



Enacting Standards for Mathematical Practices in NebraskaMATH

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Mathematical Practices

- Have you ever had two students (or do you know two teachers) who appear to know the same “facts” but for whom there is a marked difference in their ability to use that information to answer questions or solve problems?

Why?

- Do mathematical thinkers approach problems differently?
- And, if so, how do we develop the “habits of mind of a mathematical thinker” in teachers and assist them in cultivating this knowledge among their students?



Mathematical Habits of Mind

- People with productive/effective mathematical habits of mind are good problem solvers & communicators.
- “Habits of mind” refers to a way of productive mathematical thinking that is creative and persistent in solving problems.
- We see “mathematical habits of mind” as congruent to the CCSS-M Standards for Mathematical Practices



Mathematical Habits of Mind

- Across all of the mathematics courses in all of our programs is an overarching goal of helping K-12 mathematics teachers develop the habits of mind of a mathematical thinker
- Mathematical habits of mind represent an enriched view of what it means to do mathematics
 - Orientations mathematicians bring to their work
 - Expectations for mathematical understandings for preK-12 students (dimensions of mathematical proficiency, NCTM process standards, and CCSS-M Mathematical Practice Standards)



Mathematical Habits of Mind

1. Understands which tools are appropriate when solving a problem.
2. Is flexible in his or her thinking.
3. Uses precise mathematical definitions.
4. Understands there exist (therefore encourages) multiple paths to a solution.
5. Is able to make connections between what he or she knows and the problem.
6. Knows what information into the problem is crucial to its being solved.



Mathematical Habits of Mind

7. Is able to develop strategies to solve a problem.
8. Is able to explain solutions to others.
9. Knows the effectiveness of algorithms within the context of the problem.
10. Is persistent in his or her pursuit of a solution.
11. Displays self-efficacy while doing problems.
12. Engages in a meta-cognition by monitoring and reflecting on the processes of conjecturing, reasoning, proving, and problem solving.

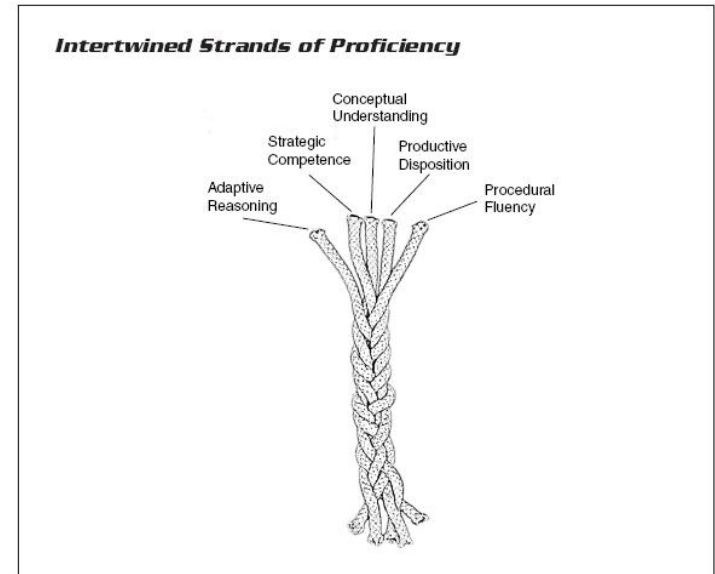


Mathematical Practices

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Box 4-1



NCTM Process Standards

- **Problem solving**
- **Reasoning and proof**
- **Communication**
- **Connections**
- **Representations**

Math in the Middle (2004-2011)

- 12 course master's program (7 math, 1 stats, 3 education, 1 integrated capstone)
 - 156 graduates from over 60 districts
- Trying to prepare “well-rounded” middle level teachers by developing their mathematical habits of mind
- We would argue these courses focus on the mathematical practice standards before this particular collection of practices was written down
- Many course materials online

<http://scimath.unl.edu/MIM/coursematerials.php>



Math in the Middle Courses

1. Mathematics as a Second Language
2. Functions, Algebra, and Geometry for Middle Level Teachers
3. Experimentation, Conjecture and Reasoning
4. Discrete Mathematics for Middle Level Teachers
5. Number Theory and Cryptology for Middle Level Teachers
6. Using Math to Understand Our World
7. Concepts of Calculus
8. Statistics for Middle Level Teachers
9. Inquiry Into Teaching and Learning
10. Curriculum Inquiry
11. Teacher as Scholarly Practitioner
12. Integrating the Teaching and Learning of Math



- New Teacher Network: for teachers in their first three years as secondary mathematics teachers
 - 24 credit hours of math & education courses across 3 years
 - Master teacher mentors/liaisons
- Nebraska Algebra: focused on Algebra 1 teachers
 - 9 credit hours: Math, Education, Educational Psychology; across 1 year
 - Coaching/mentoring support during AY
- Primarily Math: K-3 Mathematics Specialist Certificate
 - 18 credit hours: 3 math, 3 education; optional leadership course (mainly for coaches); across 14 months
 - Ongoing involvement in study groups for 2 years



Primarily Math

- Math content courses planned carefully for K-3 focus
- Planned prior to CCSS-M—utilized NCTM Focal Points to determine content
- NCTM Process Standards are deeply embedded
 - Multiple representations
 - Communication
 - Reasoning and justification
- Cooperative learning strategies
- Utilizes DMI & S. Beckmann materials



Teaching Fellowships

- STEM graduates who want to become high school math teachers in high-need Nebraska schools
- 41-credit hour, 14-month Master of Arts with an emphasis on Mathematics Teaching
- 9-month internship with Master Teacher
- Receive free tuition, \$20k during the MAmt program, \$10k/year for 4 years teaching in a high-need Nebraska school

Master Teaching Fellowships

- 24 master teachers who provide leadership to Nebraska's high-need districts to help close Nebraska's large achievement gap
- Teachers agree to continue teaching in high-need districts for 5 years
- Receive 24 credit hours free tuition, \$10k/year for 5 years, opportunity to teach on NMSSI instructional teams
- Mentor new teachers & Teaching Fellow interns



Nebraska Math and Science Summer Institutes

- Graduate education funded with Nebraska dollars held at locations across the state (began with Math in the Middle courses, adding NebraskaMATH courses, and creating new courses)
- Most courses are paired math & pedagogy
- Want to offer on-going opportunities for Nebraska teachers' professional development
- UNL has agreed to reduce NMSSI tuition by 20%; NMSSI constituted 16% of graduate enrollment at UNL in Summer 2011
- We have pursued external funds (State Farm, Pfizer, Time Warner) to provide fellowships to further reduce tuition costs to teachers



Nature of Graduate Instruction

- High expectations combined with sufficient support
- Belief that effective learning must be active
- Focus on cooperative learning (modeling effective pedagogical skills)
- Focus on building participants' *mathematical habits of mind* and *pedagogical habits of mind* (and in certain courses *educational researcher habits of mind*)



Instructional Model

- 2-5 heads are better than 1
- Most math courses involve mathematicians, math graduate students, and master teachers
- Most education courses are taught by educational researchers, mathematics education graduate students, and master teachers
- Some NMSSI courses taught by master teachers and graduate students with a faculty advisor
- Summer courses are 1-2 weeks, 8am-5pm with 3-4 hours of homework and a substantial end-of-course assignment



Habits of Mind Examples

There is a jug of wine and a kettle of tea. A spoonful of tea is taken from the kettle and poured into the jug. The mixture is thoroughly stirred and a spoonful of the mixture is taken from the jug and poured into the kettle. Is there more tea in the jug or more wine in the kettle?

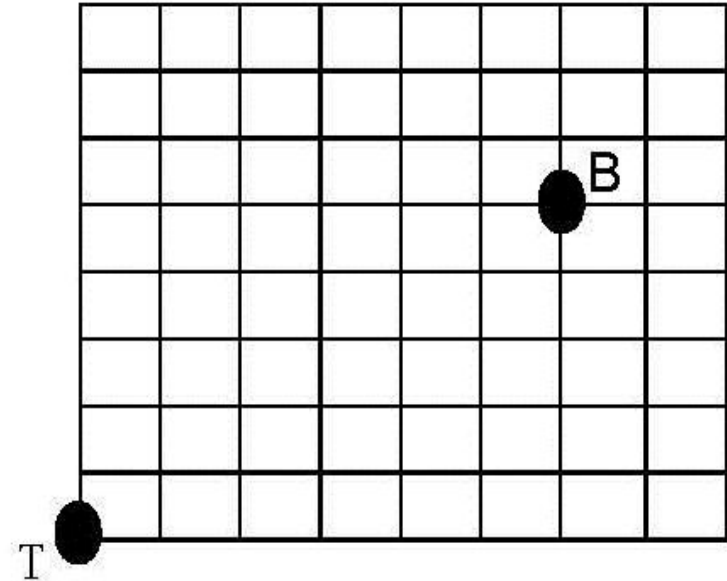


Habits of Mind Examples

Mathville is laid out as a square grid of North-South streets and East-West streets (See diagram).

Your apartment is located at the Southwest corner of Mathville. (Point T). Your math classroom is in a building that is 6 blocks East and five blocks North of your apartment. (Point B)

It is an 11 block walk to math class and that there are no short cuts. Your roommate, Curious Georgia, asks how many different paths (of length 11 blocks – you don't want backtrack or go out of your way) could you take to get from your apartment to the math class.



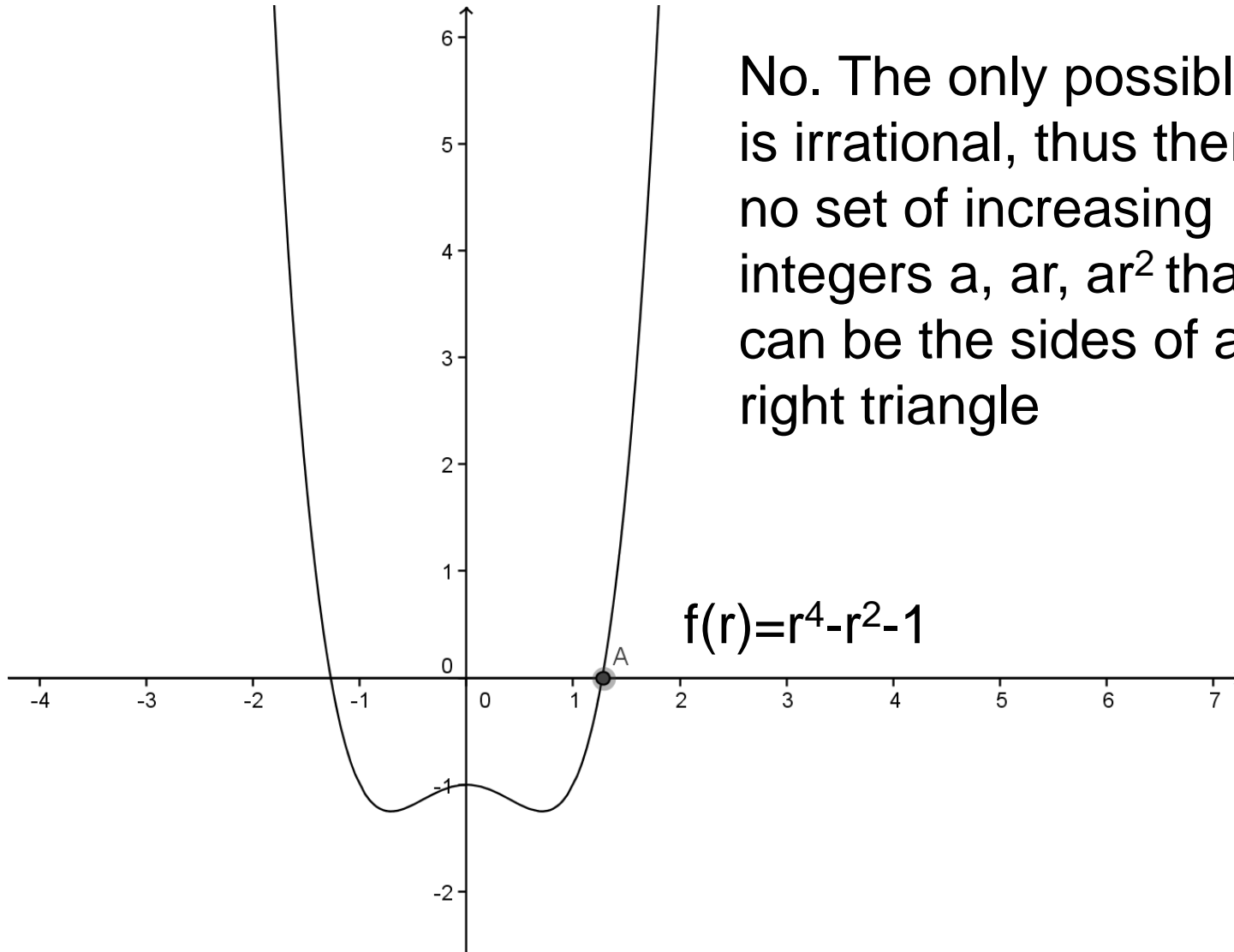
Solve Curious Georgia's math problem and give a careful explanation as to why your answer is correct.

Habits of Mind Examples

- We know how to find a right triangle with consecutive integer sides, that is, in an arithmetic progression. Is it possible to find a right triangle with integer sides which are in an increasing geometric progression, that is integer sides of length a , ar and ar^2 with $r > 1$? Explain why or why not.



Habits of Mind Examples



No. The only possible r is irrational, thus there is no set of increasing integers a , ar , ar^2 that can be the sides of a right triangle



Habits of Mind Examples

A version of this problem was asked on the NPR program
Car Talk:

Wendy went to the store to purchase ink pens for three classes of students. She found three kinds of pens. The first cost \$4 each; the price of the second kind was 4 for \$1; and the cost for the third kind was 2 for \$1.

- **The Challenge:** For each class, is it possible to make a purchase so that the cost of the pens (in dollars) equals the number of pens purchased and with the restriction that she purchases at least one pen of each type?
- Try this for 20, 15, and 33 students in a class. Try this for the number of students in your class.



Mathematical Practices

- What mathematical practices did you employ in reasoning about these problems?
- How might your students approach these problems?
- How similar are these problems to those you might pose to your students?
- What might you learn about your students in doing this type of problem in class?



Resources for Good Math Items

- Open-Ended Assessment in Mathematics
 - Example
- Create a unit of length so that the pencil would be more than 2 units long but less than 3 units long. Explain how you decided on the length of your unit.



- <http://books.heinemann.com/math/>
- Can purchase 1 license for the district

Nebraska **MATH**

*A partnership to improve
mathematics achievement*



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