Creating Opportunities for Mathematical Practices
Common Core Standards of Mathematical Practice

1. **Make sense** of problems and **persevere** in solving them.
2. **Reason** abstractly and quantitatively.
3. **Construct** viable arguments and **critique** the reasoning of others.
4. **Model** with mathematics.
5. **Use** appropriate tools strategically.
6. **Attend to** precision.
7. **Look for** and **make use** of structure.
8. **Look for** and **express** regularity in repeated reasoning.
A Serendipitous Sequence of Events

• Fostering Geometric Thinking Toolkit materials
• Invitation to Lead Seminar Series for NYC Office of English Language Learners
“For English Learners to succeed in learning mathematics, they need to be more productive in mathematics classrooms—reasoning more, speaking more, writing more, drawing more.”

Maria Santos
Former Director, NYC OELL
A Serendipitous Sequence of Events

- Fostering Geometric Thinking Toolkit materials
- Invitation to Lead Seminar Series for NYC Office of English Language Learners
- José’s group’s chart paper presentation
- “Well, duh, it’s geometry! It’s all about visual representation.”
Research Project

Fostering Mathematics Success of English Language Learners (FMSELL project)—A study of the efficacy of the Fostering Geometric Thinking Toolkit materials among teachers of ELLs.

A collaboration of EDC and Horizon Research: National Science Foundation: DRL -0821950
A problem to think about:
Before the clip:

To the questions of the researcher:

• Each of two boys says he has found 2 points as vertices, by “looking opposite the points.”
• Each says he guesses there’s a third point (vertex), but he doesn’t know how to find it.
On the clip, Wei says:

“I found 3..”

“The first one I found because I saw…..”

“I found the other side from A….Each dot…..”
On the clip, Wei says:

“I found 3..”

“The first one I found because I saw…..”

“I found the other side from A….Each dot…..”
On the clip: “The first one I found because I saw…..”

After a minute or so (post-clip) she finishes with: “I saw if I put 2 triangles together, it made a parallelogram.”
For each point you found in #1, explain how you know the four vertices form a parallelogram.

- They have the parallel
- Has the same length with opposite side
- It has 4 angles, but opposite
- Has 2 angles are the same degree.
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Geometric Reasoning Tasks: Opportunities for Mathematical Practices to Develop…. 

As long as they are attached to appropriate academic language access and academic language development opportunities
"Broadly speaking I want to suggest that geometry is that part of mathematics in which visual thought is dominant whereas algebra is that part in which sequential thought is dominant. This dichotomy is perhaps better conveyed by the words 'insight' versus 'rigour' and both play an essential role in real mathematical problems.

"The educational implications of this are clear. We should aim to cultivate and develop both modes of thought. It is a mistake to overemphasise one at the expense of the other and I suspect that geometry has been suffering in recent years."

Sir Michael Atiyah
Oxford University
1982
The figure above shows two right angles. The length of $AE$ is $x$ and the length of $DE$ is 40. Show all of the steps that lead to finding the value of $x$. Your last step should give the value of $x$. 

$2007$ NAEP $8^{th}$ Grade---Only $1\%$ Correct
Figure A and Figure B have the same area. Show why this is true, in pictures then in words.
Aim: How do we compare area of \( z - 0 \)?

**Figure A**

\[ A = \frac{1}{2} \times b \times h \]
\[ A = \frac{1}{2} \times 6 \times 6 \]
\[ A = 18 \]

**Figure B**

\[ A = b \times h \]
\[ A = 3 \times 6 \]
\[ A = 18 \]

The areas of the two figures are both 18.
Other Considerations in Ensuring Opportunities for Practice Standards
Mathematics is “a system for dealing with the quantitative, relational, or spatial aspects of human life.” (Bill Barton, *The Language of Mathematics: Telling Mathematical Tales*)

‘Dealing with’ should include the use of diagrams
Research Project

Mathematics Coaching Supporting English Learners (MCSEL project)—Develop and study materials to support coaching of mathematics teachers of ELLs.

Institute of Education Sciences, U.S. Department of Education
MCSEL Guiding Principles

• EL students should have regular opportunities to engage in mathematics tasks that are *challenging* (high cognitive demand)

• Classrooms should incorporate regular use of *multi-modal representation*, particularly students making diagrams for quantitative tasks and modifying pictures in geometric tasks

• *Academic language opportunities* woven into mathematics learning opportunities

• *Instructional coaching* to support purposeful teacher implementation of these
Two ways to integrate mathematics and language in the classroom
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<thead>
<tr>
<th>LEVEL I</th>
<th>LEVEL II</th>
<th>LEVEL III</th>
<th>LEVEL IV</th>
</tr>
</thead>
<tbody>
<tr>
<td>Newcomer with little or no knowledge of English. Pre-production or silent phase.</td>
<td>Understands basic conversational English but lacks academic English skills.</td>
<td>Engages in conversational English but needs support in reading and writing.</td>
<td>Has command of basic English. Needs support in written expression.</td>
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Listening  
Writing  
Speaking  
Reading
Language Supports of Mathematical Practices Include:

- Use of **sentence frames**, particularly those that involve the vocabulary and sentence structures commonly used to express quantitative and spatial relationships in Academic English, as well as reasoning and generalizing about such relationships, such as “All (some) (no)……are…….” “If I use a 2-dilation on a triangle, then___changes and ___stays the same.”

- Use of **teacher think-alouds** about creating or modifying diagrams to represent the quantitative or spatial relationships in a problem.

- Organizing student engagement with problems by **think-pair-share**, with sharing focused on comparing and contrasting use of diagrams by students.
A Case for Diagramming

• Since solving most mathematics problems involves understanding quantitative and spatial relationships, the *language of quantitative and spatial relationships* is central to success.

• Mathematical diagrams are facilitators of understanding quantitative and spatial relationships and also are nonverbal representations of the language in which problems frame the relationships.
A Case for Diagramming, cont’d

• In order to use diagrams as a bridge between problem statement and solution, ELLs need skills in both *creating* diagrams (important for representing quantitative relationships) and *modifying* diagrams (important for representing spatial relationships).

• Diagramming can give teachers access to ELLs’ thinking about quantitative and spatial relationships they perceive in a problem. This opens the door for various language strategies by teachers, used in order both to assess and advance ELLs’ thinking.
PENNY’S MARBLES

(TIMSS Grade 8 Assessment task)

Penny had a bag of marbles. She gave one-third of them to Rebecca, and then one-fourth of the remaining marbles to John. Penny then had 24 marbles left in the bag. How many marbles were in the bag to start with?
TASK: Solve the problem by showing your work using diagrams or other forms of representations. Explain how you arrived at the correct answer.

**Penny’s Marbles**

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\[
\begin{align*}
\frac{1}{3} \times 48 &= 16 \\
32 - 16 &= 16 \\
32 &= 48 \\
\frac{1}{3} \times \frac{4}{1} &= \frac{4}{3} = 1\frac{1}{3} \\
24 + 16 &= 40
\end{align*}
\]
Reflection

I found this problem to be hard. All you had to do is split a piece of paper into thirds, then split it into 4 parts. Take one out and the 3 squares equal 24. If you divide 24 by 3 it will give you 8 and that means she gave 16 to Rebecca and 8 to John. She had 48 marbles in the beginning.
Penny had a bag of marbles. She gave one-third of them to Rebecca, and then one-fourth of the remaining marbles to John. Penny then had 24 marbles left in the bag. How many marbles were in the bag to start with?
We had a specific way of solving this problem. First, we made a rectangle with 6 squares. Then, we represented 3 squares as Rebecca's marbles. Then, we shaded 3 squares as John's marbles. Next, we multiplied $6 \times 3 = 18$. This was the number of marbles Penny started with.
Gloria tenía una bolsa de canicas. Dio un tercio de las canicas a Rebecca, y luego un cuarto de las canicas que se quedaron a Juan. Gloria luego tenía 24 canicas que se quedaron en la bolsa. Cuantas canicas fueron en la bolsa para empazar? Utilize una diagrama para asistirte en buscar la respuesta.

Gloria

Rebecca

Juan

24

\[ x = 418 \]

33.3

66.7

16.675

49.975

50%

48c
Diagramming Creates Opportunities for:

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Instructional Strategies for Creating Mathematical Practice Opportunities

- Model for students how to use the language of spatial and quantitative relationships—e.g., to describe procedures that generalize, without referring to specific geometric figures or specific numbers
- Use tasks that require reasoning and let students engage with them Think-Pair-Share
- Ask for explanations of thinking (Will that work for every triangle?) along with explanations of procedure (What did you do?)
- Help students learn to create, use, modify, and analyze mathematical diagrams
- Ask for diagrammatic, along with verbal, explanations
- Model and encourage multiple approaches to problems
- Let students compare and contrast multiple approaches—displayed side by side