( n = 7; k = 3 \)

To find the vertices begin with 1, move 
clockwise by k each time, and reduce 
mod n. The midpoints begin with 2n 
between 1 and 1+k and move 
counterclockwise, subtracting 1 each 
time. For a heptagon, the 
Side Sum = 5k + 4.