Infants and Toddlers as Mathematical Thinkers

A Tribute to Carolyn Edwards

UNIVERSITY of NEBRASKA–LINCOLN
The principal investigators of Math Early On are Ruth Heaton, Carolyn Pope Edwards, and Victoria Molfese, along with investigators Jenny Leeper Miller, Wendy Smith, Jessica Namkung, and Lisa Knoche.

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“Questions about what teachers can and should do can never be finally answered but rather must keep returning to the starting problem: What kind of teachers are needed by our children—those real individuals in the classrooms of today?”

– Carolyn Edwards
# Table of Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Math Early On</td>
<td>3</td>
</tr>
<tr>
<td>Mathematics in early childhood</td>
<td>6</td>
</tr>
<tr>
<td>Counting with props</td>
<td>9</td>
</tr>
<tr>
<td>Counting on and back</td>
<td>11</td>
</tr>
<tr>
<td>Exploring loose parts</td>
<td>14</td>
</tr>
<tr>
<td>A feel for numbers</td>
<td>17</td>
</tr>
<tr>
<td>The concept of take away</td>
<td>19</td>
</tr>
<tr>
<td>Sorting a collection</td>
<td>22</td>
</tr>
<tr>
<td>The great outdoors</td>
<td>24</td>
</tr>
<tr>
<td>Matching and moving</td>
<td>28</td>
</tr>
<tr>
<td>Using floor books</td>
<td>31</td>
</tr>
<tr>
<td>Pathway to comparing</td>
<td>34</td>
</tr>
<tr>
<td>Leadership team and participants</td>
<td>37</td>
</tr>
</tbody>
</table>

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“Collaboration does not simply mean the distribution of work, but giving value to others’ perspectives.”

– Carolyn Edwards
MATH Early On

Math Early On, funded by the Buffett Early Childhood Fund, involves a partnership between the University of Nebraska–Lincoln (UNL) and the Educares of Nebraska, part of a national network of high-quality child care centers for low-income children from birth to age 5.

The goal of the Math Early On project is to offer professional development opportunities that build on the past successful professional development efforts of UNL’s NebraskaMATH and its Primarily Math initiative. The Primarily Math curriculum for primary teachers was adapted and redesigned for Math Early On to create new experiences for the purpose of enriching preschool teachers’ mathematical knowledge for teaching. Based on work with preschool teachers in 2014-2017, the Math Early On professional development offerings were expanded and adapted to include teachers of infants and toddlers during 2017-2019.

Math Early On identified the Early Math Collaborative at Erikson Institute in Chicago as doing important work in improving mathematics instruction for young children. Their textbook, *Big Ideas of Early Mathematics* (2013), became one key element of professional development for Math Early On. Besides math instruction, however, other key elements include: mathematically rich environments (indoors and out), integrating math with literacy activities, home-school partnerships, and pedagogical documentation. Together, these key elements provide a comprehensive approach to make a culture of math visible and alive throughout the day in programs serving young children.

In the first several years of Math Early On, during 2014-2016, Educares of Omaha at Indian Hill and Kellom were the

Teacher Jennifer introduces loose parts to Naveah, Alayna, and Linda for sorting into sets and creating simple patterns. Loose parts are a vital type of material for introducing infants and toddlers to mathematical concepts at an early age.

Teacher Lyndsay uses a loose parts approach to help toddlers explore shapes and colors. This booklet features stories from the live work in classrooms of infant and toddler teachers over the last several years at Educare of Lincoln, as well as at the Ruth Staples Child Development Laboratory at the University of Nebraska–Lincoln.
sites of the work. The team created a booklet representing that work called *Learning the Language of Nature: Young Children as Mathematical Thinkers* (2016). David Hawkins’ philosophy of “messing about” with materials and ideas inspired Math Early On’s approach to supporting the learning of both teachers of young children and their students. Given the impact of Hawkins on the team’s views of teaching, learning, and the youngest of learners, the pages of the first booklet were flanked by quotes from Frances and David Hawkins as well as other educators who were either informed by or informed Hawkins’ ideas.

The first booklet tells some of the powerful learning stories that emerged in the Educare of Omaha at Indian Hill and Kellom and the Ruth Staples Child Development Laboratory at UNL. In the mode of the Hawkins traveling exhibit, *Cultivate the Scientist in Every Child: The Philosophy of Frances and David Hawkins*, and of Italian style documentation, the stories combine images, description, and teacher interpretation to reveal, unpack, and share moments and processes of early intellectual discovery.

In 2016-2019, Math Early On began work with the teachers at Educare of Lincoln and kept the overall focus of the professional development similar. However, the team collaborated closely with teachers and leaders at the Educare of Lincoln to adapt the format and content emphases of the professional development to be responsive to the site’s needs and schoolwide goals.

The years 2017 and 2018 were a time of transition. There were staff changes at both the Educare of Lincoln and UNL. Dr. Ruth Heaton, the project’s principal investigator (PI), took a professional leave from UNL, and Dr. Carolyn Edwards, a co-PI, became seriously ill. Dr. Victoria Molfese and Jenny Leeper Miller became the project’s local leaders and enlisted additional UNL colleagues to help implement the professional development, including Dr. Wendy Smith, associate director of the Center for Science, Mathematics, and Computer Education, Dr. Jessica Namkung of the Department of Special Education and Communications Disorders—both with expertise in math education; and Dr. Lisa Knoche of the Center for Research on Children, Youth, Families and Schools, with expertise in home-school partnerships. During that same time, there was also a substantial turnover in staff at the Educare of Lincoln, a phenomenon common to early childhood school settings. Subsequently, Heaton has become chief executive officer of Teachers Development Group, a nonprofit in West Linn, Oregon, supporting preK-12 math education nationwide, and Edwards passed away after a long illness on May 31, 2018.

The leaders of Math Early On adapted the professional development to support the math learning needs of infants and toddlers and individualized professional development to meet the needs of small groups of teachers from 2017 to 2019. Some of these teachers had experienced multiple years of Math Early On professional development, and others were new to the professional development. Individualized side-by-side classroom support of teachers while engaged in the real work of teaching and a focus on documentation of classroom activities targeting math learning have characterized the most recent Math Early On activities, guided by teachers’ deepening understanding of the *Big Ideas of Early Mathematics* (2013).

This second booklet features stories from the live work in classrooms of infant and toddler teachers over the last several
Carolyn Pope Edwards (1947-2018) was a professor at the University of Nebraska–Lincoln for 18 years and a renowned expert in how young children learn. She earned both her undergraduate and doctoral degrees at Harvard University. Before coming to Nebraska, Dr. Edwards taught at Vassar College and the Universities of Massachusetts and Kentucky.

Edwards’ life mission was raising the quality of young children’s education by understanding how children develop and thrive cognitively, socially, and morally. In tireless pursuit of that mission, she traveled extensively to Italy, China, and many other countries, educating childcare professionals and giving countless public addresses, workshops, and teacher training sessions. She published 18 books and numerous articles and book chapters.

Through her study of the pioneering approach to early childhood education developed in the schools of Reggio Emilia, Italy, she became an advocate for the Reggio model and was instrumental in its adoption to improve learning environments for children. Her book *The Hundred Languages of Children: The Reggio Emilia Approach to Early Childhood Education*, written with George Foreman and Lella Gandini, documented the many ways in which children communicate their thoughts and ideas and how the schools of Reggio foster learning through that communication. *The Hundred Languages of Children* was translated into numerous languages and became a central point of reference for those who work in early childhood education. Her work has had particular impact among Chinese educators and in reforming Chinese orphanages.

Among many other honors, she won the University of Nebraska Outstanding Research and Creative Activity award, its highest research honor, as well as the Lifetime Achievement Award from the North American Reggio Emilia Alliance.

The Carolyn Pope Edwards Fund at the University of Nebraska Foundation has been established to carry on her work of advancing the quality of education of young children.

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years at Educare of Lincoln and at the Ruth Staples Child Development Laboratory. Like the earliest of work of Math Early On in Educares of Omaha, represented in the stories of the first booklet, these stories also recognize and celebrate infants and toddlers as mathematical thinkers and doers.

Not only does the Math Early On team want this booklet to serve as a celebration of the ongoing commitment and dedication to the math learning of the young children at Educare of Lincoln, but also as a tribute to friend and colleague Carolyn Edwards. The quotations in this booklet are from the writing of Carolyn and reflect not only her philosophy and approach to early childhood education, but also form the framework for the design of Math Early On professional development.

Carolyn’s vision of what Math Early On could be many years ago, coupled with her wisdom of the cognitive, social, and emotional development of young children and her expertise with the Reggio Emilia approach to early childhood education, has shaped the form and content of Math Early On professional development. Her relentless focus on the reasonableness and brilliance of the young child has permeated all of Math Early On’s work with teachers of young children.

In Carolyn’s absence, the work has continued and her presence has been felt. Her sage advice on noticing the formal and informal day-to-day math teaching and learning happening in Educare classrooms helped everyone to learn to more spontaneously participate in child-initiated mathematical activities and understand the value of material-rich classroom environments for fostering young children’s learnings.

– Ruth Heaton
Although the typical adult may assume that mathematics at the early childhood level is simple, in fact early mathematics is quite complex. For the Math Early On project, some of the main mathematical concepts included: number sense, number operations, sets, counting, patterns, and shapes. For each of these, there is a proximate mathematical learning trajectory. Although the topics are presented separately, the concepts are all interconnected in the mathematical learning of young children.

COUNTING
At its earliest stages, counting focuses on one-to-one correspondence—the idea that you say one number per object being counted. Next is the concept that when you are counting, the last number you say is how many objects there are. For 3- and 4-year-olds, counting may start to be associated with written digits, to match numbers to their symbols. Also along the counting trajectory are counting backward, skip counting (such as counting by 2s or 10s), subitizing (recognizing the sizes of sets without counting one by one), and counting on (starting with the last number counted to name the next number).

SETS
Grouping objects into sets is an important mathematical skill. Sets of objects share a common attribute, such as blocks with only straight sides, blue shirts, leaves, or the objects of a particular type that fit inside a given container. Children like to sort objects, and it is important for teachers to help provide language for sorting tasks, to help children articulate the common attributes used for sorting sets. When sets have a predictable structure (such as dots on dice), children can begin to subitize: tell how many objects are in a set without counting.

NUMBER SENSE
Early number sense helps children develop numerical literacy and intuitions, which eventually lead to a robust understanding of base 10 place value. For the smallest children, the emphasis here is on vocabulary: more, less, bigger, smaller, same. Teachers should ask children to compare (Which is more? Which is less?), estimate, and order numbers. Ordinal numbers (first, second, third) are also part of building children’s number sense and mathematical literacy.
NUMBER OPERATIONS

Early number operations are closely connected to counting, sets, and number sense. With very young children, teachers might count a set of objects (e.g., spoons, blocks), then add one or two more to the collection, and then count again with children. Preschool aged children will start being able to count on—start with the first total and count from there to the new total—and count back (for subtraction). It is appropriate for children to count on their fingers, and for number operations at this level to involve concrete objects.

PATTERNS

Mathematical patterns involve any predictable regularity involving number, space, or measure. Patterns are closely related to sets and counting. All patterns involve repetitions following a rule. Patterns can be detected via all five senses and can relate to numbers, sorting, nature, colors, shapes, rhythms, books, routines, weather, dancing, architecture, etc. Human brains look for patterns. Young children can be encouraged to sort sets by attributes and to create and recognize patterns in the world around them. The trajectory of patterns moves from simple (ABAB) to more complex patterns (ABCDABCD).

SHAPES

Everything has a shape; certain shapes have special names with precise mathematical definitions. Young children can see, label, and compare shapes in their environments, and learn basic properties of common shapes (e.g., triangles have three straight sides). Shapes are part of the spatial reasoning trajectory, and include relative position (e.g., above, below, next to, inside, under). Young children should also get practice composing shapes and recognizing both the whole and parts of composed shapes.

Tiny Tidbit

To read more about mathematics concepts and trajectories, see Big Ideas of Mathematics (2013) and Clements & Sarama's (2009) Learning and Teaching Early Math: The Learning Trajectories Approach.
“Teachers’ actions are not expected to take place in a set order, or one time only, but instead to repeat in cycles of revisiting and re-representation.”

– Carolyn Edwards
Infant teachers Jennifer and Lyndsay recognize that young children need a wealth of experiences counting collections of objects to help them begin to understand rules of counting long before they can orally count. How does this understanding happen when young children have limited oral vocabularies? Young children need to understand that: (a) there are counting words, (b) each object is counted only once, (c) objects can be counted in any order, and (d) the last counting word said is the number in the set. Teachers Jennifer and Lyndsay have found ways to support young children in this process.

A book the teachers like to use for counting is *5 Little Ducks* by Annie Kubler, illustrated by Penny Ives (2002). The five little ducks each climb on to their mother’s back one by one, providing an opportunity to count forward to 5. Then the ducks wander off, one by one, providing an opportunity to count backward from 5 to 1.

Teachers Jennifer and Lyndsay created a hand puppet of the mother duck and laminated paper ducks backed with Velcro, so that the children and teachers could act out the counting forward and backward offered by the book.

Being able to add ducks onto the back of the hand puppet

A child helps remove a duckling from Teacher Lyndsay’s hand puppet, while Emmanuel, Alayna, and Jordan, aged 13-18 months, look on. Each child took turns adding or removing ducklings, while they read the book together and acted it out with the props. The physical props to go with the book help to make the adding and subtracting concepts more concrete for very young children.
and take them off one by one is a way for a young child to show that they are beginning to understand that counting ducks means adding one duck at a time to the hand puppet and counting backward means removing one duck at a time from the hand puppet. While oral language skills are developing, toddlers may not yet have the vocabulary to express math ideas. Teachers of young children need to be keen observers and look for evidence of when young children are showing that they understand. Young children need repeated opportunities to show what they know in multiple ways.

**Tiny Tidbit**

Counting on and counting backward are segues to number operations. While one would not explicitly solve addition or subtraction equations with toddlers, the concept of counting on (i.e. plus one) and counting backward (i.e. minus one), leads young children to begin thinking about the meaning of addition and subtraction long before the formal words or symbols of addition or subtraction are introduced. Children at an early age can recognize that repeatedly adding one makes a set bigger and repeated taking away an item eventually leaves a set with none.

Teachers and parents of young children need to find ways to help children show what they know or what it is they receptively understand, when developmentally they are unable to tell you what they know. Early childhood teachers can create opportunities to help children understand counting and number operations through the use of stories and manipulatives that allow children to show what they know.

After removing ducklings, Teacher Lyndsay counts the remaining ducklings with the children, and emphasizes there are fewer ducks now, using mathematical vocabulary with children to build up their receptive language, and later, their expressive language.
along the counting trajectory, once children can mostly recite numbers, or use number words in a constant order, they need to develop the idea that when a person counts, the last number said when assigning one number name to each object in a set is the number of objects in the set. As young children develop one-to-one correspondence between saying numbers and counting objects, they eventually can reach more sophisticated counting strategies.

Counting on is when a child counts one set, and then starts from that total to count an additional set. In the book *Mouse Count* by Ellen Stoll Walsh (1995) a snake finds groups of sleeping mice to eat and counts on as, one by one, the snake adds the mice to a jar to eat later. After finding the first three mice, for example, the next group of mice, which happens to be a set of four, are counted as “4, 5, 6, 7.” For most young children, such counting on is developmentally appropriate and matches where they are on the mathematical trajectory.

Counting on and counting backward are also a segue to number operations. The concept of counting on, illustrated by adding mice one by one to the jar, leads a child toward the concept of adding. Similarly, counting backward and taking mice away one by one leads children to think about the concept of subtraction long before formal subtraction is appropriate.

Teacher Marcia used *Mouse Count* as a tool for helping toddlers make sense of number operations. Marcia developed a set of props for herself to use while reading the story and she later gave a set of props to each child to show what they understand about the
math in the story even if they are unable to clearly explain with words what is happening.

Teacher Marcia was able to adapt the activity to match the children’s levels of understanding. For example, Elijah was able to count on from the first group of three mice when a second group of four mice was found. He counted on from the last number said. Evielyn counted how many mice were in a small group, first “1, 2, 3”; then “1, 2, 3, 4” and then “1, 2, 3,” starting over with “1” each time. The third child, Rayon, was working on assigning one number name to each mouse she put in the jar, in no particular order.

The following year, Teacher Marcia worked with the younger toddlers (15-18 months) in the infant classroom. She hid single mice around the room and children used sock puppet snakes while they hunted for the mice. One by one, Teacher Marcia dropped each mouse into the jar while the children counted aloud as a group: “1, 2, 3, 4, 5, 6, 7, 8, 9, 10.”

Teacher Marcia wanted the Mouse Count activity to also encompass a school-home connection. She invited parents into the classroom and made time for parents and children to make the materials together. Teacher Marcia also read the story with the parents using the props, so they would know how children may want to re-enact the story when they got home.

Teacher Marcia introduces the book to Elizabeth (20 months) and her mother to help the mother become familiar with the book and using the prompts with Elizabeth at home.

Teacher Marcia gave each family a copy of the book and materials to make their own snake and set of mice so that they could act out the story at home. Here Raphael (22 months) and his dad make mice so that they are ready to start picking up mice using the sock snake.

Tiny Tidbit

Focusing on both oral and receptive language during Mouse Count activities are an important part of understanding number operations—counting as each mouse is dropped into the jar; talking about how many mice are in the jar as each group of mice is rounded up by the snake (first three mice, then four more mice, then three more mice); and then counting as each mouse escapes from the jar until there are no mice remaining. Teacher Marcia made mice of different sizes and colors; future activities can involve counting the number of big mice compared to small mice or gray mice compared to black mice. Teachers can get creative with the props they use by finding different materials and having an art project where children create different mice. The new materials and the children’s creations of different mice can be used in repeated Mouse Count activities.
“The teachers’ role is to help children discover their own problems and questions.”

– Carolyn Edwards
Infants and toddlers are able to explore numerous math concepts by engaging with loose parts in the classroom. Loose parts are natural and synthetic materials that do not have set directions as to their use and can be manipulated in multiple ways (Beloglovsky & Daly, 2014). Children can gain a better understanding of number sense, sorting, patterns, and shapes by using loose parts.

When loose parts are in the classroom it is important to start with a few items and work in more as children are comfortable. Loose parts should be stored in labeled containers and be placed in an area in which the children are familiar.

Teacher Mollie presented Crosby (20 months old) with new material at the Ruth Staples Child Development Laboratory. Teacher Mollie brought out a mug tree and different sized wooden rings. Without giving verbal directions, Teacher Mollie set the mug tree in front of Crosby and slowly began to put the wooden rings on the pegs. Crosby stood and watched closely. Teacher Mollie handed Crosby a large ring. He studied the large ring and looked at the other rings on the tree. He reached out and put the large ring he was holding on the peg with the other large rings. Teacher Mollie watched as Crosby continued to grab the large rings from the tray. She described his actions as he worked: “You are putting the large rings together,” “You have another big ring.”

After putting multiple large rings on the tree, Crosby began to put the smaller rings on the tree. Again, he picked up a small ring, studied the tree, and selected a peg with other small rings on it. The peg became full and Crosby tried to place one more
ring on the peg, but the rings came tumbling off. Teacher Mollie encouraged Crosby by saying, “Uh oh! They fell down. What should we do?” Crosby picked up the rings and tried to place the rings back on the peg to see if he got different results. After several attempts he splits the small rings between two pegs. Crosby was able to explore spatial relationships and sorting throughout the activity with the loose parts.

One day at Educare, the children and teachers noticed pine branches on the ground. Teacher Rachel took the pine branches into the classroom and provided the infants (ages 8-13 months) with a provocation of the pine branches placed in a paint dish. The children were then encouraged to paint using the pine branches. As the children paint, Teacher Rachel drew Harmony’s attention to the markings in the paper: “Look at the paint, look at the lines.” Harmony notices the lines and points to the paper. Teacher Rachel is preparing children to recognize lines. This skill will later develop into awareness of shapes. By adding yellow, Teacher Rachel uses two different colors to begin to help children recognize and create a visual sense of varying attributes.

**Tiny Tidbit**

When you introduce a new loose part into the classroom it is important to help orient children to the material. Be an active part of modeling for children how the loose parts can be used. Play along with them and use the loose parts in your play as a teacher. It is also important to verbally encourage and challenge children with the loose part asking questions such as, “What else can you do? What else do you notice? What do you think might happen if you tried this? What if you turn it this way?”

Crosby (in photos on page 14) uses loose parts on the mug tree to sort and classify different size rings. He attempts to place all of the large rings on one branch and all of the small rings on another branch. Harmony (in photos on page 15) is intrigued by the marks left by the paint on the pine branch. Harmony notices the change in her painting when yellow paint is added.
“Even a daily work session should ideally contain sticking points, or ‘knots.’ ... Any problem that stops the children and blocks their action is a kind of cognitive knot.”

– Carolyn Edwards
A feel for numbers

Counting is one of the most common math activities seen in early childhood classrooms and in activities at home with young children. Active engagement in counting activities multiple times throughout a day supports young children in learning to recite number names in numerical order (i.e., developing the skill of rote count), using number names to count objects in a collection (rational counting), and to answer the question, “How many?” are in a set.

Infant and toddler teacher Rachel used the book, *From Head to Toe* by Eric Carle (1999), to intentionally introduce young children to different animal actions (e.g., stomp your foot like an elephant, arch your back like a cat, thump your chest like a gorilla) and to imitate those actions as a means of working on counting. The book’s regularity is engaging for young children: “I am [animal] and I can [motion]. Can you do it?”

Teacher Rachel gathered 2- and 3-year-old children together on the rug in the classroom and brought out a large soft die numbered with large dots from 1 to 6. She asked children to take turns rolling the die and to use the number shown on the die to indicate the number of times to act out an animal’s action (e.g., three crocodile hip wiggles as a match for three dots on the die). As Teacher Rachel read each page of the book, Three-year-old Ronan is counting the dots on one face of the die using one number name for each dot as he touches the dot, while Zayli and Aspen observe. This strategy helps Ronan keep track of the dots as he counts them, “1, 2.” Knowing that each dot has one and only one number name is a fundamental skill of early counting.

Eighteen-month-old Linda matches the picture of the animal shown on the die with the picture of the animal in the book.
she stopped and had a child role the die. She supported the children throughout the activity by encouraging them to count the dots on the die, pointing to each dot while saying one number name and then counting aloud together as children acted out the animal actions that number of times.

In another activity, Teacher Rachel used a different die with pictures of a different animal on each face of the die. Playing with this die offered children opportunities to match an animal on the die to the picture of the animal in the book. Matching is an important skill for young children to develop as a precursor to later grouping objects into sets.

While the children may not yet have spoken language for all animal names, they are able to associate movements with the animals. So, if they rolled the die and it showed a gorilla, the children would thump their chests, just like the gorilla does in the book. This kind of early matching is an important precursor to later ideas of congruence.

Infant and Toddler teacher Jennifer is also using a die to help toddlers count. Here, she is using her finger to mark each dot to be counted. This child is learning about cardinality when Teacher Jennifer asks her “How many times should we arch our backs like a cat?” and the child is able to name the last number counted.

**Tiny Tidbit**

Teachers at Educare of Lincoln are intentional about reading the same books to children as they move each year from one classroom to the next. As *From Head to Toe* is read to children as they move from infant to toddler classrooms, the math concepts explored through the books can be adapted to children’s age levels. By reintroducing this activity when the children are older (i.e., the next school year), the older children are more able to use number names to count objects on the die, act out the number of movements accurately, and to answer the question “How many?” Further, with preschool-aged children, the die can be used to support subitizing—so the children can recognize how many dots are on a face of the die without counting them. The predictable patterns of dots for numbers on dice support developing the skill of subitizing.
The concept of take away

Some books immediately lend themselves better than others to math concepts for young children. *Pete the Cat and His Four Groovy Buttons* (Litwin, 2012) is one such book. Teacher Jennifer used it as a way to help young children become aware of changing quantities, and in the case of *Pete the Cat*, the expectation that there will be fewer buttons than before with each passing page of the book.

During one math experience, Teacher Jennifer read the book to a group of toddlers (20-33 months of age). As she made her way through the book she asked the children to do gestures illustrating one of Pete’s buttons popping off as they sang together, “My buttons, my buttons, my four groovy buttons… Oh no, one of the buttons popped off and rolled over. How many buttons are left?”

It is an important idea for children to understand that “take away” always means the quantity will be smaller, or the set will be decreased in size. This important idea allows children to think about the reasonableness of answers. If Pete has three buttons on his jacket and one pops off, will he have four buttons or two buttons? Knowing that take away always decreases the size of the set helps children reason about the answer to that question. Throughout the story, Teacher Jennifer helped children understand the idea of decreasing numbers in the set by holding up four fingers and putting one down every time one of Pete’s buttons popped off his jacket.

*Pete and Cat and His Four Groovy Buttons* also has repeated regularities that allow young children to anticipate what happens next. Knowing that a button popping off means a button being taken away, and that take away means the number of buttons decreases, enables children to predict what will come next as they listen to the book being read to them. Reading the book repeatedly across several weeks supports young children in learning the pattern of repeatedly taking one away.

That children understood the pattern and could predict what would happen was evident after they heard the book read several times. As Teacher Jennifer read the book, the children anticipated the loss of a button with each turn of a page and readily answered the question, “How many buttons were left?” By the end of the book, when the last button had popped off, the only button that remained was Pete’s belly button. Before Teacher Jennifer could turn the final page, Meredith (35 months) had raised her shirt to show her belly button, just like Pete the Cat does on the last page.
of the book. The children laughed loudly when Pete ran out of buttons and only had his belly button left.

In this school, teachers loop with their children so teachers have opportunities to teach the same children for multiple years. This looping allows teachers to reintroduce activities periodically, knowing that there is mathematics that can be learned from the activities, particularly as children develop richer language skills. With older infants, the emphasis on *Pete the Cat and His Four Groovy Buttons* was to hear the regularity in the story and predict what would come next. As they became toddlers, the teacher asked children to count the buttons each time and model the book by using foam shapes on paper as buttons. With toddlers, the teacher also can emphasize the vocabulary of taking away and the idea of decreasing the number in the set by one each time a button is taken away. This activity helps children understand that if they start with four buttons and take one away each time, they will eventually run out of buttons. Repeatedly taking objects away reduces the number in the set to zero, which is an important concept to understand about repeated subtraction.

Teacher Jennifer is emphasizing counting on each page by having Linda help point to the buttons while she uses her fingers to count the buttons. Alayna, Linda, and Meredith (with some support) also count on their fingers to represent the buttons on each page. The teacher is using her fingers to emphasize how taking away one button one finger away until there are no fingers held up.

**Tiny Tidbit**

By helping children apply math concepts during book reading activities, teachers can make the math concepts more interesting to children. Teachers can help children recall the story (i.e., “Remember what happened in *Pete the Cat*?”) and draw children’s attention to counting, changing the quantity of sets by adding some or taking some away. Children’s literature often embeds math concepts in obvious—and not so obvious—ways. Books allow young children to demonstrate their understanding of what mathematics are possible to work on within each individual book and think deeply, for example, about principles of counting, attributes of sets, and changing quantities. Teachers can learn to recognize the mathematical complexity and multiple math concepts frequently found within a children’s story.

What some may think is a simple, straightforward story found within a children’s book is actually an important opportunity for young children to learn about multiple, critical mathematical ideas in an engaging way.
“Although learning is a serious matter, the teacher must approach it in a spirit of playfulness as well as respect.”

– Carolyn Edwards
Sorting a collection

Sorting is one of the earliest encounters many young children have with math. Infants try to make sense of their world by attempting to group similar things together and pushing away items that are not similar. These actions are examples of early sorting behaviors. As children gain experience, their sorting becomes more meaningful, and their grouping of objects into categories becomes increasingly complex. Sorting is an important concept because grouping objects creates opportunities to recognize common attributes and compare one set to another and also allows children to start thinking about questions related to quantity. Which set has more, and which set has less?

Teachers Jennifer and Lyndsay used loose part materials with 13- to 18-month-old children, allowing the children to play with the materials. The teachers observed several toddlers sorting materials into sets. The toddlers placed milk bottle caps and large buttons into egg cartons. Some toddlers enjoyed placing lids and buttons into the egg cartons seemingly randomly, whereas other toddlers focused on sorting and making sets of a particular color (pink). One toddler began to construct a simple alternating color pattern (green, pink, green, pink, green, pink; an ABAB pattern). The teacher encouraged the toddler to recognize and continue working on the pattern by asking her, “What color comes next?” The child then successfully placed the next color in the pattern into the carton.

During the activity Teachers Jennifer and Lyndsay also supported children by helping them count the objects they were sorting. After Teacher Jennifer helped a toddler count the pink objects placed in the egg carton, the toddler counted the green items on her own. Teacher Lyndsay also helped two toddlers make patterns with the items they were placing in the egg cartons.

The following year, when the toddlers were 2 to 3 years old, Teachers Jennifer and Lyndsay looped with their children and chose to revisit this sorting activity. The teachers presented the toddlers with larger-sized egg cartons (increasing from 6-egg to 12-egg cartons) so the children could explore a wider variety of sets and patterns. Once they created the pattern, the toddlers were encouraged to count how many items of each color they have. Teacher Lyndsay extended learning by changing the colors of the pattern from pink and green to blue and green. Once again, children’s learning was scaffolded by teachers asking questions such as, “What comes next?” and “How many do you have?”

Teachers Jennifer and Lyndsay included sorting activities outdoor by having children collect things from the playground, such as plants, rocks, and bits of mulch. Then, the children were allowed to sort their collections. Talking with children as they sorted their collections allowed the teachers to scaffold children’s thinking about attributes of the objects in the set, to compare sets created by one child to those of other children, and to raise questions about set quantities.
Linda and Emmanuel point to the green and blue colored tops. Young children gesture when they want to communicate what they are thinking. While children may lack vocabulary to describe the attributes, a teacher can see what is understood about attributes by observing what children do when given a collection of objects and asked questions.

Tiny Tidbit

Some children may immediately sort collected objects into different categories based on attributes like shape, size, color, or texture; other children may create displays of the objects that reflect objects arranged in a particular order that repeats or patterns. Teachers can capture children’s work by taking photos and sharing the photos with parents in a newsletter (or to accompany other documentation) for parents. Families also can be invited to collect suitably sized items at home and bring them to school. Children can then talk about the collections and how they can be sorted in sets or arranged patterns. These collected items are useful resources in the classroom for supporting a variety of different types of learning experiences. Teachers can ask children to collaboratively sort the items into baskets for future use in art activities and other experiences in the classroom.

Children collected various plant parts (leaves, flowers, stems) outside, and then sorted them by different attributes.
Alayna can notice that she is jumping along an ABC pattern even when the color order changes. When she jumps forward (left photo), the pattern is blue, yellow, red. Then, when she turns to jump the other direction (right photo), the same ABC pattern is now red, yellow, blue.
Patterns are created by grouping or ordering objects using a repeated and predictable rule and can be found in many forms around us. Young children learn about patterns through repeated experiences sorting objects and making sets. By providing lots of opportunities for young children to arrange objects in lots of different ways, they learn to sort, make sets, and create patterns.

Teachers of young children may point out examples of patterns created with manufactured or naturally occurring objects and name them for young children (i.e., leaf, rock, leaf, rock, leaf, rock [ABABAB…] or leaf, rock, rock, twig, leaf, rock, rock, twig [ABBCABBCABBC…]). By naming the pattern, young children are supported in learning to recognize patterns wherever they occur.

When sorting sets, young children are recognizing shared attributes among objects. They often make patterns without teachers prompting them; teachers can observe closely and use questions to help understand what patterns children might be making and then ask questions to help children articulate or notice the attributes being used to generate their patterns. Teachers can ask, “Does this fit in your pattern?” as they disrupt a child’s pattern by adding a different object or creating a different order of objects, such as adding a large blue block to a child’s pattern of alternating small yellow and blue blocks or alternating a red ball with a small blue block. Teachers can ask children “What comes next?” as a way of finding out if the children understand the attributes or rules that guide a particular pattern.

One day, Teacher Lyndsay was outside with a classroom of toddlers about 24 months in age. She had tubs for children to explore, and she set them up in an ABC pattern (red, yellow, blue, red, yellow, blue). Alayna used the tubs to jump from one color to the next.

As the children were jumping, Teacher Lyndsay helped them identify the colors in the pattern by saying, “Red, yellow, blue, red, yellow, blue.” After two repetitions of the ABC pattern, Teacher Lyndsay changed the order of the buckets to blue, yellow, red, blue, yellow, red (CBA). This rearrangement of the buckets reversed the colors of the pattern but the ABC pattern remained the same, three colors in a particular order repeating. Teacher Lyndsay helped the children notice the change by identifying the colors in the new pattern—blue, yellow, red, blue, yellow, red. Teacher Lyndsay could then help the children recognize the pattern was still an ABC pattern (three different colors of tubs repeating) in both directions, even though the color order changed.

Once children can create ABC patterns, the teacher can help children see the structure of the mathematics by understanding how blue, yellow, red and red, yellow, blue are both ABC patterns. Even though the colors are in a different order, the mathematical pattern (ABCABCABC) is three different colors in an order, with that set of three repeated. Teacher Lyndsay could challenge students to each create different patterns, and then have students walk around to study one another’s patterns and decide which are truly different.
(e.g., ABAB vs. ABCABC) and which are permutations of a single pattern structure (ABCABC vs. CBACBA).

The children could recognize that jumping forward across the tubs (red, yellow, blue) versus jumping the other way (blue, yellow, red) shows that these are actually the same pattern type (ABC). To make a different type of pattern, the children could add some repetitions (e.g., ABBC), or make a pattern with only two colors of tubs (ABABAB). Teachers could also ask children to collect objects outside (such as rocks or leaves), and ask the children to make an ABC pattern with their found objects.

The jumping itself is also a pattern, and relates to developing one-to-one correspondence with children. The sound of feet jumping on a tub, one jump per tub, making an audible thump, and saying one color word per tub while jumping combines hearing, seeing, feeling, and large motor skills. Teachers could extend a jumping pattern by asking children to jump twice on the yellow tub, and once on the blue and red buckets, thus creating a new pattern out of jumps while saying color names, “yellow, yellow, blue, red, yellow, yellow, blue, red.” By having the children recognize the new jumping pattern, the teacher is supporting children in recognizing that patterns can occur in many forms.

Tiny Tidbit

One particularly helpful activity is going on a “pattern safari” at school, a nearby park, or at home. Children can hunt for objects on their safari and gather a collection. The teacher can ask children to sort their materials into sets and then make a pattern. The children can be supported by being asked to describe the attributes by which materials are being sorted, and how, or helping the children articulate the attributes in their collections or sets.

Teachers should look for any hint the child may be beginning to make patterns with the sorted materials. As children begin to create patterns, the teacher can copy the pattern the child is making. As skills increase, the teacher can provide a provocation by disrupting or expanding the pattern, inserting a new object in the pattern, removing objects from the pattern, or changing the order of objects to align with a different attribute to see how the child reacts. Does the child notice? If so, what does the child notice? Even before children can verbally express pattern attributes, they may remove an item an adult inserts into the pattern if the child sees this as disrupting the pattern.

Teachers can document the results of the pattern safari experience by photographing the patterns children created and using the pictures in later lesson plans to talk with children about the patterns they created and to learn more about what is or is not a pattern and extending or changing patterns.

Alayna collects leaves and twigs on her pattern safari. Back in the classroom, the children used their collected items to sort objects into sets and to make patterns with leaves, twigs, and rocks.
“Listening is the basis for any learning relationship.”

- Carolyn Edwards
  discussing the work of Carlina Rinaldi
Everything around us has a shape, and some shapes have names. It is important for infant and toddlers to have experiences in exploring different shapes and opportunities to learn the names of those shapes. Teachers can help children to learn the common properties of shapes—such as, what makes a shape a “triangle”—and, through exploration, how to compare shapes.

During one afternoon in a younger toddler room, Harmony (13 months old) was seated on the floor and saw the plastic lids for the stamp pad containers. While the teacher intended for the children to use the stamp pads to create some art, Harmony was more interested in the lids. She picked up several of the hexagonal lids and examined them.

Harmony turned the lids over and around and then ran her finger around the lids to feel the shape. Once Harmony had explored the lids individually, she began placing the lids inside one another. She rotated the lids until their sides
matched. Her happy smile reflected her feelings of success when she was able to make some of the lids fit together. While this young child did not yet know the name of the shape she was manipulating, she was exploring the concept of congruence or matching, by fitting the lids together.

Teacher Rachel noticed what Harmony was doing and chose two ways to connect her exploration to mathematics. First, Teacher Rachel helped Harmony apply language to her task, by pointing out, “You’re matching. You noticed you can make those lids match.” Then, following up with the concept of matching, Teacher Rachel offered Harmony a circular lid to go with her set of hexagon lids. Harmony picked up the circular lid and compared it to a hexagon lid. After rotating the lids, Harmony flung the circular lid away. While nonverbal, Harmony clearly had decided the circle lid did not match the attributes of the congruent hexagon lids. While probably not yet understanding the properties of hexagons, Harmony could determine that the circular lid did not have the straight sides that the hexagonal lids did. Early matching experiences like this will help Harmony to later understand object congruence.

Tiny Tidbit

Sometimes teachers create experiences for children based on an idea or a concept that is not necessarily focused on math, such as this activity where the teacher was targeting children’s fine motor skills through using the stamp pads to create art. However, by noticing the child’s play with the lids and allowing the child time to explore the lids, the teacher followed the child’s lead, which did involve a mathematical exploration.

Early ideas of matching lead to more complex mathematical concepts of congruence and properties of objects and sets. Disrupting a child’s pattern or set of matched objects by offering a different shape can be a way to explore mathematics with the very youngest children.

Opportunities to learn math can happen throughout the day and by looking for math in children’s explorations, teachers help children see, name, compare, and create math concepts as they play.

Harmony (in photos on page 28) is discovering properties of the lid. She runs her fingers around the outside of the lid to feel the shapes. Harmony (in photo on page 29) discovered that she could nest the lids together if she got them aligned just right.
“Yet another version of responsive teaching involves providing a next occasion for the children to follow their conjectures or probing children’s drawings of how something works to help them clarify their theories.”

– Carolyn Edwards
An important part of providing meaningful math experiences for children is documenting what happened. The teachers at the Educare of Lincoln have been using floor books adapted from Warden (2006) to highlight the math experiences in their classrooms. Floor books are a mechanism to engage children, teachers, and parents in the learning process through the use of pictures, open-ended questions, and the child-produced artifacts. Children are able to participate in the documentation of classroom activities by using markers, stamping with painted shapes, and working with teachers to add comments to pictures. Floor books provide opportunities to engage children higher-level thinking activities and collaborative learning.

Using floor books

The floor books are on the floor so that children can easily use shapes and paint to make their own pictures, and the teachers document their work in photos. For infants, noticing pictures in floor books started in different ways—for one infant, sitting on the floor book provided an ideal perspective while others looked at pictures while seated on the floor.
In the infant classroom, the children and teachers were very interested in examining size. The children explored items that were different sizes using classroom materials. As the children were exploring, the teacher helped the children notice objects that were big and those that were little. Teacher Rachel’s pictures of the activity were added to the floor book. She then placed the book on the floor and let the children explore the book. The children scribbled in the book, others scribbled on the pictures of themselves, and some children were content with long looks at the pictures. Teacher Rachel drew the children’s attention to what they were doing in the picture and talked with them about the big and the little objects.

Teacher Rachel discovered new ways for the infants to explore the floor book by putting glue on the pages and encouraging infants to tear colored tissue paper and put the colored pieces on the pages. As the children were tearing the tissue paper, Teacher Rachel drew their attention to the different sizes of the paper by saying, “Look, that blue piece is very big and that yellow piece is very small.” They also discussed the colors of the pieces and wrote the names of the colors on the page as the children were gluing on the pieces. 

The infants used their floor book to further explore the mathematical experiences in their classroom. As the children scribbled on and around the pictures, Teacher Rachel drew their attention to different sizes by using the words “big” and “little.”

This floor book page illustrates how infants explored colors and sizes. Teacher Rachel put glue on the page and, as children were tearing the paper, she drew their attention to the different sizes and colors of the pieces of paper.
“Amongst the many goals of professional development, fostering a culture of reflection and teachers’ abilities to reflect is an important one. Reflection takes time.”

- Carolyn Edwards
Math concepts can occur spontaneously during classroom activities. The sequence of events during activities creates opportunities for teachers to follow the children’s lead and to help them find materials in the classroom to explore their ideas. By incorporating many of the big ideas together, teachers are able to teach many math concepts at once to young children. This celebrates each math idea, while following the child’s lead.

One day in the older toddler classroom, Teacher Caron created a road using masking tape and a pond using colored butcher paper, covered with transparent contact paper. The contact paper extended beyond the butcher paper, and some air bubbles were trapped between the contact paper and the floor and butcher paper. The road had a straight segment and a roundabout, with a pond at one end. The teacher created some paper “fish” that were all the same size and color and taped them on the pond. The classroom floor already had some shapes taped on it, and the road crossed and went nearby some of these shapes. The roundabout is a traffic phenomenon that the city has been increasingly installing. Children would go through a roundabout near the school every day on their way to comparing.
to and from school; thus, the concept was one within the children's realm of experiences.

Teacher Caron created this situation to encompass many different mathematical ideas. Teacher Caron's intent of the lesson was to have the children count the fish in the pond. However, Ayham, Ky'mani, Monica, and Haevan noticed other mathematics that then led to richer mathematical questions. Because Teacher Caron had a focus on incorporating mathematics into classroom activities, the shapes on the floor were part of the routine for lining children up (e.g., go stand on the orange triangle), and she intentionally used position words (“Can you park on the star?”). The shapes on the floor provide a rich mathematical environment for children lining up: not only are there shapes that are different colors, but teachers also can use ordinal language (“The child standing on the star is first”). After setting up a mathematically rich environment, Teacher Caron was able to follow the activity lead of a child (which actually involved physically climbing in a box with the child) and to intentionally and repeatedly use mathematical vocabulary words with children.

Haevan (26 months old) asked Teacher Caron to go on an adventure with him. They both squeezed into a cardboard box (car) and drove it down the road. When they reached the roundabout, the teacher asked the boy to park on the star and to get a toy boat off the shelf.

They then continued to drive toward the pond. Once they reached the pond, Haevan “parked” his car, and then drove his boat on the pond. There, the teacher and Haevan counted the fish in the pond. They repeated the car trip down the road to the pond several times. The teacher continued to use position words, such as the car on the road, parking on the star and then next to the pond, the boat on the shelf and then float on the pond, and fish in the pond. On one visit to the pond, Haevan noticed there were bubbles between the contact paper and the floor. Teacher Caron helped Haevan count the bubbles. Haevan noticed that there were bubbles of different sizes—some small and some...
large. When the teacher and Haeven were counting the fish (all similar), a main question to prompt mathematical thinking is “How many?” When the teacher and Haeven were counting the bubbles with different sizes, there were opportunities for more types of questions: “How many big bubbles are there?”, “How many little bubbles are there?”, “Are there more big bubbles or little bubbles?”, or “Are there enough bubbles so each fish can have its own bubble?” Comparing two different sets is an important mathematical skill, and a child who can count (as Haeven can), is ready to advance along the mathematical trajectory by using numbers to compare the relative sizes of sets (which has more?).

On their last trip to the pond, Teacher Caron brought over some sorting items depicting aquatic life. Haeven immediately found all of the whales and lined them up in the pond. Teacher Caron noticed he had sorted a subset out of the larger set, and extended his play by stating, “You picked out the whales and placed them in the pond. How many do you have?” The boy then counted the whales to see how many were there. The teacher could also ask the child to compare: “Are there more whales or more fish?” In this case, Haeven has lined up five whales, and there are also five fish in the pond. For a child who is not as strong of a counter, the teacher could ask the child to match up the whales with fish, physically moving the whale blocks in one-to-one correspondence with each fish. “Can you make pairs of whales and fish? Can you give every whale a fish to eat? Are there enough fish? What does that tell you about how many whales and how many fish?” The teacher could prompt the child to add more of the aquatic objects, further counting and comparing the sets.

Throughout the experience, the teacher talked about the different math concepts, such as spatial awareness by noting the car’s position along the road and the location of the triangle, heart, and star shapes; counting the objects placed in the pond; using words to identify size and shape names; and the grouping of objects into sets.

After several trips to the pond Teacher Caron took a basket of animal creators off of the shelf and encouraged Haeven to add them to the pond. Haeven selected all of the whales and lined them up in the pond. Teacher Caron encouraged Haeven and once he was done they counted the whales.

Tiny Tidbit

By following the child’s interests in the driving game and commenting on the experiences they were sharing, the teacher was able to introduce a variety of different math concepts into the activity. The availability of a range of open-ended materials in the classroom helps children integrate these materials with their activities to add both richness and variety of concepts into their experiences. By allowing time for the driving game and the pond activity to be repeated, the teacher provided opportunities for the child to show and extend his learning.
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Infants and Toddlers as Mathematical Thinkers:
A Tribute to Carolyn Edwards

References


p. 16 (quotation on page 2), p. 15 (quotation on page 33)


p. 3 (quotation on page 27)


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