James Tanton’s Remarkable Paper Folding

Katie Soto  Grand Island Public Schools
Anne Schmidt Culler Middle School, Lincoln Ne
Have you seen this before?
This Fractal Starts from this…

Transformations

level 1 → Transforms to level 2
then to level 3

and so on…
Let’s start with something a little lighter today…

ONE FOLD

- Take a strip of paper and tape its left end to the table
- Pick up the right end to fold the strip in half
- A “valley” is created. Valleys have a value of 1
Two Fold

- What happens when we do this again?
- How many valleys?
- Are there any mountains?

- The sequence for the two fold is: 1, 1, 0
Now you try

- What is the sequence for a three fold?

- What about a four fold?
Do any patterns emerge?

- How many numbers are in each sequence?

- Can we predict how many numbers will be in a five fold?
So....

- One Fold: 1
- Two Fold: 110
- Three Fold: 1101100
- Four Fold: 110110011100100
Just a little change...

<table>
<thead>
<tr>
<th>Fold</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>One Fold</td>
<td>1</td>
</tr>
<tr>
<td>Two Fold</td>
<td>1 0</td>
</tr>
<tr>
<td>Three Fold</td>
<td>1 0 1 1 0 0</td>
</tr>
<tr>
<td>Four Fold</td>
<td>1 1 0 1 1 0 0 1 1 0 0 1 0 0</td>
</tr>
</tbody>
</table>
Question to ponder:

- What is the sequence of 1s and 0s for five folds?
- How many digits are in the one-hundred fold sequence? How many digits are 1s? How many are 0s?
- What is the 112th digit of the one-hundred fold sequence?