**Slide 1**

“Evoking Effective Mathematical Practices”

A Break-out Session at the Conference

*Enacting the CCSSM Standards of Mathematical Practices*

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Thanks for this opportunity to talk with you today. I have been a professor of math for 40 years. For the last 25 years or so, much of my effort has gone to making undergrad education and graduate education work better at Rutgers. For just over 10 years I have been trying to make the mathematical education of teachers work better.

As a mathematician, I am accustomed to starting talks by stating a problem and laying out the underlying results and assumptions. Today my problem is has two parts: (1) how can we understand the CCSSM Standards of Mathematical Practice and (2) how can we help teachers to help students to make good use of these practices.

**Slide 2**

**N. J. Partnership for Excellence in**
**Middle School Math (NSF 0934079)**

Prepare a cadre of mid-career middle school math teachers to

- Understand mathematics more deeply
- Engage students more effectively in studying and learning math
- Take on roles assisting their colleagues

I am reporting on joint work by the full staff of the NJ PEMSM Institute. I'd particularly like to thank Cecilia Arias for technical help in editing the video-clips, Lou Pedrick for running the video camera, Raul Furnaguera for taking notes on enactments of the Core Standards, Lynda Ginsburg for helpful discussions, and all the teacher participants who have worked so hard with such good humor.

My assumptions will commonly – but not always – be introduced by “I believe”. My beliefs are based primarily on my own experience. In some cases there is even research evidence to support them. Of course, many of my beliefs have been modified by reports at meetings and by discussions in the profession.

The CCSSM call for students to become proficient in using certain mathematical practices. We might even call them the “Core Math Practices”. They are listed in the Standards for Mathematical Practices; I'll hand out a summary soon. When I say that these practices are effective, I mean that I believe that they help students learn mathematics and – more importantly – learn to use it not only to do textbook exercises but also to apply their mathematical knowledge to problems that arise not just in academic math or science or engineering, but also in the workplace, in the family, and in civic discourse.

Some writers have referred to these practices as “habits of mind”. But I prefer to use the term “effective mathematical practices” since two of those three words connote actions with outcomes, not just mental states.

I also believe that students will appreciate the value of these practices and make use of them to their own benefit only if their teachers do the same. I believe that we should not ask teachers to “model” the mathematical practices recommended by the CCSSM. If I had to do so to keep my job, I might model newly fashionable clothing that makes me uncomfortable. But my doing would not sell the fashions – potential buyers would see my discomfort. Instead,

I want teachers

- to **experience the power** of these practices in their own work,
- to **recognize their value**, and
- to **demonstrate that power and value** honestly and comfortably.

So can teachers employ these practices? And can teachers learn to recognize and evoke these practices in their classrooms? I have heard some folks answer “no”. I will offer some evidence that middle school math teachers can indeed do these things.

This evidence comes from the Summer Institute of the NSF-funded MSP Institute at Rutgers, of which I am the PI.

Our goal is to deepen the mathematical understanding of our participating teachers – not just to help them become more effective in their own classrooms but also to prepare them to take on roles such as math coach or math mentor to their district colleagues. Our partner districts – now 15, originally 7 – face both educational and economic challenges. They are losing their math specialists and their math supervisors due to budget cuts. The teachers you will see do not come from wealthy districts, but rather from districts ranging from blue-collar to poverty-stricken.

**Slide 3 - A still picture of Cohort 1 during a summer institute**

We bring in cohorts of about 25 mid-career teachers for a structured program of seven courses applicable to a Master of Education in Mathematics Education. Each cohort is in the program for about two years. Five of the seven courses are math content courses. The video-clips I will show and discuss come from the second course taken by the second cohort – it is called Number, Operation and Algebra.

We had not discussed the CCSSM standards explicitly with them at the start of the course. Instead, we had a daily schedule involving some exposition, some exploratory exercises, but lots of workshop problems to be done in groups of 4 or 5 teachers with time set aside for classroom connections.

**Side 4 -- A still picture of Cohort 2 during a workshop session**

Each morning we have presentations of two or three of the prior day's problems. Each group gets to present one problem every other day. Groups are encouraged to include several different points of view, methods, and explanations of their work. They are encouraged to think of variants and extensions of the problem – and to include comments on classroom connections – especially on differentiated instruction.

My first clip comes from Cohort 2 and from the second morning. It excerpts a report on a problem dealing with place value taken from Bassarear's text *Mathematics for Elementary School Teachers*.

## Slide 5 and Handout

**A Place Value Game** (From Bassarear: Exploration 2.10)

*Materials*

- A six-sided die with the 6 covered by a 0
- Game boards provided below. The grids have spaces for
  - a 4-digit number, a 3-digit number, a 2-digit number, and a 1-digit number.

*Rules for the competitive version*

1. Each player has a separate game board.
2. After each roll of the die, players decide where on the grid to write digit.
3. After ten rolls, each player determines the sum of the four numbers – that is, the 4-digit, 3-digit, 2-digit, and 1-digit numbers. The player with the largest sum wins.

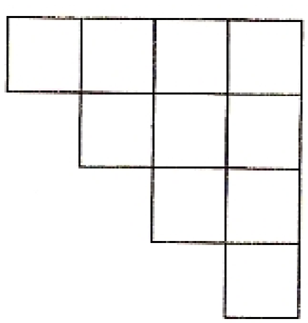
1. Play the first game as follows:
   a. Do not speak during the game.
   b. In the blanks provided on the game board, record the order of the ten throws of your die.
      For example, 5 3 4 0 2 4 5 2 3 4.
   c. After the first game, take some time to think about your choices. Also think about whether you learned something from playing the game or from watching the other players. Can you express what you learned in a way that would make sense to someone who has not played the game? Write down the strategies that you learned from the first game.
2. Play the second game as follow:
   a. Re-do the order of the ten throws of your die.
   b. Every time you write a number in the grid, explain why you chose that location.

3. Summarize your strategies for this game, and briefly justify them.

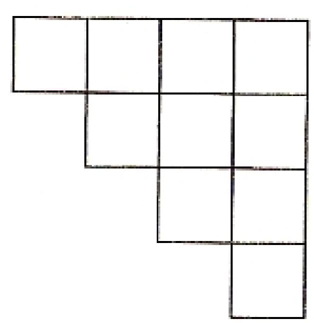
4. What did you learn from playing this game?

**GAME BOARDS**

1.



2.



Sum: ________________________ Sum: ___________________________

Let me distribute some handouts for the rest of the talk. [not included here] Before the bit of the presentation I will show you, the group ran through several ways to make sense of the problem and to suggest variations for differentiating instruction. Your handout includes a grid for you to follow one round of the game, so you can make sense of it yourself.

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**Slide 6 Play Clip 1** In this clip Kathy presents two mathematical extensions of the place-value game. First she wants to find the expected value of the sum of the four numbers; then she wants to find a strategy with the highest probability of winning. She admits that she doesn't recall much probability from college. Then she talks about an expected value of 2.5 for any one role of the die and suggests that the expected outcome will be the sum of

2.5 thousands + 2.5 hundreds + 2.5 tens + 2.5 ones
+ 2.5 hundreds + 2.5 tens + 2.5 ones
2.5 tens + 2.5 ones
+ 2.5 ones

for a total of 2500 + 500 +75 + 10, which is 3085.

She then describes her attempts to find the probability of getting exactly *k* fives in a sequence of ten rolls of the game die. She talks of thinking about this problem as she drove home – taking notes, but only at traffic lights! – then, thinking at a party and writing notes on a napkin, and finally thinking more as she drove back to Rutgers in the morning again taking notes as she drove.

If that doesn't suggest persistence, I don't know what would. [Lawyers' chorus:

The names are pseudonyms.] I'll show you bits of the rest of Kathy's presentation; it lasts just a few minutes.

**Slides 7, 8 -- T**hese clips show Kathy describing her false starts and her eventual decision that she was most interested in the probability that at least one 5 comes up in the ten rolls. Eventually she finds that the probability of at least one five is

$$(1 - \text{ probability of no 5's}) = 1 - (5/6)^{10} \text{, which is approximately } 1 - 0.16$$

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The computation of the probability of “no fives in ten rolls” provoked an energetic discussion of whether at least one five “must” come up in a string of 10 rolls of the die. But that is a topic for another day.

This group's presentation suggested to me the possibility that our teachers' presentations would show many of the Core practices. We took notes and found them all. Your handout lists the headlines of the eight CCSSM standards of mathematical practices.

We did eventually make and display a poster of these standards.

**Slide 9 - Listing of the CCSS Math Practices**

Occasionally I would point out a bit of a presentation that illustrated one or another of these practices. We never asked teachers to be sure to enact this or that effective practice because of the CCSSM. So in this context I claim that the teacher behaviors are relatively spontaneous.

[Note: The next two paragraphs describe what I intended to say – since my videos wouldn’t run, it is not what I actually said.]

Now the next clip I'll show is longer – in total about 7 minutes. This is still Cohort 2, only a week later. I will play the clip in sections and make some comments as we go. Please take some notes of what practices you see. To make it a bit easier, those on the left may want to concentrate on the first 4 practices on the list and those on the right may want to concentrate on the remaining four. Those in front might concentrate on the even numbered practices and those in back on those with odd numbers. When we are done I will ask for some reports from you and then add my comments if you haven't already made the remarks I have planned.

After that, I will wrap up my presentation by talking about what I think elicits these good practices and what that suggests about the mathematical education of teachers, both pre-service and in-service. I have planned to leave time for discussion at the end of our period.

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**Play Clips, [not available here]**

<u>Comment on David</u> – note his comfort in mentioning errors and false starts and changing points of view. CCSSM doesn't say so, but I believe it was Edison who noted that if you don't have lots of mistaken ideas, your aren't generating enough ideas in the first place. Mathematical practice is aimed at getting a correct answer and having good reasons to believe it – but we value the honesty to recognize errors and the willingness to learn from them and correct them.

David's presentation started very concretely but ended with a summary in algebraic notation and used some of the algebraic reasoning from the summer course.

Comment on Miguel -- We all enjoyed his starting to cover up our poster of the Standards of Mathematical Practices and his decision not to do so. He never looked at that poster again! He mentioned how in his Special Education math classes he would try to make the problem very concrete with an actual bucket of pencils – at which point one of his group-mates laid out two cups of pencils you see in the screen shot. He didn't want to handle the actual pencils – so he represented them as hash-marks as you see on his poster. 15 at the end needed to be ¾ of some number of marks – so he added the five to make the 20. These 20 pencils had to be 2/3 of some number of pencils – so he added another ten marks to get 30. These 30 marks had to represent half the original number of pencils – so he multiplied by 2 to get his answer 60.

Miguel's presentation was quite concrete. He intentionally avoided any algebra, but did engage in explicit quantitative reasoning.

Comment on Kathy – She worked hard to write out algebraic representation of the number of pencils at each step starting with the original unknown number x. She used handwriting to display her own thoughts. Then she talked about how she would present her reasoning to her students – with algebraic simplifications at each step. Later you may want to debate the relative merits of letting students struggle with messy notation and protecting them from the experience of struggle.

Comments on Alice – Here we saw an entirely different approach – rationalized guess-and-check. She pointed out that to start she would need a number bigger than 15 so there could be 15 pencils left at the end. Then she tried out 20 as a first guess – and introduced the idea of a decision tree to record the actions and the acceptability of the guess and subsequent check. After removing half of the 20, she was left with 10 and could not remove one-third of them without violating the implicit assumption that the problem required only integer numbers of pencils. She tried 30, 40, 50 with reference to and increasingly familiar structure of reasoning. Finally she hit on 60 as the solution to the Pencil Problem. She noted that in class she would make connections ideas of common multiples and arithmetic progressions and so on.

Alice made use of several of the Core Math Practices, especially use of appropriate tools and recognition of patterns of reasoning.

**Closing remarks**

I will argue on very partial data, that we should offer more opportunities for mid-career teachers to reconsider the mathematics they teach and to deepen their understanding. A new teacher is necessarily concerned with the mechanics of classroom management, following pacing guides, etc. New teachers must believe that they know enough content to do their job acceptably. In many cases – certainly not in all cases – teachers may not be “ready” to reconsider what they know and how they know it until they have several years teaching experience and the self-confidence they have built up during those years. We should try to find ways to use Professional Learning Communities, or Math Teacher Circles, or whatever opportunities can be found in

AfterSchool Teacher Academies, to help teachers help each other deepen their mathematical knowledge for teaching.

While I believe that knowing the math itself and using the Core Math Practices are necessary conditions for a teacher successfully helping children become proficient with these practices themselves, I must quickly add that a necessary condition is rarely a sufficient condition. Teachers must also acquire extensive instructional practices to help kids value and use good mathematical practices in their learning and later in their adult lives.

We have a few more minutes for discussion. Who'd like to say something?

For more info on my own grant go to **www.math.rutgers.edu/NJPEMSM**

Thanks!