Learning the Language of Nature: Young Children as Mathematical Thinkers
The principal investigators of Math Early On are Ruth Heaton, Carolyn Pope Edwards, and Victoria Molfese. The grant is supported by the Buffett Early Childhood Fund. Additional support is provided by the University of Nebraska-Lincoln, including the following: College of Education and Human Sciences; Center for Science, Mathematics, and Computer Education; Nebraska Center for Research on Children, Youth, Families, and Schools; Department of Teaching, Learning, and Teacher Education; Department of Child, Youth, and Family Studies; and the Ruth Staples Child Development Laboratory.
“In so far as nature and our experience exhibit discreteness and multiplicity, arithmetic is a mirror of nature. It abstracts, and exhibits, an essential character of things.”

- David Hawkins
David Hawkins, philosopher, and his wife, Frances Hawkins, early educator, influenced a generation of progressive educators in the United States through the 1960s and 1970s. Especially in the fields of science and mathematics education, they made a lasting mark for learning based on wonder, curiosity, and engagement. David Hawkins felt strongly that children have a built-in affinity for the natural world; that is, the physical world that is all around us, including the human-built environment. He believed that mathematics, properly understood, is the language of that world, holding the key to the discovery of the wonders within it.

Cultivating Curiosity

David Hawkins made two trips to Reggio Emilia, a city in northern Italy known for its innovative and excellent system of public early childhood education. Under the leadership of the founding director, Loris Malaguzzi, the system evolved in the aftermath of World War II from a parent cooperative movement into a city-run system of preschools serving children from birth to age 6 and their families. The system exercises a leadership role in educational innovation in Italy and Europe, and now, increasingly, the world.

Malaguzzi credited the influence of David Hawkins in the development of the Reggio Emilia approach. Through their dialogue, they affected philosophy, still resonant today in early educators’ values and practice; for example, about creating intellectually rich, aesthetic, open environments, careful observation, and visual documentation.

An exhibit interpreting their work with young children, Cultivate the Scientist in Every Child: The Philosophy of Frances and David Hawkins, created by Hawkins Centers of Learning in Boulder, Colorado, is touring the United States and came to Lincoln, Nebraska, in spring and summer, 2014. It was the occasion of “Messing About” teacher workshops that opened the Math Early On project at the University of Nebraska-Lincoln.

Messing about, for both children and adults, means to provide time and space for learning experiences to evolve. With a flexible agenda, using play and exploration, guided inquiry, and reflective discussion, participants co-construct thoughts, ideas, and understandings. These workshops engaged teachers in exploring a variety of natural and found materials, such as tiles, rods, stones, fabric squares, bottle tops, and buttons, as an avenue for understanding science and math concepts in order to support their work with young children.

Photos on pages 2-3 by Alex Cruickshank and Lixin Ren
Math Early On, funded by the Buffett Early Childhood Fund, involves a partnership between the University of Nebraska-Lincoln (UNL) and the Educare of Nebraska, part of a national network of high-quality child care centers for low-income children from birth to age 5.

The goal of the Math Early On project is to offer professional development opportunities that build on the past successful professional development efforts of UNL’s NebraskaMATH and its Primarily Math initiative. The Primarily Math curriculum for primary teachers was adapted and redesigned to create new experiences for the purpose of enriching preschool teachers’ mathematical knowledge for teaching.

This booklet tells some of the powerful learning stories that emerged in the Educare of Omaha at Indian Hill and Kellom and the Ruth Staples Child Development Laboratory at UNL. In the model of the Hawkins exhibit, and of Italian-style documentation, the stories combine images, description, and teacher interpretation to reveal, unpack, and share moments and processes of early intellectual discovery.

Math Early On early identified the Early Math Collaborative at Erikson Institute in Chicago as doing important work in improving mathematics instruction for young children. Their textbook, *Big Ideas of Early Mathematics* (2014), became one key element of professional development for Math Early On. Besides math instruction, however, other key elements include: mathematically rich environments (indoors and out); integrating math with literacy; home-school partnerships; and pedagogical documentation. Together, these provide a comprehensive approach to make a culture of math visible and alive throughout the day in programs serving young children.

Building on strengths and shared perspectives, the university team and Educare leadership have planned every step of the project together. The collaborative partnership creates, nurtures, and spreads the ideas and insights emerging within the professional development. Firsthand understanding and engagement with the goals and daily practices of Educare, as well as hopes and dreams of Math Early On, are critical to sustainability.
“... what we start in school is only a beginning, and not an ending.”

– David Hawkins

Families influence children’s development in many ways. With mathematics, parents can support learning through everyday activities and routines at home; the games, books and puzzles they play with children; communications with teachers; and the values they model and teach.

Yet many of these opportunities are lost because parents may be uncomfortable with math and feel they were never good at it. The anxiety of parents of young children with math sometimes is mirrored by teachers’ own discomforts of working with parents. Teachers may not know how to partner with families around counting, measuring, patterns, and all the other topics of early childhood math.

The Week of the Young Child provided just the occasion for the Educare of Omaha to showcase the pleasure of mathematics school-wide. Teachers wanted to share with parents some ways to integrate math into everyday life at home and show parents how much their children loved learning about math.

Family Math

Photo by Jill Dibbern-Mahlert

Images above from video by University Communications
Teachers decorated their doors and walls, devised games and activities to play with children and families, incorporated math into music and art, cooking and nutrition, block-building, and literacy. A table was set up for a scavenger hunt with pictures of the objects families were supposed to find and bring to the table (i.e. shapes, patterns, size comparisons, numbers and objects).

One evening was designated Family Math Night, and teachers signed up for activities geared toward the age group of their students. Many of these activities focused on a teacher’s favorite book, such as one teacher who loved *Mustache Baby* and devised a matching game for infants.

Teachers used the week as a time to communicate to families the many ideas for what they could do at home. Teachers took care to make these communications clear yet specific and to use precise math vocabulary. For example, one handout was called, “Turn Mealtime into Learning Time,” and had these suggestions:

- Count how many children are in the room. How many adults? How many apple slices are left on the plate?
- Apply one-to-one correspondence as you and your child set the table with one plate, cup, and napkin for each child.
- Make and name geometric figures. Fold the square napkin in half. What shape does it look like now?
- Understand and use positional prepositions such as next to, under, over, near, behind, between, on, off, in, out, below, beside, and in front of.
- Create and understand fractions while cutting a whole sandwich in half or in quarters.
- Measure liquids. The pitcher has enough milk to fill six cups.
- Discuss shapes, sizes, and attributes of foods. Do you want a large piece or a small piece? Which is smaller, the orange or the clementine? How are oranges and clementines alike? How are they different?
Curious toddlers in the arms of adults leap at the chance to explore a game with plastic ducks. A teacher hands the child a duck and asks, “Where does Duckie go?” . . . .

Even before being prompted, one eager toddler, Gage, held by his mother, placed it on the yellow cut-out duck directly in front of him. He answered the question, “Where does Duckie go?” by pointing to where he had placed the duck. Still held safely in his mother’s arms, he then reached for another duck and continued the game.

Duck, Duck, Match

The Game:
Teachers at the Educare at Indian Hill have created a matching game involving a familiar animal—the duck. The materials were placed on a low table for toddlers to access. Each plastic duck had a symbol, number, or shape that was taped on its underside, and this image matched to one of the cut-out paper ducks on the table. The plastic ducks each had a distinctive characteristic, such as a hat, crown, or goggles, but these were not relevant to the game, as the child quickly discovered.

Moving out of his mother’s arms, but still holding his blanket, Gage then took another duck from the teacher and placed it on the correct match, the rectangle. He showed he has studied the visual cues on the bottom of the plastic ducks and found the corresponding images on the paper ducks.

Becoming more and more engrossed in the matching game, his blanket fell unnoticed to the floor. He selected ducks on his own and inspected their undersides. Finding the numbers on some of them, Gage matched them up correctly and beamed and clapped with satisfaction.

His mother watched intently as he demonstrated his early mathematical competence. “I didn’t know he could do that! I just didn’t know,” she said with surprise.

"We must provide for children those kinds of environments which elicit their interests and talents and which deepen their engagement in practice and thought."

Frances Hawkins

Math with Imagination

Magnet tiles are beloved in all the classrooms because they are so versatile and open-ended, satisfying in the way they click together, and beautiful in their transparency. They can be composed and decomposed into other shapes, and are constructed to be proportionate to one another.

In Linda Wegner’s classroom at Educare of Omaha at Kellom, children arranged magnet tiles on the table and the floor. A child arranged triangular pieces into a hexagon that she laid on a square, and then she created a cube. Some children composed rectangular arrangements out of squares, while other children became entranced with using them to make a long, long line.

When Linda showed her photos, other teachers wondered why young children so often like to make very long, or very tall, arrangements. It might have something to do with the mathematical process of adding another, adding another — the sense of being able to continue forever, an intuition about numbers they cannot articulate in words.

Another teacher noted a developmental sequence she observed. The youngest children compose in flat two dimensions, then as they get older they move on to discover how to create in three dimensions, and suddenly their imaginations go wild and they begin to create “houses, castles, all sorts of elaborate things.”

Photos on page 13 by Lindsay Augustyn (recreated at Trinity Infant and Child Care Center, Lincoln, Nebraska)
One day in Brittani Laski’s Kellom classroom, it was time for the children to put away their toys. Four of the children had been playing with magnetic shapes: large and small squares, two sizes of right triangles. When it was time to clean them up, rather than just tossing them into their box, the children categorized them by shape, each child specializing on one kind of shape and stacking them one congruent shape precisely on top of another, forming a tower.

Anthony specialized on the small squares. Ke’Ajah took a pair of triangles and clicked them together like two halves to make a square the size of the smaller squares. She wanted to add it to the part of Anthony’s stack she was holding.

“Anthony, look, I made a square. It’s a square,” Ke’Ajah told him as she placed her construction on his stack. He shook his head no, and tried to make her take the two triangles off.

“No,” Ke’Ajah insisted. “No you can’t take them apart. It’s a square. See, Anthony, I made a square.”

Anthony adamantly told her, “No, it’s not a square. It’s two pieces – like a pizza.”

They continued to talk like this with each other until their teacher Brittani told them the clean-up song was ending and they needed to be finished. At this point, Ke’Ajah layered the two triangles on top of the square pile. She left, and Anthony removed it from the squares in the box before going for story time.

Evidently, Ke’Ajah still believed she had constructed a square out of two triangles, and Anthony still believed it did not belong. The children’s deep familiarity with the tiles’ possibilities allowed them to conduct a 3-year-old version of a reasonable mathematical discussion without involving a teacher or getting angry.

Recap: Ke’Ajah took a pair of triangles and clicked them together like two halves to make a square the size of the smaller squares. ... “See, Anthony, I made a square.” Anthony adamantly told her, “No, it’s not a square. It’s two pieces – like a pizza.”

“If conflicts don’t arise, if there are no confrontations, if there aren’t moments in which there is a losing of equilibrium, if the certainty doesn’t leave room for the uncertainty, if a child doesn’t accept the flux of insecure moments, the climbing up stops.”

– Loris Malaguzzi
Maegan Heimes, a master teacher at Kellom’s Educare, observed in Sherry Loyd’s classroom one day as the children were exploring geometric shapes by lying on the ground and seeing what their bodies produced. The girls’ group of five could make several shapes – square, pentagon, triangle – using different numbers of girls. But the boys’ group of three found they could only make triangles (see photo, page 17). They really wanted to make a square, so girls joined them.

After making the shapes with their bodies, the children became fascinated with using objects to make shapes. They made shapes with crayons and with some long, clear cylinder tubes. With the tubes, they realized they could make triangles, rectangles, and squares, but they really wanted to make a circle. They tried getting some other small objects to fill in the gaps, but they were still using straight-edged objects, so it didn’t work. The children stuck with this problem a long time, but weren’t able (that day) to break through to a solution.

Cognitive sticking points, or “knots,” are more than negative moments or confusion and frustration. They contain positive possibilities for regrouping, hypothesis testing, and intellectual comparison of ideas. They can produce interactions that are constructive not only for socializing but also for constructing new knowledge. The teacher’s task is to notice these knots and help bring them to center stage for further attention – launching points for future activities.

Teachers Maegan and Sherry later talked about having string available the next time the children explored geometric shapes with their bodies or objects.
“To have respect for children is more than recognizing their potentialities in the abstract, it is also to seek out and value their accomplishments—however small these may appear by the normal standards of adults.”

- Frances Hawkins
Maliyah got out the transparent cylinders and built her tower on the table. She positioned them carefully and corrected the alignment of the middle blocks to stabilize her stack. Cherish came and watched her intently, holding her body at a safe distance as she leaned in to see.

A contagion of interest

Toddler Engineers

A contagion of interest

Toddler Engineers

Cherish stacked transparent blocks on the light table for a long time. Judah watched for a while though he did not build. Even when her stack fell over, Cherish continued and built again.

Maliyah got out the transparent cylinders and built her tower on the table. She positioned them carefully and corrected the alignment of the middle blocks to stabilize her stack. Cherish came and watched her intently, holding her body at a safe distance as she leaned in to see.

She smiled as she finished her tower. Was she smiling with satisfaction at what she had done? Was she impressed with the beauty of her tower glistening with light? (Notice how the color of light from the table changes from pink to yellow to green to blue; it is on a timer. This suggests how long Cherish stayed with the experience.)

Toddlers understand and appreciate mathematical concepts in action

During the children’s long engagement with stacking, Kathy Paradies observed intently, taking photos to capture the experience. She didn’t interrupt the children, because she knew something important to them was going on and did not want to disturb the flow of it. Later, she posted the photos on the board outside her classroom and sought others’ perspectives at the final meeting of Math Early On. The documentation is rich and invites analysis. Here is what can be observed:

• Children each picked materials of the same geometric shape for stacking: cylinders or rectangular prisms.
• They sought to extend them in one dimension: length, in a vertical direction.
• They experimented with concepts of balance, stability, and symmetry around the fulcrum.
• Far from being the egocentric creatures scientists used to write about, they showed deep interest in each other’s work and care in not disturbing it. They evidently grasped the engineering and math concepts involved because they adapted them to their own choice of materials.
• Finally, the toddlers’ faces suggest joy at the logic of their action, the order and aesthetics, and what they themselves could create and discover.
Many children’s books have strong literary quality but also math content that can be extended into math activities. Stacey Puett used the storybook, “ROAR! A Noisy Counting Book” (by Pamela Duncan Edwards and Henry Cole, 2000), in this way one day with her 3- and 4-year-olds at Educare at Indian Hill.

In the story, a little cub goes to look for playmates; he invites first one red monkey, then two pink flamingos, and so on, but nobody wants to play because he roars at them. Finally, 10 lion cubs are happy to play. Stacey read the book slowly, stopping on each page to have the children act out the plot by putting that many animal crackers on their counting mat.

The children were familiar with this because they use the mats for other counting activities. Most children were able to count all the way to 10, though on a few occasions an error was made, and Stacey helped that child to re-count the crackers.

At the end of the story, she held up number cards and had the children put that many crackers on their mats.

One of Stacey’s co-teachers was taking photographs throughout (shown here on pages 22-23), and Stacey used them to make a documentation panel outside her classroom to show the course of the activity. In this way, she shared with the school and families how she extended book reading into a counting lesson. She used the panel to talk with other preschool teachers in her wing about how they could use books for math teaching. She showed how she let children play with math concepts without destroying the fun of children roaring like a lion and imaginatively entering the world of the lion cub and jungle animals.

In a later interview, another teacher, Katie Arnold, remarked that now when she is doing one-on-one reading with children, they often ask math questions, or they spontaneously start counting things on the page of the book. Is this the result of all the experiences they have had with math this year?
“Children’s curiosity and investigative talents can lead them into genuinely mathematical subject matter.”

– David Hawkins

Scientists see mathematics as the language in which they frame hypotheses and evidence, but when it comes to early childhood, the connection between “cause-and-effect” and “more-versus-less” is still murky for children – something still being figured out. As a result, early science and math educators don’t always make meaningful connections in the curriculum.

But Katie Arnold at Educare at Indian Hill ventured into an integrated approach to math and science with 3- to 5-year-olds. She thought about how children see their own little bodies as a force making things happen. How much? How big? How high? How far? What demonstrable force can their bodies exert?
Katie’s classroom first investigated how far they could throw hand-made paper airplanes. Young children are intrigued with things in motion – how and when they move, what makes them move. Katie assigned each child a number, to be carefully written on a paper. In the hall, they went in that order to try to send their planes as far as they could. They each ran to be next to where their plane had landed. Looking around, they could directly experience the comparative magnitude of their results. Katie gave them a new numerical ranking, according to how far their plane had flown. They talked about “close,” “closer,” “far,” “very far,” “farthest,” to practice spatial concepts.

They then threw again, now taking turns in order of the new ranking. (Learning about both ordinal and cardinal numbers is important for preschoolers). They continued their experiments many times, indoors and out, and Katie talked with them about the real airplanes they saw flying overhead in and out of the Omaha airport.

Katie constructed a large documentation panel outside her classroom portraying the project. Along the bottom, she placed small cutout figures from photos she had taken of the children outside, hurling their paper airplanes.

On a later day, the children experimented with the force they could exert in jumping down on something. Using empty pop bottles, duct tape, and tubing, Katie constructed a bottle rocket launcher. Each child selected and placed a rocket on one end of the pipe, then prepared to jump on the bottle attached to the other end. Every rocket had its own number – indicated by a numeral and the corresponding number of dots – which the children counted and said out loud while taking their turn. These numbers were just for practice in number identification.

The group waited with excitement, then sang the launching song and countdown, “We’re going to the moon. We’re going to the stars. Five, four, three, two, one, BLAST OFF!” The child leaped onto the bottle, and lo and behold, the rocket soared up and hit the school ceiling. No explicit measuring was involved, but the children talked about whether if they jumped higher as they leaped onto the bottle, then would their rocket go higher than the others had? They noticed that some rockets shot higher than others and started picking only the best rockets. The children continued to talk about the bottle rockets long afterward, singing the blast off song and asking if they could do the experience again, including all the children in nearby classrooms.
“The domain of mathematics is to be consciously expanded to include all those junctures in the lives of children, their working contact with the great world of nature and of human society, out of which mathematics in the usual restricted sense can be seen to evolve.”

– David Hawkins

The idea of sets is basic to children’s thinking and learning and is fundamental to mathematics. Even the youngest children make simple distinctions between objects, and, as they grow older, they realize that attributes can be used to sort collections into sets and the same collection can be sorted in different ways.

Eventually, they understand sets can be compared and ordered; for example, how many things are in one set versus another. These are all “big ideas” to little children.

While student teaching in a classroom of 2- to 3-year-olds at the UNL Ruth Staples Child Development Laboratory, Kayla Hass met with her master teacher, Erin Hamel, to discuss plans for a sorting activity one morning. It is common for teachers to use plastic manipulatives for such activities; however, these items rarely offer more than one attribute for sorting, such as color or size.

Rather than present children with plastic bears for sorting, they wondered what would happen if children were presented with a more complex collection of materials.

How would children explore and sort diverse materials? Where would the teachers find such an assortment?

Erin, master teacher, suggested using a variety of natural materials from the outdoor classroom and garden space. Kayla gathered sticks, leaves, flowers, and garden vegetables from the outdoor space for her lesson. She placed the items in the center of the table and encouraged children to take a basket and put items that they thought belonged together into it. Through observation, some of the children’s rules for sorting were easily identified. Kayla commented on their work and confirmed their perceptions through conversation.
Jasper (photo below left) collected edible items. He felt each one and brought them up to his nose to smell, too. He even tried to bite the green pepper. Jasper squished up his face with dislike and put the pepper into his basket. Consequently, he did not try to taste any of the other items but continued to collect the fruits and vegetables in his basket.

Yang was drawn to the flowering plants and quickly scooped them up. She was frequently found in the art area drawing flowers with pencils and crayons; therefore, it was no surprise to her teachers when she accurately identified the colors of each flower and placed them in her basket.

After children had the chance to explore and sort the items creating their own rule, Kayla presented colored paper to the children. She told them she wanted help sorting the items by color. Children selected a paper and worked together to sort the items that matched their sheet. They worked together handing each other items and identifying colors. They debated the placement and colors of items including a cherry tomato, which Owen (below, in blue) pointed out could be both red and green because of the ripe red skin on the outside but the greenish seeds that spilled out when he pinched it.

Amelia gathered up several items into her basket. She touched the items to her face and noticed something. The items in her basket varied by texture, color, and shape. Her teacher tried to identify Amelia’s sorting rule but it was not obvious.

What commonality did Amelia see in the items?

Did she simply select items she found appealing?

Amelia picked up a lemon and gently touched it to Kayla’s cheek and then her chin. Kayla allowed the exploration and questioned Amelia on this approach. Amelia revealed to her teacher that the item was cold. It was then that Kayla discovered Amelia had sorted her objects by temperature! How sensory young children are when they explore and categorize the world around them.
The children at the UNL Ruth Staples Child Development Laboratory engaged in a long study of patterns. They learned about repeating patterns and growing patterns, and looked for patterns all around their school and classroom. They went to the International Quilt Study Center to examine patterns in the textiles. During one visit with her class Jenny Leeper Miller, master teacher, observed children labeling sequences and identifying rules of patterns in the quilts. She was surprised at how much they noticed.

Many of the children noticed repeating patterns in the designs. Charlie studied and observed the flag quilt. He looked at it with an interested and puzzled gaze. Many children were drawn to this quilt’s unique design. They stood in front and began to have conversations with a teacher about what they noticed. It was as if they wondered. Why does each flag look different – or are they the same? Charlie began to describe his observations to the group: “I see four flags and brown eagles.” Leo interrupted, “The flags are different, not the same.” Emily contributed to the description: “Some eagles are flying and some are laying down, and the flags are laying down, too.” Charlie thought about Emily’s comment. He began again to describe his observation: “Upside down, not upside down, upside down, not upside down.” He pointed to the flags, reading them in two rows starting with the first flag in the upper left, going to the next on the upper right, moving and pointing to the flag in the lower left ending with the lower right. “Charlie, what is that called?” teacher Jenny asked. Charlie happily labeled his description of the flag as a “pattern.”
Upon invitation to families, children wore all different types of patterns one day. They began to comment on them, going up to each other and putting their patterns next to each other to seek matches. One child was very excited to have on the same pattern as Taariq Allen, his student teacher.

“"We will not be in a hurry, and no one will deny the children the time they need for thinking and doing, for finding, creating or changing ideas and applications.””

- Loris Malaguzzi
One day, Taariq Allen, student teacher in the 5-year-old classroom at the UNL Ruth Staples Child Development Laboratory, built a tall pyramid of red plastic cups to illustrate a simple, linear “growing pattern,” one that increased by one from row to row. The children watched intently.

Grace (pictured below) said: “That looks really neat. Just by adding to each row, the tower got bigger and bigger. That looks hard.”

The next day, Grace tried to recreate the pyramid of cups. Jenny Leeper Miller, master teacher, observed and documented the process. Would Grace be able to describe the pattern and thus reveal her mathematical thinking? Grace began with determination and the simple material of red plastic cups -- a LOT of cups. She started out making a row of six, paying careful attention to the placement of each cup. She built up from the base of six and then added another row of five on top. Grace moved quickly from row to row, each time adding one less cup. She mostly worked silently, as if she was thinking through the process as she went about her next step.

After the construction of each row, Jenny prompted Grace to stop and think about the number of cups and represent them on a paper nearby the structure. With no struggle at all, Grace drew the right number of circles to represent each row of the structure.

As she worked on the second row of five cups, Grace stopped and said, “Now I have to do four. I know because you are counting backward.” Jenny said, “So the pattern is you are counting backward?” Grace replied, “Now it’s three. That’s easy. Next has to be two.” Jenny could see Grace’s thought process of calling this a growing-backward pattern because as she adds another level, it is one less number of cups in the pattern but the structure is bigger in height – growing up in the structure and backward in number of cups. 

-- Jenny Leeper Miller and Erin Hamel

Preschool Patterns
- Rules of patterns can be defined and supported by using representation tools.
- Children can build on their understandings by slowing down and drawing, writing, acting out their thoughts about numbers and patterns.
- Children need to demonstrate their understanding in mathematical skills to build confidence.
the sense that the world around is suffused with numerosity.

– David Hawkins