Proceedings of the Eighth Annual Mathematics Teacher Education Partnership Conference

The MTE-Partnership: Transformation. Equity. Leadership.

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Preface

These proceedings are a written record of the presentations and papers presented at the Eighth Annual Mathematics Teacher Education Partnership Conference held in St. Louis, June 23–25, 2019. The theme for the conference was "The MTE-Partnership: Transformation. Equity. Leadership." We are pleased to present these Proceedings as a resource for the mathematics and mathematics education community.

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[1] Robert N. Ronau participated as a member of the MTE-Partnership Planning Team while serving at the National Science Foundation.

[2] Any opinion, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation.

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Introduction

The Eighth Annual MTE-Partnership Conference: The Beat Goes On

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The Mathematics Teacher Education Partnership (MTE-Partnership) was formed by the Association of Public and Land-grant Universities (APLU) in 2012 to address a major problem in secondary mathematics teacher preparation: a shortage of secondary mathematics teachers entering the profession who are well prepared to ensure their students can meet rigorous state mathematics standards for college- and career-readiness, with an initial focus on the *Common Core State Standards for Mathematics* (CCSS-M) (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) and other documents. This consortium of over 90 universities and over 100 school systems has a common goal of transforming secondary mathematics teacher preparation using the networked improvement community (NIC) design (Bryk, Gomez, Brunow, & LeMahieu, 2015). This paper provides a brief overview of the development of the MTE-Partnership and how the goals for the Eighth Annual MTE-Partnership Conference, held in June 2019, supported its continuing progress.

An Overview of the MTE-Partnership

The APLU's 2011 Annual Conference of the Science and Mathematics Teaching Imperative (SMTI) focused on how higher education might respond to the just-released CCSS-M, with a particular focus on changes necessary in teacher preparation. A group of attendees at the conference submitted a white paper to the SMTI Executive Committee proposing the formation of an initiative focusing on secondary mathematics teacher preparation. This proposal was accepted, and a planning team was formed to organize the initiative, which became the MTE-Partnership, with the goal of transforming secondary teacher preparation with the following aim:

> to build a national dialogue around guiding principles for the preparation of mathematics teachers; promote partnerships among all sectors throughout the teacher development process, with a focus on promoting institutional change; develop and coordinate a networked research and development agenda; serve as a clearinghouse for model programs and practices; and advocate for change at university, state and national levels. (MTE-Partnership, 2014, p. 1)

One of the central features of the MTE-Partnership is its focus on partnerships with secondary mathematics teacher preparation programs across its stakeholders. Representatives from university programs that wished to participate were asked to form teams including K–12 school districts and other partners involved in secondary mathematics teacher preparation, with a requirement that teams engage mathematics teacher educators, mathematicians, and K–12 personnel in their activities. As stated in the *Guiding Principles for Secondary Mathematics Teacher Preparation Programs* developed by the MTE-Partnership (2014), successful transformation requires a focus on "develop[ing] and promot[ing] a common vision and goals for how to best prepare teacher candidates who can promote student success in mathematics" within a program, as well as engaging in mutual learning and sharing responsibility across the MTE-Partnership (p. 2). There are currently 40 partnership teams across 31 states in the United States (see Figure 1).

Figure 1. Participation in the MTE-Partnership. Stars represent lead institutions for a team, and small circles represent other participating universities and colleges.

Design

The MTE-Partnership adopted the NIC design at the end of its first year. This design is consistent with the collaborative intent of the MTE-Partnership, and the focus on disciplined inquiry aligns with the mission of the universities involved in the MTE-Partnership (Martin & Gobstein, 2015). A brief outline of the design based on four essential characteristics of a NIC (Bryk, Gomez, Brunow, & LeMahieu, 2015) follows; further discussion of the MTE-Partnership design can be found in Martin and Gobstein (2018).

Focused on a well-specified common aim: As stated by Bryk et al. (2015), "an improvement aim articulates the specific problem to be solved and the measures of accomplishment to which the community will hold itself accountable. It imbues the community with purpose" (p. 150). The MTE-Partnership is focused on the aim of increasing by 40% the quantity of teacher candidates who meet a gold standard of preparedness by 2020. Progress toward this aim animates the work of MTE-Partnership.

Guided by a deep understanding of the problem and the system that produces it: Soon after its launch, the MTE-Partnership developed its *Guiding Principles,* which were updated in 2014 and have served as the central organizing document for the MTE-Partnership, describing critical aspects of secondary mathematics preparation. Since that time, the Association of Mathematics Teacher Educators (AMTE) has released *Standards for the Preparation of Teachers of Mathematics* (2017). These standards, which address K–12 mathematics teacher preparation, are now being used to augment the MTE-Partnership's (2014) *Guiding Principles*; the visions of the two documents generally align since they build on the same general theoretical and research base.

Further analysis of the problem space led to the development of a driver diagram, which is a primary tool for analyzing problems and explicating ideas for specific work to address them. Driver diagrams are not comprehensive descriptions of the system; instead, they depict the drivers, or hypothesized actions that will change the state of the system, moving toward the desired aim (Bryk et al., 2015). The MTE-Partnership driver diagram has been frequently revisited over the years; the most recent driver diagram is shown in Figure 2. Note that Research Actions Clusters (RACs) and Working Groups have been formed to address these drivers.

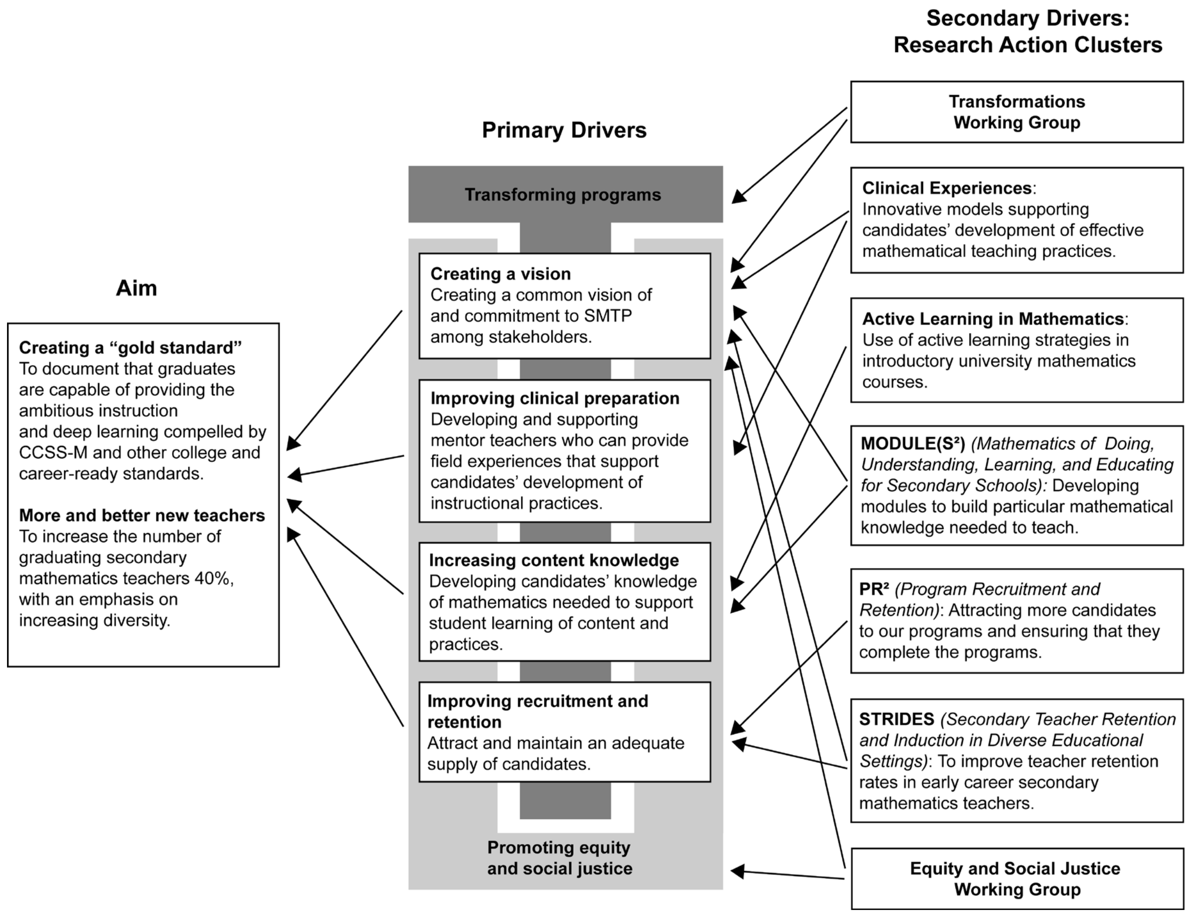

Figure 2. The MTE-Partnership driver diagram (Martin & Gobstein, in press).

Disciplined by the rigor of improvement science: The use of evidence to guide the development of interventions ensures that the changes being proposed are actual improvements. The RACs and Working Groups employ Plan-Do-Study-Act (PDSA) Cycles (see Figure 3) to iteratively prototype, test, and refine interventions; the use of PDSA cycles has the potential to lead to timely solutions to important problems (Bryk et al., 2015). Without the incorporation of evidence, changes may not actually be improvements.

Networked to accelerate progress: NICs are designed to marry precepts of improvement science with precepts of networked improvement, so that the improvement work is carried out across a range of contexts (Bryk et al., 2015). Thus, partners are mobilized to work in a parallel and coordinated manner to address critical sub-problems in secondary mathematics teacher preparation, as shown in Figure 2. The NIC design embraces variation across contexts to study how interventions need to be adapted to respond to the differing conditions under which they are used. As they are tested and refined, interventions can gradually spread across the network, supporting scale up (Bryk et al., 2015). The networked organization further allows a divide and conquer approach in which subsets of teams can address different problem areas.

The Annual Conferences: A Trajectory of Growth

Beginning in 2013, the MTE-Partnership has convened an annual conference to further its work that brings together many of those active in the MTE-Partnership to reflect on the progress that has been made throughout the past year, make concentrated progress on specific issues, and set forth plans for the coming year.

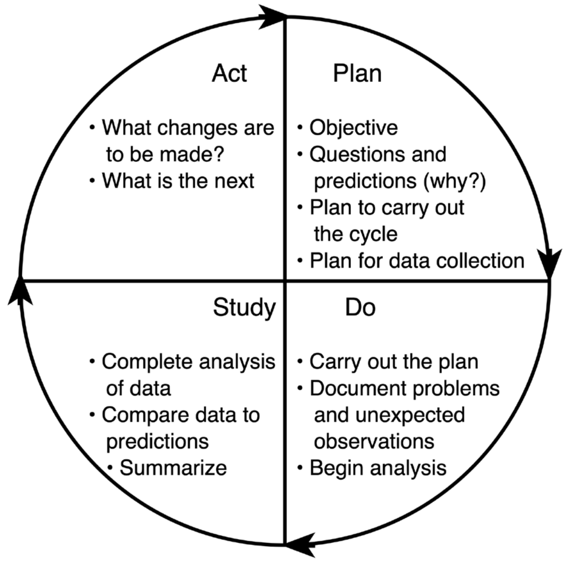

Figure 3. The Plan-Do-Study-Act (PDSA) Cycle. (Adapted from Langley, 2009)

With the Eighth Annual MTE-Partnership Conference held in June 2019, we are proud to say that the work is progressing. A brief outline of the growth of MTE-Partnership through the lens of its annual conferences follows; which builds on an account given in the 2018 *Proceedings* (Martin & Gobstein, 2018).

2012 Conference: The first conference, held in April 2012, focused on creating an initial draft of guiding principles for the MTE-Partnership, which led to *Guiding Principles*, since updated in 2014. A first attempt was also made at identifying central challenges in meeting the guiding principles; follow-up work led to the identification of the four initial areas of inquiry, shown as primary drivers in Figure 2. The decision to adopt the NIC design followed this conference.

2013 Conference: The second conference focused on learning more about the NIC design, which had been adopted following the 2012 conference, and developing the problem space in alignment with that design. Initial concepts were written for a set of 13 RACs, which were later narrowed down to an initial set of five that were launched in the fall following the conference. Teams were invited to join the RACs, and an initial boot camp organized by representatives of the Carnegie Foundation was convened in the fall following the conference to initiate their work. The Carnegie Foundation played a key advisory role throughout the launch of the RACs.

2014 Conference: The third conference was focused on establishing the work of the RACs. RAC members met in small groups to review their initial work in forming an aim and driver diagrams and to begin planning specific improvement efforts to be undertaken in the coming year using PDSA Cycles in which evidence would be gathered to guide their continued development and refinement. Additional sessions focused on increasing understanding of the NIC design and exploring issues related to secondary mathematics teacher preparation. The RACs continued their work throughout the following year.

2015 Conference: The fourth conference continued a primary focus on accelerating the work of the RACs. A new RAC on improving the retention of program graduates in the profession also was launched, replacing an earlier RAC. This conference saw the incorporation of all 22 campuses of the California State University system that offer teacher preparation, greatly increasing the capacity of the MTE-Partnership. The 2015 conference also introduced an emerging emphasis on program transformation, reflecting the challenges programs face in moving beyond making changes based on the one or two RACs in which they are actively engaged, to aggregating the findings of multiple RACs to undertake the broad-scale changes needed to ensure both the necessary quantity and quality of secondary mathematics teacher candidates.

2016 Conference: The work in the RACs was again the focal point of the fifth conference. A newly formed working group on program transformation presented a panel discussion of issues related to transformational change and continued its work throughout the following year. In addition, a new focus on equity and social justice was launched; while these issues are embedded in the *Guiding Principles* and in the work of many of the RACs, members of the planning team noted that this was not visibly a part of the Partnership aim or drivers. A work session was held at the conference to discuss how to make equity and social justice a more explicit focus of the MTE-Partnership. In addition, a series of refereed brief research reports were included in the conference to enhance the sharing of ongoing work across the MTE-Partnership. For the first time, Conference Proceedings (Lawler, Ronau, & Mohr-Schroeder, 2016) were released to provide an accessible record of the work of the MTE-Partnership at the conference and throughout the past year.

2017 Conference: The overall trajectory of work by the MTE-Partnership continued at the sixth conference. The work of the RACs was highlighted, along with the themes of program transformation and equity and social justice. The theme of program transformation was addressed in a keynote by Jennifer Russell, fellow at the Carnegie Foundation for the Advancement of Teaching, who discussed the power of networks for program improvement, and a working dinner organized by the Transformations Working Group. A panel discussion addressed various aspects of equity and social justice related to secondary mathematics teacher preparation; Nicole Joseph, noted scholar on issues of equity, served as a reactant to the panel and to the conference at its conclusion. A new working group of social justice and equity was launched prior to the conference, and work sessions were organized by both the Transformations Working Group and the Equity and Social Justice Working Group (ESJWG). The series of refereed research reports was expanded, again appearing in a Conference Proceedings (Smith, Lawler, Bowers, & Augustyn, 2017).

2018 Conference: These themes continued at the seventh conference, with a focus on the work of the RACs along program transformation and equity and social justice. The conference was kicked off by a keynote in which Susan Elrod, noted author in the area of institutional change in the STEM disciplines (cf. Elrod & Kezar, 2016), interacted with Marilyn E. Strutchens, noted scholar in issues related to equity in mathematics education on the two conference themes and their interaction. Work sessions were organized by both the Transformations Working Group and the ESJWG. The series of refereed research reports was continued, again appearing in a Conference Proceedings (Smith, Lawler, Strayer, & Augustyn, 2018).

Goals of the 2019 Conference

A new approach was taken to enhance the work of the 2019 Conference: a series of eight webinars were presented in the two months prior to the conference. These webinars were designed to provide information about the MTE-Partnership, including its RACs and Working Groups, prior to the conference. The webinars were intended to enhance understanding of what is happening across the MTE-Partnership, as well as help to orient new participants to the ongoing work. Each webinar consisted of a 20- to 30-minute presentation, followed by time for questions and discussion, to allow for easier entry into the work of the conference.

While in many ways the themes of the past several conferences were continued in the eighth conference, this conference marked a heightened focus on program transformation, corresponding to the launch of NIC-Transform. NIC-Transform is a new NSF-funded project (DUE-1834539 and DUE-1834551) in which five MTE-Partnership teams are focusing on how to better promote and document local program transformation. The slide in Figure 4 was shown in the opening session and every general session throughout the conference. Over the past few years, much of the energy of the MTE-Partnership has focused on the work of the RACs to provide solutions to problems of practice in secondary mathematics teacher preparation; the message in Figure 4 is that transformation of local programs must become a co-equal aim. Until local teams begin to utilize the solutions developed by the RACs, the aim of the MTE-Partnership will not be fulfilled.

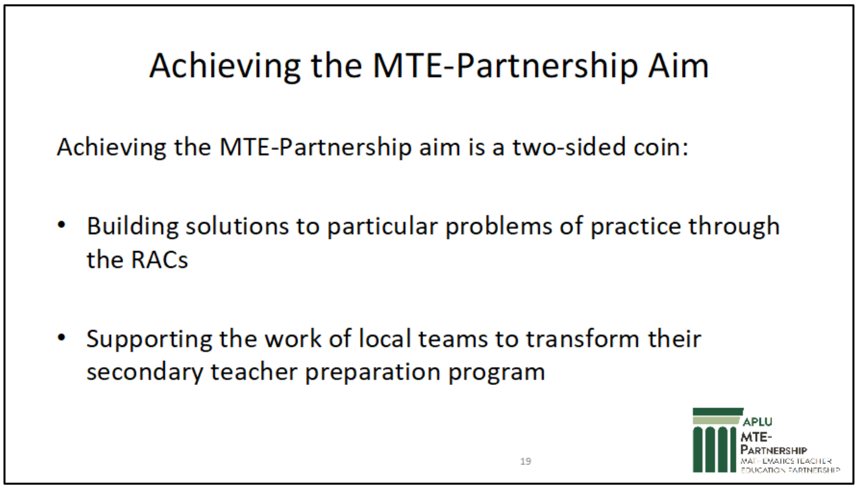

Figure 4. Slide depicting the foci of the 2019 Eighth Annual MTE-Partnership Conference.

Including program transformation, the Eighth Annual MTE-Partnership Conference had four primary goals that are discussed in turn, along with how the structure of the conference supported each goal.

Partnership/institutional teams will plan next steps in transforming their programs: As aforementioned, program transformation was a central focus of the 2019 Conference, supported by four sessions throughout the program. A working dinner Sunday evening included a keynote presentation by Kathryn Chval, dean of education at the University of Missouri Columbia, who offered reflections on how to approach program transformation from an administrative point of view. Her talk was paired with a presentation by Etta Hollins, a distinguished researcher into equity issues in teacher preparation, who addressed the infusion of equity across efforts to improve mathematics teacher preparation. The theme of program transformation was picked up in a panel discussion Monday morning, which consisted of participants from NIC-Transform sharing their efforts at local program transformation. The panel included time for reflection and discussion within and across teams on how they might enhance their efforts at program transformation. Monday late afternoon included a set of breakout sessions focusing on issues related to program transformation, such as stakeholder engagement and policy, how the NIC design can support these efforts, and equity as the focus. The closing session on Tuesday included time for local teams to develop an action plan for furthering their efforts toward program transformation, both prior to the start of the new school year and by the end of the first semester.

The RACs will continue their work to improve aspects of secondary mathematics teacher preparation, including considering how they share their work in order to contribute to additional teams' transformational efforts and to the knowledge of the field: Despite the sharper focus on program transformation, the work of the RACs continues to be important to the progress of the MTE-Partnership. The RACs spent more than eight hours working across the conference, central to their work in progressing toward their respective aims. However, increased focus was placed on how they might begin to disseminate their work, both within the MTE-Partnership to support teams' transformation work and to external audiences. In the closing session, RACs were asked to present opportunities for other teams to engage in or learn from their work. Updates on their progress can be found in the Research Action Cluster Reports section of these proceedings.

The Partnership as a whole will grow its sense of joint purpose and identity as a NIC-supporting program transformation: While engagement in the specific work of the MTE-Partnership in the RACs and locally may be at the forefront for MTE-Partnership participants, it is also critical that the MTE-Partnership maintains a sense of common purpose and identity across these efforts in order to thrive as a community focused on improvement (Martin & Gobstein, 2015). The project co-directors emphasized the defining characteristics of the MTE-Partnership as a NIC in the opening session, again emphasizing the importance of those characteristics for the continuing success of MTE-Partnership. The webinar series presented prior to the conference was designed to build that cross-cutting purpose and to enhance communication across the RACs, and a roundtable discussion following the opening session provided opportunities for participants to follow up with several of the RACs in which they are not involved. Brief research reports were included to build understanding of the work going on across the partnership, both by the RACs and working groups and by local teams. The closing session included a celebration of the progress made by MTE-Partnership over the past eight years and reflections on where the MTE-Partnership might continue to grow.

Specific focus on equity and social justice will be included throughout the proceedings: The theme of equity and social justice was threaded throughout the conference. As previously mentioned, this focus was included in the working dinner and was included as one of the foci in the Monday late afternoon breakouts. The ESJWG is continuing to build a liaison structure in which members of ESJWG, who are often also members of a RAC, are specifically designated to provide a link between ESJWG and the RACs in which they are engaged. For the 2019 conference, these liaisons were asked to highlight two issues within the RAC worktime at the conference: (a) initiating critical conversations about equity issues, and (b) confronting deficit ideologies. All of the RACs included a discussion of their response to these issues in their reports in the closing session. Finally, a number of research sessions focused on work around equity and social justice.

Moving Forward: MTE-Partnership 2.0

The closing session launched the need for the MTE-Partnership to re-examine its underlying framework as a NIC—its aim, driver diagram, and priorities. Adjustments to this foundation have been happening incrementally, but the MTE-Partnership has reached “a pivotal moment where its foundations need to be reconsidered rather than tweaked to both incorporate what MTE-Partnership members have learned as a community over the past years and to address the changing circumstances that programs face” (Martin, Lawler, Lischka, & Smith, in press). New foci may be needed to infuse the knowledge generated by RACs into program transformation efforts, as well as to support efforts by local teams at program transformation. In addition, new foci may be necessary to address the dramatic changes that have occurred in the landscape for secondary mathematics teacher preparation since the launch of MTE-Partnership: the significant national decrease in students entering teacher preparation, the rise of non-traditional programs for teacher preparation, and new technologies that might be addressed, such as virtual teaching and learning environments (Martin et al., in press).

To begin this process of reformulating the foundations of MTE-Partnership, its leadership will be engaging members in critical conversations over the coming months. The following questions were posed in the closing session to launch that discussion: *What is most needed to support program transformation efforts? What new RACs are needed? What former foci can diminish?* It is our hope that these efforts will launch a new era of work by the MTE-Partnership to promote the improvement of the preparation of secondary mathematics teacher preparation—the “MTE-Partnership 2.0” ready to thrive for another eight years.

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Overview of the Conference

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The Eighth Annual Mathematics Teacher Education Partnership (MTE-Partnership) Conference was held at Drury Plaza Hotel at the Arch in St. Louis, Missouri, from June 23–25, 2019. With 81 registrants from 22 of the 39 MTE-Partnership teams, the theme for this year's conference was "The MTE-Partnership: Transformation. Equity. Leadership."

In support of the partnership aim, the goals for the 2019 annual conference were:

- The MTE-Partnership/institutional teams would plan next steps in transforming their programs.
- The Research Action Clusters (RACs) would continue their work, including considering how they share their work, in order to contribute to additional teams' transformational efforts and to knowledge of the field.
- The MTE-Partnership as a whole would grow its sense of joint purpose and identity as a networked improvement community supporting program transformation.
- A specific focus on equity and social justice would be included throughout the conference.

Prior to the MTE-Partnership conference in June, a series of webinars were scheduled across April and May to provide introductions to each RAC and Working Group, as well as to describe ways people could get involved in each. With the shuttering of Trellis Science in fall of 2018, the MTE-Partnership shifted to Open Canvas as its platform for project information and collaboration. The MTE-Partnership webinars are all archived in the Open Canvas site.

The Eighth Annual MTE-Partnership Conference opened on Sunday afternoon, June 23, with a brief welcome, followed by lightning-round discussion tables. The lightning-round format allowed conference attendees to ask questions of RAC representatives and learn more about recent work and opportunities for involvement, before the afternoon moved into RAC work time (see the RAC section of these proceedings for what each RAC worked on during this year's conference). The RACs worked throughout the conference: Sunday afternoon, Monday morning, Monday afternoon, and Tuesday morning.

During the Sunday dinner hour, two plenary speakers focused on different aspects of program transformation. Etta Hollins, professor and Ewing Marion Kauffman/Missouri Endowed chair for Urban Teacher Education at the University of Missouri-Kansas City, spoke first, on equity and social justice in secondary mathematics teacher preparation. Hollins' talk spoke about a wide variety of approaches to social justice; her presentation abstract summarized her main message:

> Equity and social justice are cross-cutting themes incorporated in the design of pre-service programs and in all aspects of the professional preparation of teacher candidates. In the design of pre-service programs, equity and social justice are evident in the philosophical stance, the application of the theoretical perspective on learning teaching, framing the curriculum for professional preparation, and the application of professional knowledge to practice in field experiences. In the professional preparation of candidates, equity and social justice are evident in content knowledge, pedagogical content knowledge, knowledge of learning, and knowledge of students. This discussion will provide examples of approaches and evidence for incorporating equity and social justice in both teacher preparation program design and in the preparation of teacher candidates. This discussion will include approaches for college mathematics

instruction that prepares candidates for teaching students from diverse cultural and experiential backgrounds.

Second, Kathryn Chval, dean of the College of Education at the University of Missouri-Columbia and professor of mathematics education with the Joanne H. Hook Dean's Chair in Educational Renewal, spoke from an administrator's perspective on transforming secondary mathematics teacher education during the age of disruption. Chval discussed how to approach transformation efforts with university administrators and related some of Missouri's transformation efforts in broadening participation in teacher preparation. Her presentation abstract summarized her main message:

> Imagine you were part of education systems that had leadership, vision, strategy, infrastructure, and capacity to:
>
> - address problems of practice;
> - strengthen communities;
> - develop, understand, and expand effective models to scale; and
> - pursue research and innovation so that every student at every level was successful in learning mathematics.
>
> Building these education systems requires partnerships that engage strategically and collectively. As faculty, administrators, and partners engage on design teams, reciprocity, positioning, resilience, interdependence, and persistence are critical for successful program transformation. How do we engage with others and lead transformation in mathematics teacher education during the age of disruption? Our students are waiting.

Monday, June 24, began with a plenary session, facilitated by Gary Martin, that featured four panelists discussing transformation efforts at their local partnerships. The panelists' talks are based on their work with the NIC-Transform project, and are included in these proceedings: Dana Pomykal Franz (Mississippi State University), Marilyn E. Strutchens (Auburn University), Margaret J. Mohr-Schroeder (University of Kentucky), and Wendy M. Smith (University of Nebraska–Lincoln). Monday afternoon and Tuesday morning each included two sets of concurrent presentations by MTE-Partnership members, in 21 different presentations. Each of these presentations (abstract and/or full article) are included in these proceedings. The Transformations Working Group hosted three discussions late Monday afternoon all focused on program transformation, with an additional focus on equity, stakeholder engagement, and NIC design. Finally, Tuesday's closing session featured Gary Martin and Howard Gobstein and focused on program transformation efforts and discussing next steps for the "MTE-Partnership 2.0."

Evaluations of the 2019 Conference were positive, finding the various sessions useful and strongly agreeing the conference was a good use of time. Among all the sessions, the Sunday evening plenaries of Chval and Hollins were most highly rated as useful. When asked about the most valuable aspect of the conference, responses centered on work time with RACs and networking/collaboration. Attendees appreciated how the organization and content of the sessions helped advance understanding of program transformation efforts in a NIC framework, with a focus on initiating local transformation efforts. As one responder to the evaluation survey wrote, "This was useful to my program because MTE-P facilitated honest conversations about difficult work."

Panel Talks and Working Groups

Transformations Panel[1]

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Since its inception, the primary goal of the Mathematics Teacher Education Partnership (MTE-Partnership) has been the transformation of secondary mathematics teacher preparation. However, much of the initial work of the MTE-Partnership has focused on the work of Research Action Clusters (RACs) and Working Groups, which address particular problems of practice in alignment with the driver diagram developed as a part of its networked improvement community (NIC) design. See the introduction of these proceedings and Martin and Gobstein (2015) for more information about the overall design of the MTE-Partnership. One section of these proceedings describes the work of the RACs and Working Groups; many of the research reports also relate to this work.

As the work of the RACs has progressed, emphasis on supporting the work of local teams to use the emerging knowledge to transform their secondary mathematics teacher preparation programs has increased. A working group on program transformation was formed in 2016, which organized a new theme on program transformation at the 2016 MTE-Partnership Conference. That theme continued in the following conferences in 2017 and 2018, but reached a new level of emphasis in the 2019 MTE-Partnership Conference, where the case was made that the work of local teams to transform their secondary teacher preparation program must receive parallel billing with building solutions to particular problems of practice through the RACs. The transformations panel was the second in a sequence of four sessions across the 2019 conference focusing on program transformation.

The transformations panel session was organized by leaders of institutions participating in a project funded by the National Science Foundation (NSF) called NIC-Transform, which stands for Using Networked Improvement Communities to Design and Implement Program Transformation Tools for Secondary Mathematics Teacher Preparation (DUE-1834539 and DUE-1834551). The panelists described their progress in transforming their secondary mathematics teacher preparation programs. The institutions within the project operate as a NIC, focusing on coordinating efforts to support program transformation (Martin, Smith, & Mohr-Schroeder, in press); its driver diagram is shown in Figure 1. The conference itself was supported by the NIC-Transform project in response to the key change idea "convenings/communication across programs," and RACs were encouraged to include address this change idea related to thinking about ways the resources they are developing can be effectively shared with other teams focusing on program transformation.

The final change idea is built on an additional premise of this project, that the NIC design can be adapted to support transformation efforts at the local level. Each institution participating in the NIC-Transform project has identified drivers related to transformation of its program and is conducting Plan-Do-Study-Act (PDSA) Cycles to address those drivers. The purpose of this session was to help local teams think about how they might use the NIC

[1] Work on this chapter was supported in part by a grant from the National Science Foundation Directorate for Education and Human Resources, Division of Undergraduate Education (DUE) – Improving Undergraduate STEM Education (IUSE), Exploration and Design: Institutional and Community Transformation, Grant ID #s 1834539 and 1834551. Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.

design to help organize their efforts at program transformation. Four of the NIC-Transform institutions included on the panel provided a short description of their efforts at program transformation—including their local aim, the primary drivers they posit will lead toward their aim, and examples of change ideas on which they were working—as a means to promote participants' reflection on their own contexts. An overview of each institution's presentation follows in the expectation that they will both serve as a record of what happened at the conference as well as the basis for other MTE-Partnership institutional teams to reflect on their efforts to promote program transformation.

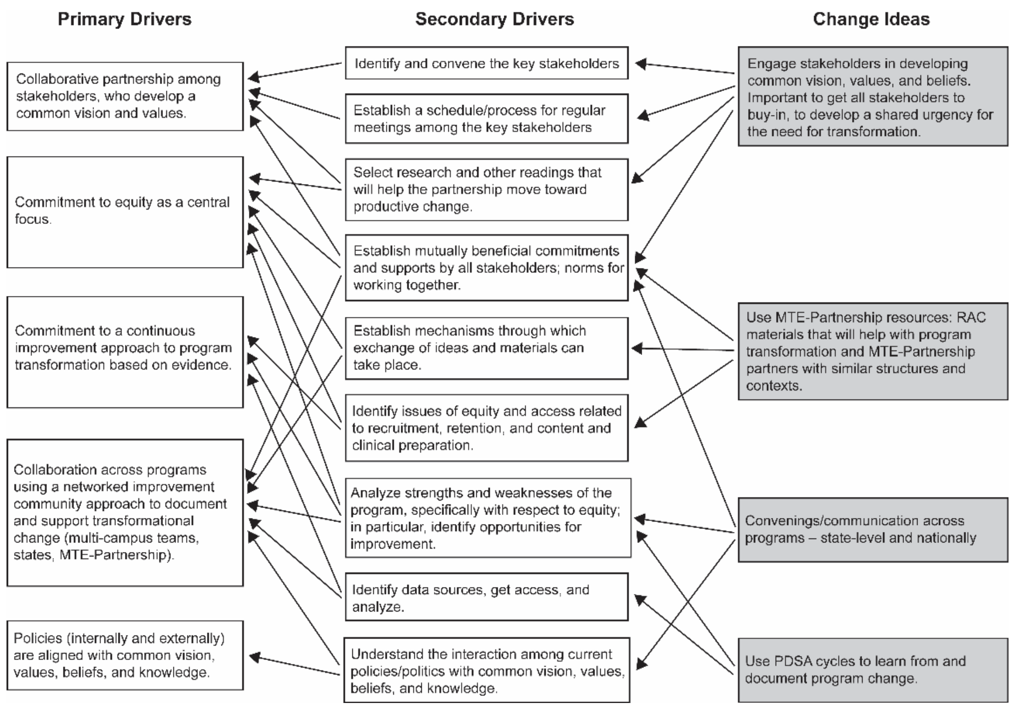

Figure 1. Driver diagram guiding the work of the NIC-Transform project.

Mississippi State University

Context

Mississippi State University is a land-grant institution with a distinction of being a global research institution. Mississippi State is designated "R1: Doctoral Universities – Very high research activity" and is the leading research institution in Mississippi. Student enrollment was 22,201 students for Fall 2019. The secondary mathematics program is housed in the Department of Curriculum, Instruction, and Special Education. Students earn a degree in secondary education with a concentration in mathematics, although the degree program is essentially a degree in mathematics. A Master of Arts in Teaching is also available through the department. The program graduates 10 to 15 students each year. The program has been continuously nationally accredited since the early 1970s. Mississippi State University educates a majority of teachers in the state and surrounding region.

The Mississippi State Partnership, as part of the MTE-Partnership, is hosted at Mississippi State with other partners being the Mississippi Association of Mathematics Teacher Educators and K–12 partners from Starkville Oktibbeha Consolidated School District and Golden Triangle Early College School. The Mississippi State University team consists of two mathematics educators, a science educator, and three mathematicians.

Aim and Primary Drivers

The aim of our work was to increase the number of students completing a degree in mathematics education and/or science education. Given the nature of our degree programs and the rural nature of our state, students could easily earn dual certification in mathematics and physics, chemistry, or general science. Our primary driver was to identify groups of students to target for recruitment. We need to understand how to effectively recruit incoming first year students, transfer students, and undeclared students. We have college recruiters that work effectively with incoming first year students, but we need to develop closer relationships with our partnering community college partners. Finally, we have a large number of undeclared students on campus. We need to understand how to reach out to these students.

Change Efforts

Our first PDSA Cycle concentrated on the undeclared students. We met with the director of undeclared students to learn about the characteristics of these students, effective communication with these students, and how many of these students already had interests in a STEM field. After study and discussion, we held a recruitment event for undeclared students. We had moderate success with our recruitment event. We plan to hold a similar recruitment event every year prior to the university's advising period. We will expand our recruitment advertisement to mathematics majors and other majors that have a heavy emphasis in mathematics.

Lessons Learned

We learned several valuable lessons. First, recruitment events need to be easily accessible to students. We will continue to hold these events in the student union during the primary lunch-time. Second, having students assist with the events is crucial. Students talking with other students serves to peak interest in what is being presented. Our students would initiate contact and then bring interested students to one of the professors/advisors working the event. Third, recruitment can be time-consuming and administrators, while they want more students, may not provide the resources or compensate your time for these events.

Next Steps

Through talking with students, we believe that students may be attracted to a double major in mathematics and mathematics education. Since the mathematics requirements for the two majors are similar, students could earn the double major easily. We currently are developing a prescribed dual degree that will be formalized for Fall 2019. We plan to have posters and flyers developed prior to the fall advising period.

Auburn University

Context

Auburn University is a comprehensive land, sea, and space grant institution – among the few that hold this distinction. Auburn's enrollment for the 2018–19 academic year was 30,440, which composes 24,628 undergraduate students and 4,706 graduate students. Auburn serves as the lead, and the public land-grant member, of the Central Alabama Mathematics Teacher Education Partnership (CAMTEP) of the MTE-Partnership. Other institutional partners of CAMTEP include Tuskegee University; Alabama State University; the Alabama Mathematics, Science, and Technology Initiative; Auburn City Schools; Tallassee City Schools; and Dadeville City Schools. The secondary mathematics program at Auburn University is housed in the Department of Curriculum and Teaching in the College of Education and has 12 to 20 bachelor's and/or alternative master's program completers each year. Each of these programs are initial certification programs and are Specialized Professional Association (NCTM, 2014) accredited with the Council for the Accreditation of Educator Preparation (CAEP). Thus, the programs have to meet specific standards for CAEP and the Alabama State Department of Education. Auburn

University also grants a doctoral degree, an educational specialist degree, and a master of education. Currently Auburn has 34 undergraduates at different points in their program and no alternative master degree students.

The programs are run by two nationally known mathematics teacher educators who strive to ensure that their teacher candidates receive the highest quality mathematics education that they can, based on research and innovative practices in the United States and the world. In addition, teacher candidates are provided with field experiences that are in alignment with their program goals and state and national standards.

Teacher candidates seeking initial certification at Auburn University take 42 hours of mathematics courses, which enable them to have a double major in mathematics and mathematics education. Other courses outside of mathematics education courses include foundation and core curriculum courses. Members of Auburn's local transformation team include mathematics teacher educators; mathematicians; mentor teachers; teacher candidates; the regional in-service director; the Alabama Mathematics, Science, and Technology Initiative director; the curriculum and teaching department chair; and members of the Office of Students Services of the College of Education.

Aim and Primary Drivers for Change

The aim of the team is to increase the engagement of stakeholders in secondary mathematics education program improvement. Our primary drivers for change are as follows. First, we would like to institutionalize the paired placement model for student teaching. We have university administrative support to do this but need to educate stakeholders. Second, we would like to increase engagement with clinical experiences for mentors in the methods courses. We communicate with mentor teachers but need to do more activities that ensure that mentor teachers' practices are in alignment with what is taught in methods courses. Third, we would like to redesign the math content sequence to become more aligned with the AMTE Standards (2017). We generally offer a capstone course, but this is an elective. Fourth, we would like to increase recruitment and retention of students. Our number of students has dropped to about 50% of what it was five years ago.

Change Efforts

Our first PDSA Cycle took place during initial stakeholder meetings. We planned to have meetings with stakeholders, such as the department chair of curriculum and teaching, the dean of the College of Education, representatives of the mathematics department, and the regional in-service center director. Overall the stakeholders were supportive of suggested program improvements and provided good advice on next steps. We will have a follow-up meeting with the mathematics department to further discuss suggested mathematics courses for teachers. Also, a mathematician attended the MODULE(S^2) training during the 2019 MTE-Partnership Conference. We plan to meet with the regional district leaders in September or October 2019 about mutual concerns related to mathematics teacher shortages and preparing new teachers.

Lessons Learned

The initial lessons that we learned through this process are as follows. We learned that we need to act quickly due to rapidly changing circumstances, such as new policies, new administrators, and other unforeseen events. We also need to take advantage of opportunities that can be used to improve the program, such as departmental initiatives that increase research productivity and visibility. We need to capitalize on intersecting needs of the program and other stakeholders, such as the need for more mathematics teachers in the region.

Next Steps

Our next steps include continuing to work on replacing the traditional student teaching model with the paired placement model. In addition, we plan to convene a meeting with district leaders to discuss program changes and needs of the schools related to mathematics education. Continuing discussions with the mathematics department about mathematics content related to teaching is also on our agenda. Finally, we are planning to develop a master's degree program to recruit and support career-changers.

University of Kentucky

Context

The University of Kentucky is a land-grant institution situated in the urban context of Lexington, but given its central location, it serves a large rural population across the Commonwealth. There are over 30,000 students at the university, which is home and leader to the Kentucky MTE-Partnership. While the Kentucky MTE-Partnership focuses on the improvement of secondary mathematics teacher preparation and retention, the programs at the University of Kentucky are housed within the Department of STEM Education in the College of Education. As the college's newest department (formed in 2011), it focuses on using a transdisciplinary framework to prepare mathematics and science teachers for the future STEM classrooms. There are two routes for initial secondary (Grades 8–12) teacher preparation in the department—a four-year, 120-credit-hour, undergraduate, double-major program, and a Master of Arts in Teaching program. While both program majors are called STEM Education, preservice teachers focus on their content area via a major in the area in which they wish to teach and is certifiable (e.g., mathematics). The programs, combined, graduate 10 to 15 students per year and are guided by program faculties that are made up of: STEM education and content faculty, K–12 teachers and administrators, the Kentucky Department of Education, and alumni. The Kentucky MTE-Partnership is active in the Program Recruitment and Retention (PR^2) RAC, co-leads the Secondary Teacher Retention and Induction in Diverse Educational Settings (STRIDES) RAC, co-leads the Transformations Working Group, and is a piloter for the Mathematics of Doing, Understanding, Learning, and Educating for Secondary Schools (MODULE(S^2)) RAC.

Aim and Primary Drivers for Change

The Kentucky MTE-Partnership is committed to producing STEM teachers who will function as professionals, lifelong learners, and leaders in their schools and communities. Candidates will gain hands-on experience in a variety of unique educational and research activities that will foster their interest, expertise, and enthusiasm for STEM education.

The primary drivers for change for Kentucky include: improving clinical experiences for students by creating and implementing more embedded course and clinical field experiences; be more intentional about infusing equity throughout all of our coursework and not only the methods experiences; understanding recruiting patterns of the students who apply and accept in order to increase the number of incoming students; expand programming opportunities for future teacher candidates so they can choose a program that fits their needs the best; and expand our partnerships, including schools, businesses, and industry.

Change Effort

Our former post-baccalaureate program was combined with another department, and there was little focus on STEM content, except within the one methods course, and little flexibility in the timeline for completion of the program. In order to give students more of a mathematics (and science, as applicable) focus and program flexibility, a new MAT program was created. Taking feedback from alumni, cooperating teachers, and other program faculty members, the new program was a complete transformation from the old one. For example, instead of traditional field experiences, candidates now get the opportunity to include informal learning experiences in their field experiences, as well as embedded methods courses at one of our school partnership sites. There are now full time and part time options available. Equity is a focal point, with strategies woven throughout all of the coursework, including practice and infusion of strategies for difficult and courageous conversations. The use of storylines frames the content, and professional noticing is the framework infused throughout, which provides a lens for observation, reflection, and professional growth.

Lessons Learned

The process took six months to plan and nine months to get through the university's approval processes. Throughout the process, the challenges and roadblocks were mainly from faculty rather than from partnership processes or an obligation to adhere to particular standards; change is difficult. The Kentucky MTE-Partnership uses a shared leadership model, especially in leading the programs, but we learned along the way that you still need someone to be *the* point person. While we used networked improvement communities (NICs) as a foundation group for our work, not everyone liked the idea of a NIC. Finally, even with a fancy, new, gold-standard-infused program, negative images of teachers still exist that can make recruiting a challenge.

Next Steps

Our next steps will focus on major recruitment and retention efforts. Universities are getting creative with the recruitment of students into the university, but we need to capitalize on this to promote teaching as a viable major and career choice. We are starting with the Educators Rising Partnership with high schools across Kentucky and partnering with our Educators Rising Collegiate group. We are looking at dual credit models and also unique admissions scenarios that help to take away barriers to students going into the teacher education program. We know our retention in our mathematics courses is not the greatest, especially in Calculus II. Already an active learning class, we are exploring additional ways that we can provide our students with the support and resources they need to successfully complete Calculus II.

University of Nebraska–Lincoln

Context

The NebraskaMATH Secondary Teacher Education Partnership (NebraskaMATH STEP) includes the University of Nebraska system campuses in Lincoln (which is the flagship), Omaha and Kearney, and the public school districts located in those same cities. The state of Nebraska only has 16 colleges and universities that prepare teachers; nearly all mathematics and mathematics education faculty have PK–12 responsibilities for preparing teachers. Thus, the focus of NebraskaMATH STEP has broadened to PK–12, rather than only secondary mathematics, and also involves others who prepare teachers of mathematics, such as special education and English Language Learners (ELL). Members of NebraskaMATH STEP include mathematicians, mathematics educators, special educators, district mathematics, special education (SpEd), and ELL coordinators, and other school, district, and regional PK–12 administrators. Members of NebraskaMATH STEP are quite active in the MTE-Partnership, including involvement in the Active Learning Mathematics Research Action Cluster (ALM RAC); Mathematics of Doing, Understanding, Learning, and Educating for Secondary Schools (MODULE(S^2) RAC), Equity and Social Justice Working Group (ESJWG), and Secondary Teacher Retention and Induction in Diverse Educational Settings (STRIDES RAC).

For this panel presentation, the focus is on efforts in Lincoln (at the University of Nebraska–Lincoln [UNL] and Lincoln Public Schools [LPS]). Lincoln is the second largest city in Nebraska, with about 26,000 students at UNL and over 40,000 students in LPS. UNL has two overlapping teacher preparation programs: a traditional undergraduate program and an 18-month master's level program for those with substantial undergraduate mathematics coursework. Recently, across both programs, UNL graduates about 25 new secondary mathematics teachers per year.

Aim and Primary Drivers for Change

The aim of Nebraska's efforts is that all programs for preparing teachers of mathematics (PK–20, including SpEd and ELL) will work to align their programs to meet the AMTE Standards (2017), including (a) a deep and integrated focus on equity, (b) foundation for career-long learning, (c) focus on mathematics, (d) shared responsibility across stakeholders, and (e) commitment to improving effectiveness

The primary drivers for change include (a) understanding mathematical dispositions of students (engagement, anxiety, etc.); (b) understanding teacher prep programs statewide; (c) improve clinical experiences; (d) focus on cooperating teachers; (e) strengthen school-university partnerships; and (f) transform programs with a focus on equity.

Selected Efforts: Infusing Equity

Although the NebraskaMATH STEP team is working on all of the drivers, for the transformations panel presentation at the 2019 MTE-Partnership Conference, we highlighted the efforts toward *infusing equity.* In partnership with the ESJWG, UNL and LPS worked together to revamp the mathematics methods courses for future secondary teachers to focus on the AMTE Standards (2017). This course redesign has an explicit and central focus on equity. At UNL, the Department of Teaching, Learning and Teacher Education engaged in department-wide efforts to infuse a focus on equity throughout secondary teacher preparation. It was recognized that the responsibilities for equity infusion cannot solely be on the methods instructors; equity infusion needs to be part of all other education courses future teachers take at UNL. At the same time, LPS ended the low math track for K–8 students. All students are now in either on grade level or above grade level in mathematics courses. Students who struggle to be successful in on grade level mathematics courses are provided additional support through mathematics intervention time daily.

Lessons Learned

Across the Nebraska partnership, we learned that higher education faculty and PK–12 personnel have a genuine desire to collaborate, despite official competition among some of the institutions. However, keeping all those engaged in the work focused across all partners is hard. Transformation efforts are an add-on to most people's current responsibilities, so it can be hard to make time for the work. We realize we need better ways to share progress across sites and partner locations as well as a more explicit focus on PDSA Cycles as part of our NIC.

Next Steps

One next step for the Nebraska partnership is to share the UNL/LPS equity work across the rest of the partner institutions. In related work, UNL and two Nebraska community colleges recently received an NSF S-STEM award for funding undergraduate scholarships for STEM majors; this project will help to bring community colleges into the partnership efforts. Finally, we will supplement online and email communication by continuing to meet in person at least once per year, during our state meeting (Nebraska Association of Teachers of Mathematics) in the fall.

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Equity & Social Justice Working Group

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A well-prepared beginning secondary mathematics teacher embraces the goal of academic success for each and every student, and also understands how social, historical, and institutional contexts affect teaching and learning (Association of Mathematics Teacher Educators [AMTE], 2017). As the members of the MTE-Partnership consider the preparation of future secondary mathematics teachers, we must respond to these challenges to produce knowledgeable and committed advocates for each and every child.

Problem Addressed and General Approach

The Equity and Social Justice Working Group (ESJWG) was formed to support each Research Action Cluster (RAC) as well as the MTE-Partnership as a whole in order to achieve the aims identified in each of the driver diagrams. Specifically, the aim of the ESJWG is to attend to both the equity-driven dispositions of future secondary mathematics teachers as well as ensure they are well equipped to implement equitable teaching practices. This aim has two foci, for the future secondary mathematics teachers to see themselves as advocates, and for them to have the tools to achieve that goal. Along with this aim statement, a first draft of a complete driver diagram for the ESJWG was completed in early 2018; see Figure 1. The driver diagram identifies the primary drivers necessary to advance the aim as well as the secondary drivers, which serve to describe the system components that we hypothesize will activate the primary drivers. Finally, the change ideas for the ESJWG, as listed in the driver diagram, are initial thoughts on processes to be tested relative to the associated primary and secondary drivers through Plan-Do-Study-Act (PDSA) Cycles.

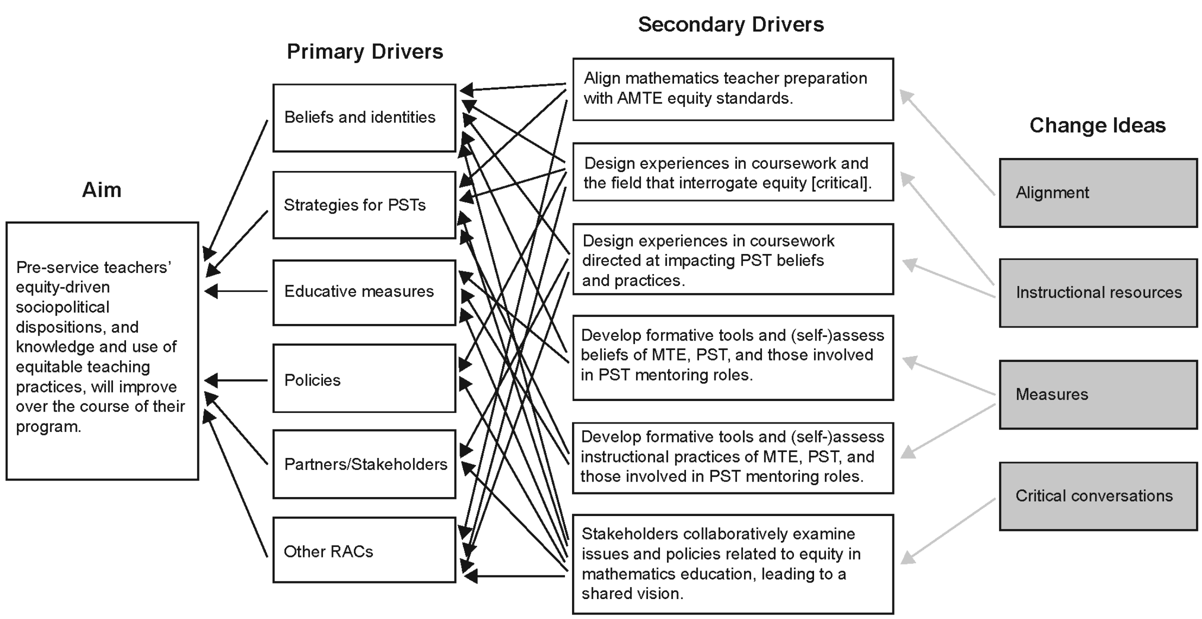

Figure 1. ESJWG driver diagram.

With the first draft of the driver diagram (i.e., theory of change) established, ESJWG also sought to refine its role within the MTE-Partnership. It was recognized that the ESJWG buttresses the five current primary drivers in the driver diagram of the MTE-Partnership: transforming programs, creating a vision, improving clinical preparation, increasing content knowledge, and improving recruitment and retention. Furthermore, equity and social justice issues are underlying values in all elements of the preparation of future secondary mathematics teachers. Thus, equity and social justice stands apart as a secondary driver distinct from the other RACs. With these dual roles in mind, this refined understanding of the role of the ESJWG led to a revision of the MTE-Partnership driver diagram, shown in Figure 2. Added are the gray U-shaped figures expressing the roles of equity and social justice in each of the other primary drivers, as well as the ESJWG added as a secondary driver.

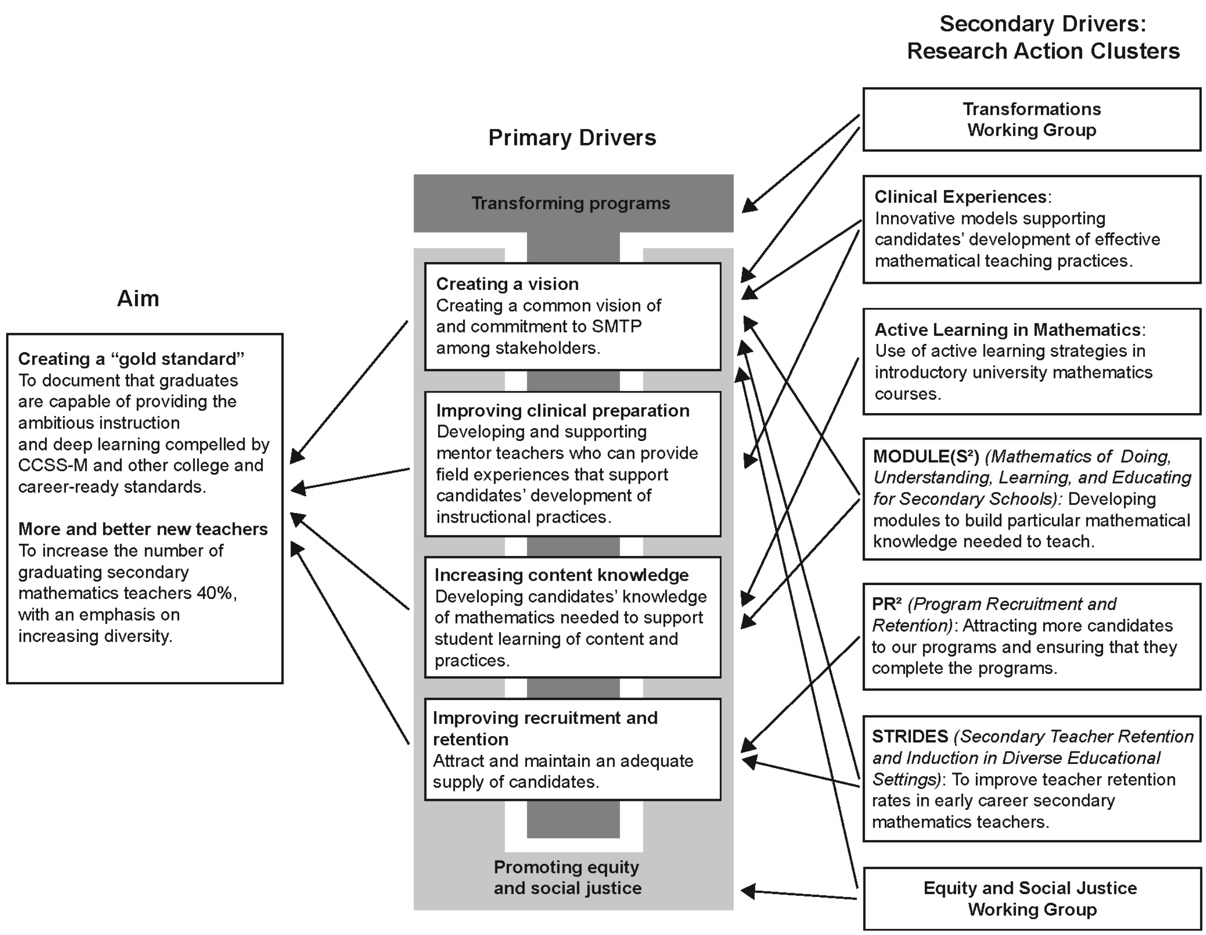

Figure 2. MTE-Partnership driver diagram after ratification of the ESJWG.

With the aim identified, and drivers established, the ESJWG subsequently initiated liaison structures with the other RACs and launched a small number of PDSA research cycles during the 2018–19 academic year. Two of these research projects are especially worth noting, as they seem to have gained some ongoing interest. One subgroup of the ESJWG seeks to identify resources that could support teacher educators attending to equity within their practices. In particular, how these might be catalogued in such a way to make them accessible for interested mathematics teacher educators. A second subgroup began to consider how we might measure our aim, specifically the equity-directed dispositions. They began to explore the EQUIP tool developed by Reinholz and Shah (2018), which will be piloted with future secondary mathematics teachers. A final, significant project of the 2018–19 academic year was to contribute a chapter to the forthcoming MTE-Partnership monograph (Martin, Lawler, Lischka, & Smith, 2019). The chapter highlights the development of the group's driver diagram, and identifies factors that impact the extent teacher educators, future secondary mathematics teachers, and in-service teachers attend to equity within their instructional practices.

Accomplished During 2019 Conference

The 2019 Conference had social justice and equity emphasized in a number of different ways. The theme for the conference ("The MTE-Partnership: Transformation. Equity. Leadership.") included an explicit emphasis on equity. Moreover, equity and social justice were mentioned in six different presentation titles, and seven different presentations were facilitated by members of the ESJWG on the program. Furthermore, keynote speaker Etta Hollins explicitly addressed issues of equity and social justice in regard to program transformation and evaluation. Similarly, Kathryn Chval's keynote address discussed the notion of disruption and what it means in terms of education and program transformation. Based on the presentations and conversations among members of the various RACs, it is clear that issues of equity and social justice are permeating through the entire partnership in different ways and are being viewed as of central importance to the MTE-Partnership. There was also a program transformation discussion session prompted by MTE-Partnership membership input, which considered how issues of equity and social justice are integral parts of the various mathematics teacher educators' local program transformation efforts. Therefore, we are now reflecting on how the ESJWG can support and interact with teams and their goals to attend to equity and social justice in their program transformation efforts.

The ESJWG also discussed the liaison structure because most members of the working group are also members of RACs and serve as liaisons to the RACs they work within. We also have liaisons who are not active members in the ESJWG, yet these individuals have volunteered to serve as liaisons between the RACs they are active members within and the ESJWG by occasionally joining monthly meetings calls and communicating between the group and their RACs. Thus, we have two different types of liaisons, which we are still working on clearly defining and differentiating in terms of their specific roles. The emphasis on considering program transformation within teams also increased, which means future consideration needs to be placed on how the ESJWG might liaise with both RACs and institutional teams.

Additionally, we considered the structure of communication between the RACs and the ESJWG. Particularly, during a discussion session held at the conference, members of the Mathematics of Doing, Understanding, Learning, and Educating for Secondary Schools (MODULE(S^2)) RAC suggested having some formal structures in place. MODULE(S^2) wondered when would be the best time to contact the ESJWG and through what means. To make our interactions with other RACs more robust we are planning to have each RAC have their liaison sign up for one of our monthly meetings and present issues they are facing and would like to obtain feedback on, or to share their current scholarly efforts related to issues of equity and social justice. There was also discussion related to possible PDSA Cycles and research projects that are collaborations between RACs and the ESJWG. For example, members of the ESJWG can create a measure relative to the group's overarching aim, which could be refined and validated through work being done in the various RACs. The ESJWG can also collaborate with other RACs to examine a phenomena from an equity and social justice stance.

During the 2019 MTE-Partnership Conference, the ESJWG invited liaisons to facilitate dialogue with their RAC focusing on deficit ideologies (Aguirre et al., 2017) and courageous conversations (Boyd & Glazier, 2017). The various RACs were asked to: reflect on the extent they addressed deficit ideologies; review their driver diagrams and aim statement for the extent it focused on deficit ideologies; describe challenges that they may face during efforts to address deficit ideologies; discuss the extent their RACs facilitate and nurture courageous conversations; and explicate how ESJWG can help the RACs address challenges faced. All of the RACs provided feedback on the emergent themes of their discussions related to the prompts.

To address deficit ideologies, it was suggested that messaging campaigns ought to focus on reframing the narrative to promote inclusive and equitable practices in mathematics teaching and learning. The public discourses, resources, and other forms of communications utilized within school settings also should seek to acknowledge inequities and opportunity gaps that exist. Additionally, it was recommended that professional development training, which focuses on non-productive discourse and deficit language, can help individuals become aware of implicit bias, deficit ideologies, and strategies that can empower individuals to be change agents (Hernandez, Morales, & Shroyer, 2013; Martin, 2012; Priestley, Edwards, Priestley, & Miller, 2012; Riegle-Crumb & Humphries, 2012; Stein, Engle, Smith, & Hughes, 2015; White, Crespo, & Civil, 2016).

The extent to which RACs attend to deficit ideologies varied. In some RACs, the concept is not directly addressed, while, in other RACs, attending to deficit ideologies is embedded in the team's PDSA Cycles. For instance, in the Clinical Experiences RAC, the team plans to examine the nature of clinical experiences with a lens of equity and will consider how equity is attended to in various mathematics teacher education programs. The Active Learning Mathematics RAC also noted that they considered gradual introductions of the ideas presented and the use of strategic hiring to help to advance change ideas at their institutions.

The RACs acknowledged that they are challenged to attend to deficit ideologies due to time constraints, and lack of professional development training for them and their colleagues, or resources that could be used. Therefore, the ESJWG can seek to provide training modules on deficit ideologies that can be used to educate individuals of means to facilitate messaging campaigns, as well as facilitate faculty learning opportunities that are time sensitive.

Moreover, to support courageous conversations, there is a need to engage in conversations with faculty and students. The conversations with faculty can focus on how the programs can support diverse candidates, and how to foster community partnerships that focus on addressing disparities. The conversations with students should focus on challenges various groups face, which may mirror the institutional climate and culture. It was also suggested that space should be created for students (i.e., future secondary mathematics teachers) to work with faculty to achieve various RACs goals and engage in courageous conversations. Furthermore, the networks formed within the RACs provide opportunities for various team members to navigate difficult circumstances. Nonetheless, the extent to which the various RACs attend to courageous conversations and deficit ideologies could be enhanced and could be a point of further focus by the ESJWG in liaising with RACs.

Plans for the Coming Year

At the 2019 Conference, the ESJWG charted the course for the upcoming year. One of the goals identified is to document all aspects of the PDSA Cycles and report lessons learned within the group. Particularly, members were asked to record two PDSA Cycles per semester and share at least one of those cycles with the group during the monthly online meetings. Another related goal is for members of the ESJWG to expand their research activities related to issues of equity and social justice, which may be independent of their RAC work. Additionally, for the upcoming year, the ESJWG will plan to develop a survey that documents secondary teacher preparation programs' structures and their policies and practices that attend to equity and social justice. Surveying programs will help to get a general sense for how programs are structured, what role issues of equity and justice play in programs, and what proportion of programs incorporate issues of equity and justice. The survey instrument also can help to identify exemplary programs that seek to prepare future secondary mathematics teachers to support equitable learning opportunities, address social justice issues, and be agents of change. After identifying exemplary programs, the ESJWG will subsequently and purposefully select programs to investigate how their programs were transformed, and the barriers the programs faced and how they overcame those barriers (or did not), in an effort to gather insights into program transformation. The information garnered could be translated to make similar transformations across the membership of the partnership.

Opportunities for Engagement

Membership in ESJWG is by individual, not by team, like in each of the RACs. There are two types of members in the ESJWG: those who are *active,* and those who serve as *liaisons*. The active members are fully invested in the research project defined in the ESJWG driver diagram. This includes supporting the development of PDSA Cycles, responding to inquiries from other RACs, and considering the structures for effective liaising with the RACs. There are also members of ESJWG who are exclusively liaisons to the RACs to which they belong. They serve primarily as a point of contact between ESJWG and their RAC. Occasionally they will bring a problem of practice to ESJWG, and occasionally ESJWG will do the same. Most of the active members also participate in other RACs, and in that way serve as liaisons as well.

The ESJWG meets monthly via an online video conference call for one hour. These meetings are attended by its active members. Liaisons are welcome as well; however, liaisons typically participate when there is a relevant agenda item.

During the upcoming academic year, 2019–20, the ESJWG monthly meetings will be structured to devote a significant portion to advancing a research agenda. Five of the meetings will be led by a liaison in order to examine the development or outcome of a PDSA Cycle from their RAC. The remaining meetings will have a similar focus on analyzing the design or result of a PDSA cycle, but for research conducted by active members of ESJWG rather than the other RACs.

All individuals in the MTE-Partnership are welcome to join the ESJWG at any time. Individual members are welcome because unlike RACs, ESJWG does not require team-based membership. ESJWG is a place to have focused conversations, specifically about advancing equity and justice issues in each of the RACs. If you are interested in this sort of role, we encourage you to become a liaison or an active member. Please contact Brian R. Lawler at brian.lawler@kennesaw.edu to express your interest and join.

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RESEARCH ACTION CLUSTER REPORTS

Clinical Experiences

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Teacher preparation programs face significant challenges in providing secondary mathematics teacher candidates with quality clinical experiences. The problem is two-fold:

1. There is an inadequate supply of quality mentor teachers to oversee clinical experiences. Too few teachers are well versed in implementing the *Common Core State Standards for Mathematics* (CCSS-M; National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010), and teachers are especially inexperienced with embedding the standards for mathematical practice into their teaching of content standards on a daily basis. Further, many veteran teachers do not implement the mathematics teaching practices as discussed in *Principles to Actions: Ensuring Mathematical Success for All* (National Council of Teachers of Mathematics [NCTM], 2014) on an ongoing basis.
2. Bidirectional relationships between the teacher preparation programs and school partners in which clinical experiences take place are rare. Such relationships that reflect a common vision and shared commitment to college and career ready standards and other issues related to mathematics teaching and learning are critical to the development and mentoring of new teachers.

The work of Clinical Experience Research Action Cluster (CERAC) encompasses a number of the principles and principle indicators from the 2014 Mathematics Teacher Education Partnership's (MTE-Partnership) *Guiding Principles for Secondary Mathematics Teacher Preparation Programs*, including fostering partnerships between institutions of higher education, schools and districts, and other stakeholders, such as state departments of education, and is focused on preparing teacher candidates who promote student success in mathematics. Moreover, the 2017 Association of Mathematics Teacher Educators' *Standards for the Preparation of Teachers of Mathematics* (AMTE Standards) state:

> An effective mathematics teacher preparation program includes clinical experiences that are guided on the basis of a shared vision of high-quality mathematics instruction and have sufficient support structures and personnel to provide coherent, developmentally appropriate opportunities for candidates to teach and to learn from their own teaching and the teaching of others. (p. 26)

In the CERAC, higher education faculty and partner school districts and schools work together to actively recruit, develop, and support in-service master secondary mathematics teachers who can serve as mentors across the teacher development continuum from pre-service to beginning teachers. Moreover, the CERAC helps to ensure that teacher candidates have the knowledge, skills, and dispositions needed to implement educational practices found to be effective in supporting all secondary students' success in mathematics as defined in the CCSS-M and other college- and career-ready standards.

The CERAC consists of 26 university-led teams, each consisting of at least one mathematics teacher educator, a mathematician, and a school partner. The CERAC is divided into three sub-RACs based on the three types of field experiences that we are implementing to meet the goals that we set forth in our primary drivers and our aim statement. The sub-RACs are: Methods, Paired Placement, and Co-Planning and Co-Teaching. Each sub-RAC is implementing Plan-Do-Study-Act (PDSA) Cycles based on their goals and objectives. Teams work together via conference calls, email, and the Canvas platform. They use Dropbox, Google Drive, and Canvas as ways of sharing files and materials. Additionally, they have held face-to-face meetings as a RAC that included breakout meetings for sub-RACs. The sub-RACS have overlap areas that drive and focus the RAC, such as the emphasis on the mathematics teaching practices (NCTM, 2014) and other equitable teaching practices, professional development for mentors related to the *Standards for Mathematical Practice* (National Governors Association & the Council of Chief State School Officers, 2010) and mentoring mathematics teacher candidates, and outcome measures. There are also specific goals to be attained within each of the sub-RACs, and each sub-RAC has developed its own specific research questions.

Update on the Collective Work of the RAC

Since the 2018 MTE-Partnership Conference, the CERAC has been busy implementing the work related to the National Science Foundation-IUSE grant, *Collaborative Research: Attaining Excellence in Secondary Mathematics Clinical Experiences with a Lens on Equity* (DUE-1726998, 1726853, 1726362). The project is led by principal investigators from Auburn University, the University of South Florida, and the Association of Public and Land-grant Universities (APLU). We are implementing an improvement science study to answer the following question: *How does a continuum of collaborative and student-focused clinical experiences, including co-planning /co-teaching and paired placement fieldwork models, impact pre-service teachers' equitable implementation of the Mathematics Teaching Practices (MTPs; NCTM, 2014) across multiple institutional contexts?* The research is being conducted by a consortium of 26 universities, along with their school partners engaged in APLU's MTE-Partnership, which is currently developing and testing three alternative models for clinical experiences using a networked improvement community (NIC) design (Bryk et al., 2015).

Throughout the 2018–19 academic year, members of the RAC continued implementing the project. During the 2019 Conference, RAC members reflected on their data collection plan; were led by the Equity and Social Justice Working Group liaisons in a discussion about deficit ideology and courageous conversations, which are terms related to equity lens of work; revised our driver diagram to make equity issues related to the students with whom the teacher candidates interact more explicit; discussed what RAC members gleaned from the conference that could help them in ensuring that teacher candidates across the 26 teams are developing equitable teaching practices and other skills that the teacher candidates need in order to facilitate their students' mathematics growth; and worked on populating Canvas with our materials and acclimating RAC members to the Canvas platform. RAC members also discussed challenges related to the goals that they have set for themselves as a RAC and for the grant and found some solutions. Figure 1 shows the revised driver diagram.

In addition to working on the grant during the academic year, members of the RAC submitted six chapters to a MTE-Partnership book (Martin, Lischka, Smith, & Lawler, in press) related to clinical experiences. The submissions are listed in Table 1.

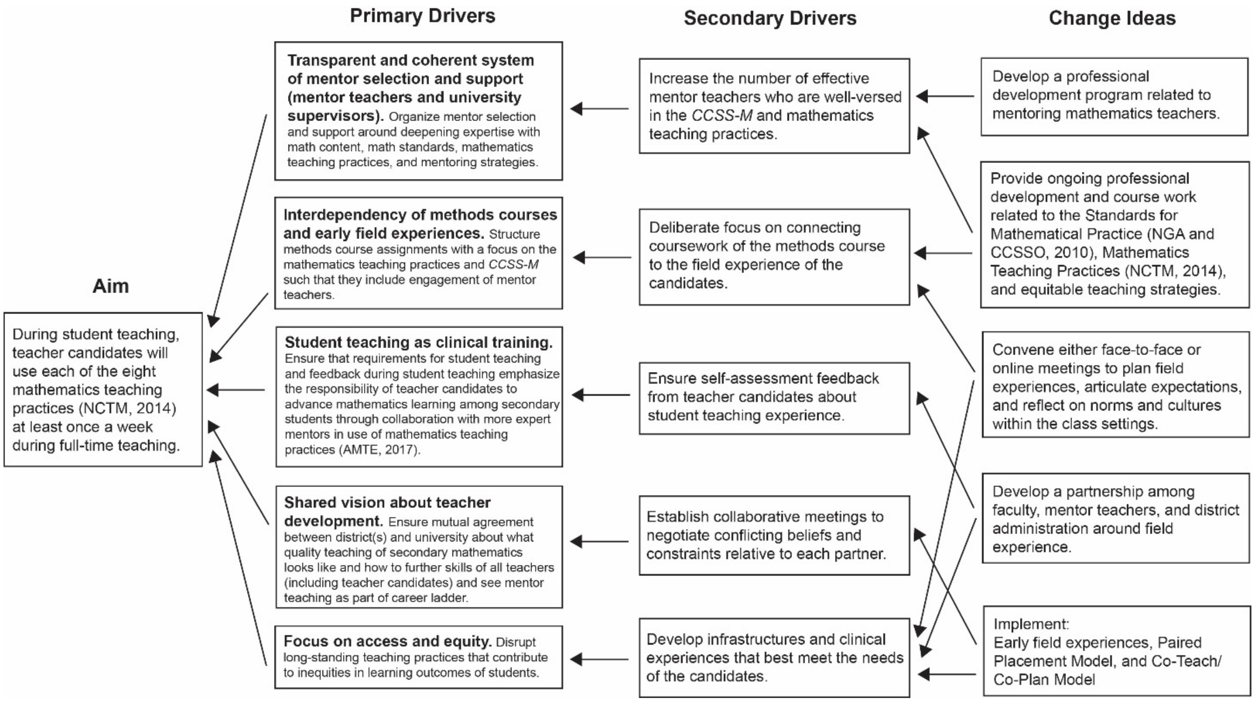

Figure 1. The revised CERAC Driver Diagram.

Table 1

CERAC Chapters in MTE-Partnership Book

Authors	Title
Strutchens, M. E., Erickson, D., Sears, R., & Zelkowski, J.	Clinical experiences for secondary mathematics teacher candidates
Strutchens, M. E., Sears, R., & Zelkowski, J.	Improving clinical experiences for secondary mathematics teacher candidates
Zelkowski, J., Yow, J., Ellis, M., & Waller, P.	Engaging mentor teachers with teacher candidates during methods courses in clinical settings
Grady, M., Sears, R., Stone, J., & Biagetti, S.	Using co-planning and co-teaching strategies to transform secondary mathematics clinical experiences
Strutchens, M. E., Whitfield, J., Erickson, D., & Conway, B.	Fostering collaborative and reflective teacher candidates through paired placement student teaching experiences
Mangram, C., Clarke, P. A. J., Waller, P. P., Ellis, R. L., & Castro-Minnehan, C.	Focus on improving clinical experiences in secondary mathematics teacher preparation

Members of CERAC also participated in a poster session, presentation, and webinar about the RAC as a whole:

Strutchens, M. E. & Sears, R. (January, 2019). *Collaborative research: Attaining Excellence in Secondary Mathematics Clinical Experiences with a Lens on Equity* (Poster). Joint Mathematics Meeting, Baltimore, MD.

Amick, L., Franz, D., Lischka, A., McNamara, J., Smith, W., & Strutchens, M., Discussants: Lawler, B. & Martin, W. G. (April, 2019). *Research Symposium: Collaborating to Improve the Preparation of Secondary Mathematics Teachers*. National Council of Teachers of Mathematics Research Conference, San Diego, CA.

Strutchens, M. E., Sears, R., & Zelkowski, J., (May, 2019). *Clinical Experiences Research Action Cluster.* MTE-Partnership Webinar Series.

Members of the CERAC also submitted a proposal to present our work at the Association of Mathematics Teacher Educators 2020 annual conference. We will probably do another poster session at the Joint Mathematics Meeting.

Consistent with the whole RAC goals, each of the sub-RACS worked on materials that they had already been developing and began thinking about PDSA Cycles that they would like to run in the fall to continue improving their products and processes. What follows are brief summaries of the work of each of the sub-RACS since the 2018 MTE-Partnership Conference.

Methods Sub-RAC

The methods sub-RAC has focused our work on the development of modules that educate teacher candidates on critical components of the teaching and learning of mathematics, as well as adding a critical component of engaging the mentor/cooperating teacher in an activity that culminates the modules' activities. Each module has three activities, providing methods course faculty flexibility in how much class time needs to be devoted to the module.

Mathematical Practices Module #1: The Methods sub-RAC finished the multi-year effort to develop the Mathematical Practices (NGA & CCSSO, 2010) Module #1 in 2017. It was made available for use across the entire MTE-Partnership for the 2017–18 academic year. Jan Yow of the University of South Carolina is providing methods faculty the information and materials. She began tracking MTE-Partnership use of this module in late 2018 as a means to understand contextual usage of the model. This module focuses on faculty engaging teacher candidates in a quadrilaterals' activity in which teacher candidates understand what engaging in the Standards for Mathematical Practices look like as students, watching a related short video with their cooperating teachers, and discussing the mathematical practices enacted in the classroom with students. Multiple surveys are included for faculty to collect pre-service teacher work, as well as a survey from their mentor teacher.

Lesson Planning Module #2: The work on this module has been through three PDSA Cycles. The module team solicited pilot sites for the 2018–19 academic year, collecting data for study, with revisions to the module ongoing in July 2019. Data analysis is not complete but we anticipate providing the module for MTE-Partnership use for the 2019–20 academic year. This module focuses specifically on lesson planning featuring the mathematics teaching practices of *Principles to Actions* (NCTM, 2014) accompanied by a lesson planning rubric focused on the Mathematics Classroom Observation Protocol for Practices (MCOP^2; Gleason, Livers, & Zelkowski, 2016). The lesson planning rubric is used by teacher candidates and mentor teachers to evaluate the planned lesson, revised lesson, and the implemented lesson. Minor revisions were made from Fall 2018 data with additional modifications from Spring 2019 data ongoing.

Student Feedback Module #3: The work on this module has gone a year beyond the initial development in the PDSA Cycle. It is being developed by Belinda Edwards, Patrice Waller, and Holly Anthony. Pilot sites are being solicited for the 2019–20 academic year in order to collect data to study and make revisions. The pilot sites include methods instructors from four different university partnership teams. The timeframe for full partnership rollout is planned for the 2020–21 academic year. This module focuses specifically on the value of providing students high-quality feedback as a teaching practice in relationship to mathematical goals to improve student learning outcomes.

Co-Planning/Co-Teaching Sub-RAC

During the 2019 MTE-Partnership Conference, the Co-Planning and Co-Teaching (CPCT) sub-RAC studied the results of the previous PDSA Cycle, determined how to proceed into the next PDSA Cycle, reflected on means to refine data collection efforts, and considered dissemination efforts for the 2019–20 academic year. Additionally, the team members acknowledged institutional constraints and considered how to sustain and improve on collaborative efforts for the next year.

The CPCT team members reviewed the PDSA objective for 2018–19 that focused on the extent professional development training increased the likelihood that instructional pairs (collaborating teachers and interns) used the six co-planning and co-teaching strategies, and enacted equitable teaching practices during clinical experiences. The team acknowledged that providing professional development training on the co-planning and co-teaching strategies, and means to attend to equity in secondary mathematics, was quite helpful. The professional development training clarified what was expected and modeled the desired mathematical processes and practices. Moreover, instructional pairs that were not able to attend the face-to-face meetings were encouraged to view videos (http://cream.coedu.usf.edu/Research/Attaining_Math_Excellence-Videos.html) of the training to enhance the nature of their enactment of co-teaching strategies during clinical experiences. The results from the MCOP^2 suggested that few participants obtained above-average ratings on all items; hence, the attention instructional pairs place on enacting equitable teaching practices could be enhanced. To support the teacher candidates' professional growth, one institution (Georgia State University) used the result of the MCOP^2 to facilitate cognitive coaching and reflection activities such that the candidates identified means to enhance their instructional practices.

As a result of the lessons learned from 2018–19 PDSA Cycle, the 2019–20 cycle will place greater attention on co-planning that promotes equitable learning opportunities. Particularly, the team will create training materials that focus on co-planning and will reflect on their actions as mathematics teacher educators as well as how their teacher preparation programs support equity. Thus, they will create videos that focus on co-planning, and they will engage in a self-study as mathematics teacher educators. To capture how they attend to equity at their institutions, they will audio-record their CPCT meeting discussions. The CPCT team also will monitor the data garnered from the MCOP^2 and consider means to improve the ratings teacher candidates receives.

To further streamline the data collection the team suggested that the Mathematics Teaching Practices survey be combined and collected as part of the pre-service teacher survey pre-survey, pre-service teacher post-survey, and as a standalone survey, at least once in the middle of the clinical experiences. Thus, the Mathematics Teaching Practices survey will continue to be collected three times; however, the schedule as to when the data will be garnered was refined. Additionally, the team agreed to discontinue the use of the CPCT Equity Checklist because the instrument did not adequately account for variance in the nature of clinical experiences across institutions. Therefore, the team sought to ensure they collected the data required for the grant (MTP survey, MCOP^2, completer survey, and focus group interviews), while also reflecting on instructional pairs use of co-planning and co-teaching strategies with a lens on equity via the CPCT pre-survey and post-survey.

The CPCT team identified conferences to attend, and articles they intend to submit during the 2019–20 academic year. Thus far, the team has submitted conference proposals to the National Council of Teachers of Mathematics and the Association of Mathematics Teacher Educators. The team also intends to submit a proposal to the International Congress of Mathematics Education. Additionally, the team plans to publish two articles focusing on developing teacher candidates' equitable teaching practices, and on how the apprenticeship of learning framework could be used to support the implementation of co-teaching during clinical experiences.

As discussed above, the CPCT team considered new objectives for their upcoming PDSA Cycle for the 2019–20 academic year, and the CPCT teams are striving to advance their research and scholarly activities. Being cognizant of institutional constraints, the team members have decided to embed some of the data collection instruments into course assignments and syllabi in order to sustain the work of the sub-RAC over time. Moreover, since some team members are being assigned other professional responsibilities within their local context, it is critical to develop means to sustain the change ideas at the institutional level, rather than at the faculty level.

Paired Placement Sub-RAC

Since the 2018 MTE-Partnership Conference, members of the paired placement sub-RAC have been implementing the paired placement model within their universities using instruments and protocols related to the CERAC IUSE grant. These included facilitating orientation sessions and workshops for teacher candidates and mentor teachers, updating syllabi based on previous PDSA Cycles, and updating other resources for implementation of the model. The paired placement team conducted PDSA Cycles and collected data to answer their questions relative to partnering with regional schools, co-teaching and co-planning, and the observational task protocol. In February 2019, Charmaine Mangram and Basil Conway IV joined the paired placement sub-RAC leadership team.

During the MTEP 2019 Conference, members of the paired placement sub-RAC made contributions to the conference's presentations and proceedings. Furthermore, during the RAC breakout sessions, members of the paired placement sub-RAC moved materials for implementing the model to the Canvas platform and are currently rearranging sub-RAC materials in a way to increase efficiency of shared cloud space by integrating Google Drive with CANVAS. Currently, the sub-RAC is creating a website (or living document) that will help us disseminate materials to a broader audience and help us to determine how well institutions are able to implement the model with integrity within their context.

Overall the CERAC has had a productive year and has made plans for a productive 2019–20 academic year. Others can get involved in implementing the materials developed by the sub-RACs by contacting the sub-RAC leaders: Marilyn Strutchens, Charmaine Mangram, and Basil Conway (Paired Placement); Jeremy Zelkowski and Belinda Edwards (Methods); and Ruthmae Sears (CPCT).

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Active Learning Mathematics (ALM)

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Problem Addressed and General Approach

The Active Learning Mathematics Research Action Cluster (ALM RAC) was formed to address the ongoing problems of undergraduate student success in first-year mathematics courses (Precalculus through Calculus 2—P2C2). Student success in first-year mathematics courses (or lack thereof) can prompt changes in decisions to pursue STEM majors; student retention from first to second year and the four- and six-year graduation rates are highly correlated with grades in first-year mathematics courses (in large part because mathematics courses are a near-universal requirement for graduation). Active learning strategies can improve student engagement and learning outcomes, but instructors need professional development and ongoing support to positively change their teaching practices. Further, different in-class materials (activities) are needed to better engage students.

The ALM RAC activities are detailed in our driver diagram (see Figure 1). Related to curriculum and assessment, ALM RAC partners work to develop and share materials that can support active learning, and also promote local coordination of assessment, through common homework, exams, and grading. Instructor capacities are addressed through initial and ongoing professional development; graduate student instructors are a unique (rotating) population of P2C2 instructors who need targeted supports. Student dispositions are measured via common surveys and other outcome measures. Focusing on a common vision entails significant will-building and local leadership to navigate policies and barriers, and to activate change levers (such as hiring and empowering a course coordinator).

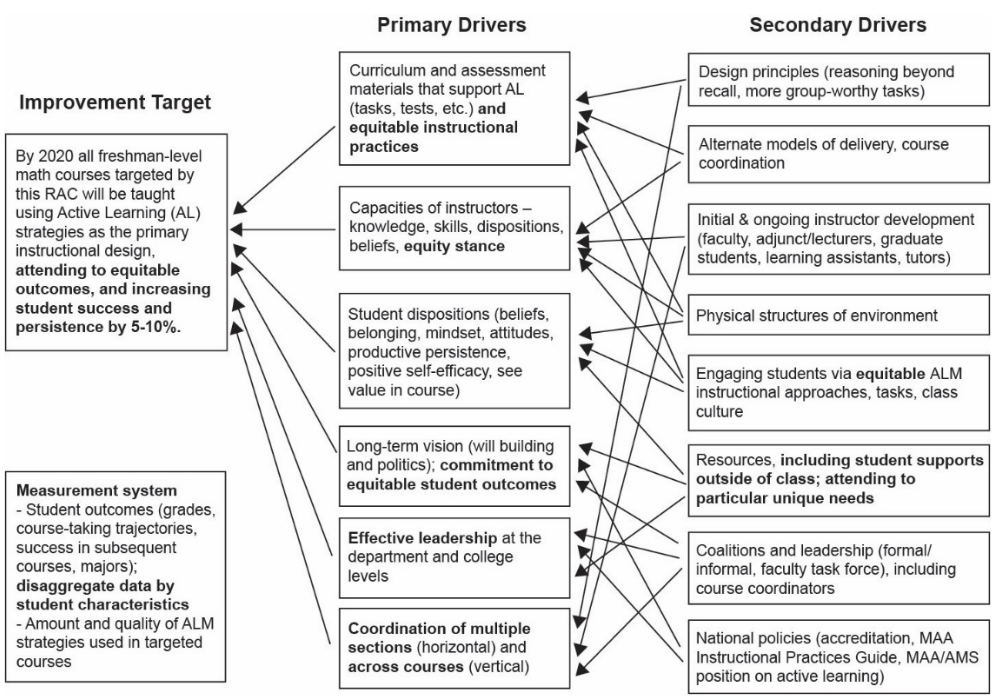

Figure 1. ALM RAC Driver diagram, as revised in 2019 to include leadership as a primary driver, and to update the secondary drivers.

Smith, W. M., Callahan, K. M., Strayer, J. F., Jones, R. S., & Augustyn, L. C. (Eds.). (2019). *Proceedings of the eighth annual Mathematics Teacher Education Partnership conference*. Washington, DC: Association of Public and Land-grant Universities.

Whereas ALM RAC members are focused on their own transformation efforts, a related coalition is studying how to effect departmental transformation to adopt and sustain active learning strategies. The Student Engagement in Mathematics through an Institutional Network for Active Learning (SEMINAL) project is a collaborative grant among the Association of Public and Land-grant Universities, the University of Colorado Boulder, the University of Nebraska–Lincoln, and San Diego State University (DUE-1624643, 1624610, 1624628, 1624639). Now in Year 3, SEMINAL's research findings related to change levers for active learning, and sustaining departmental transformation efforts are aligned with ALM RAC efforts.

Current Progress

We spent time during the 2018–19 academic year discussing how to organize and share the work of the ALM RAC. The issues related to hosting a dynamic repository of active learning materials are quite complex; we are working closely with the Transformations Working Group and the NIC-Transform project to learn from their work in knowledge generation and management systems. We launched a Google Drive folder with a spreadsheet as a dynamic table of contents, and we created space in an Open Canvas site for our ALM RAC materials, but we learned that neither of these places quite met our needs for the dynamic repository we seek to establish.

Across 2018–19, the ALM RAC members spent time collecting and making sense of local data. We continue to work on building a common database and some common guidelines for types of data to request from an institution, which could potentially show the impact of active learning reform efforts. Several ALM RAC members collaborated to write a summary of our work in the new MTE-Partnership book (Martin et al., in press).

At our ALM RAC work time in June 2019, we made further revisions to our 2018 version of our driver diagram, to better align it to our current work. We spent time discussing the intersections of equity and active learning, taking the stance that issues of equity need to be highly prioritized in active learning. Invoking active learning by itself does not guarantee more equitable student outcomes. Rather, instructors must explicitly focus on improving equitable student experiences and outcomes, as part of the process of actively engaging students.

We summarized our 2018–19 progress across our member sites, organized around our primary drivers (see Table 1). We particularly discussed in 2019 the need for leadership to be elevated as a separate driver. All sites have been finding that leadership is key, particularly getting formal leaders as advocates. It feels impossible to scale-up change efforts without the chair as an active proponent for the reforms. When external funding ends, an ongoing issue is to sustain practices that take resources (e.g., coordinator course releases, hiring learning assistants). Long-term vision also involves the need for campus administrators above the department who buy into sustaining efforts, including those with power to change policy power (such as promotion and tenure guidelines).

Finally, we spent time in summer 2019 setting up three lesson study groups within the ALM RAC. Members selected topics for initial focus, and worked to develop detailed lesson plans for fall 2019 implementation. Following implementation, the lesson study groups will meet for reflection and revision. We plan to then have a second round of lesson study in 2020.

Opportunities for Engagement

The ALM RAC welcomes additional partners who want to engage, from helping to develop a dynamic repository of materials, to engaging in lesson study for P2C2 lessons. We are increasingly convinced how much contextual features and personal relationships impact the successful implementation and institutionalization of ALM efforts, so we appreciate having diverse partners whose collective experiences can better span the many variations.

We note that the 2018 publication by the MAA of an *Instructional Practices Guide*, has many excellent principles for actively engaging students in learning mathematics. This publication is a great resource for helping to start local conversations about mathematics teaching and learning and has many practical tips for increasing student engagement. Local teams can implement or increase course coordination; coordination can help to sustain improvements and address inequitable student experiences and outcomes. Finally, those interested in improving P2C2 teaching and learning need to approach departmental transformation systemically, recruiting key leaders within and above the mathematics department in order to effectively initiate, implement, and sustain changes.

Table 1

ALM RAC 2018-2019 Progress on Primary Drivers

Curriculum and assessment materials that support AL (tasks, tests, etc.) and equitable instructional practices
Each campus is building own set of materials; many pull from University of Colorado materials. Textbook selection can be contentious and supports or inhibits ALM adoption. Building local materials can be a way to get people on board (ownership); sharing materials in useable form is an ongoing consideration (OneDrive, Google Drive, WikiSpace, Dropbox)
Capacities of instructors – knowledge, skills, dispositions, beliefs, equity stance
ALM RAC members each doing PD of some type (formal or informal) with instructors (including graduate student instructors, undergraduate learning assistants)
Student dispositions (beliefs, belonging, mindset, attitudes, productive persistence, positive self-efficacy, see value in course)
Some ALM RAC members are surveying students. Campuses engaged in comprehensive transformation efforts seem to be improving student outcomes.
Long-term vision (will building and politics); commitment to equitable student outcomes
Each ALM RAC member working on this; a key focus of ALM RAC meetings is sharing current lessons learned. Challenges to scaling up are often due to lack of buy-in. In some cases, collecting local data is (or is the foundation for) getting more people on board that there is a problem.
Coordination of multiple sections ("horizontal") and across courses ("vertical")
Each ALM RAC member is working on better coordination, along with hiring/designating coordinators. Getting buy-in for common assessments and common grading is a tough sell in some locations. Coordination can be argued as a structure for increasing equitable outcomes.

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The Mathematics of Doing, Understanding, Learning, and Educating for Secondary Schools[1]

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Overview and Problem Statement

The Mathematics of Doing, Understanding, Learning, and Educating for Secondary Schools (MODULE(S^2)) Research Action Cluster (RAC) is focused on the development of prospective secondary mathematics teachers' mathematical knowledge needed for teaching (MKT; Ball, Thames, & Phelps, 2008; Rowland, 2013) within upper-level content courses. The work of the RAC aims to address the identified problems that (a) prospective secondary mathematics teachers often do not find connection between upper-level mathematics content courses and teaching secondary mathematics (Goulding, Hatch, & Rodd, 2003; Zazkis & Leikin, 2010), and (b) prospective secondary mathematics teachers must deeply understand the mathematics they are going to teach and learn it in a way that is consistent with expectations of them as teachers (Banilower et al., 2013).

In response to these problems, the MODULE(S^2) RAC has collaborated with mathematicians, mathematics educators, and K–12 teachers to design 12 educative curriculum (Davis & Krajcik, 2005) modules in the content areas of Geometry, Algebra, Statistics, and Mathematical Modeling. Each module includes opportunities for PSMTs to engage in mathematical tasks that are set in explicitly pedagogical settings, for the purpose of developing prospective secondary mathematics teachers' MKT. The MODULE(S^2) RAC iteratively pilots and revises the materials in order to understand how to support instructors in implementing the materials, understand the ways in which dissemination of the modules across a wide range of institutions can vary, and improve the quality of the modules.

Current Progress

The MODULE(S^2) RAC has made progress on our work in several areas over the last year: drafting materials, piloting materials, attending to issues of equity and social justice, and organizing Plan-Do-Study-Act (PDSA) Cycles to document and refine our progress. Both the Mathematical Modeling and the Statistics writing teams have created all modules for their materials. With a complete set of materials across all four content areas, the RAC spent time at the annual meeting in June discussing variations in common features (e.g, Simulation of Practice activities) across the materials and considering affordances for learning among the variations. Through this discussion, the RAC members developed a plan for cross-team review to work toward building more cohesion among the materials.

In June 2018, the RAC began its first pilot year with a professional development session for nine faculty implementing Geometry and Algebra materials. The RAC then supported these piloters throughout the 2018–19 academic year as they implemented the materials in their courses across the United States. In June 2019, the RAC began its second pilot year with professional development for 12 faculty implementing Mathematical Modeling

[1] The work of this RAC is supported in part with a grant from the National Science Foundation (DUE-1726744, 1726707, 1726098, 1726252, 1726723, 1726804). Any opinion, findings, and conclusions or recommendations expressed in this material are those of the authors and do not necessarily reflect the views of the National Science Foundation.

and Statistics materials in the 2019–20 academic year. Piloting faculty are supported throughout the year through online discussion boards and video-conference meetings to discuss teaching practices along with specific questions about the materials.

In addition to drafting materials and supporting piloters, members of the RAC have engaged in PDSA Cycles focused on the development and implementation of activities focused on equity and social justice. These cycles were shared as brief reports at the 2019 MTE-Partnership Conference. This process has enabled RAC members to more effectively include equity-based activities and teaching practices in the modules. Other PDSA Cycles are planned to aid the RAC in (a) improving the Simulation of Practice activities and implementation, (b) taking up feedback from 2018–19 piloters to improve the materials, and (c) to improve the effectiveness of the planned professional development activities of the RAC.

Opportunities for Engagement

The MODULE(S^2) RAC is excited to share materials with any interested educators. At the local institutional level, implementing any or all of our course materials can initiate and foster discussion between mathematicians and mathematics educators involved in teacher preparation programs. This discourse across disciplines can support partnerships and move a department toward its goals for program transformation.

For the 2020–21 academic year, we are seeking formal piloters to join the second piloting of our Geometry and Algebra materials. Every formal piloter joining this experience will be supported by a stipend, professional development, and communication with the RAC and fellow piloters throughout the academic year. In addition to formal piloting, we welcome anyone who is interested in informally piloting materials in all content areas. You can help the MODULE(S^2) project move forward by reaching out to your colleagues about these opportunities. For more information regarding piloting, please connect with us on the MTE-Partnership MODULE(S^2) webpage.

Acknowledgement

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Program Recruitment and Retention (PR^2)

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Problem Addressed & General Approach

The Program Recruitment and Retention (PR^2) Research Action Cluster (RAC) engages in studying the recruitment of qualified and diverse prospective teachers and to retain these prospective teachers through completion of the program. Across the nation, the number of students entering teacher education programs or students seeking credentialing through fifth year or Master of Teaching programs are declining. The continued improvement target for this RAC is: *to increase the number of well-prepared secondary mathematics teachers entering the mathematics teaching workforce by at least 40% from each participating program by Summer 2022, reflecting the diversity targets of each program.*

Members of the PR^2 spent the past academic year studying recruitment strategies across institutions. We attempted to collect data across institutions to begin to form demographics about students self-selecting into mathematics education. This project was halted as it became increasingly clear that programs, demographics, and even student motivation varied greatly across the United States. The members revised the approach to gathering information about "what works" for recruitment across the institutions in the RAC. Our goal was to share these strategies with the larger mathematics education community.

Current Progress

In 2015, the RAC released the *Secondary Mathematics Teacher Recruitment Campaign Implementation Guide* (Ranta & Dickey, 2015). This guide consisted of nine modules designed to provide examples of recruitment and marketing related resources for recruiting teacher candidates. The RAC has begun revisions and updates to this implementation guide. We are also discussing the platform for permanently housing the guide. As this work continues, the RAC plans to release a second edition of the guide.

Additionally, the RAC members reviewed and revised the secondary drivers and change ideas. Two change ideas were prioritized for this upcoming academic year: courageous conversations and digital marketing. Readings aligned to equity and social justice priorities of MTE-Partnership were discussed. These readings will serve to help the RAC members develop model conversations as related to recruitment and retention. These conversations would be appropriate to use with faculty and staff across institutions. Additionally, the RAC members are developing positive messaging about becoming a mathematics teacher, being a teacher, and loving mathematics. The goal is to have a weekly pushout of positive messages that can be utilized by the larger mathematics education community.

Opportunities for Engagement

Three specific opportunities exist for engagement with the PR^2 RAC. We welcome all members to share images and messages that may be used in our digital marketing of positive messages. The RAC members hope to develop messaging that can be shared across various platforms. Second, we need bright, creative people to help design these messages. Finally, we plan to have in-depth reading about issues of diversity, equity, and social justice within the realm of recruitment and program retention. Anyone interested in becoming a member of our reading and studying group is welcome. We need many voices to understand this complex issue.

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Secondary Teacher Retention & Induction in Diverse Educational Settings (STRIDES)

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Problems Addressed and General Approach

- The U.S. faces a continuing shortage of well-prepared secondary mathematics teachers, among the worst of any subject (Malkus, Hoyer, & Sparks, 2015).
- The quality of teacher preparation, particularly related to pedagogical practice, significantly impacts new teacher attrition (Ingersoll, Merrill, & May, 2014).
- 50% of all teachers leave the profession within the first five years (Foster, 2010), and the rate of departure for mathematics teachers is highest in high poverty schools (e.g., Goldring, Taie, & Riddles, 2014).
- According to the Learning Policy Institute, 40% of newly hired mathematics or science teachers are underprepared, and underprepared teachers are far more likely to teach in schools serving students of color and low-income students (Carver-Thomas, 2018).

STRIDES members strive to create a sustainable and cohesive system of professional support (from pre-service through early years in the profession) to retain high-quality secondary mathematics teachers in the field.

Current Progress

The past year:

- Data analysis from the survey results:
    - Survey data was analyzed and two key findings were that teachers benefit most from on-site mentors/colleagues and that they want more meaningful relationships with their administrators.
- Intervention development and implementation:
    - Based on the survey findings, two sub-RACs were created within STRIDES. The first focused on developing and implementing teacher collaborative groups that focus on professional collaboration and knowledge, and the second targeted support for administrator/first-year teacher relationships.
        - Teacher collaborative groups: First- and second-year teachers and their mentors participate in monthly professional development sessions such as online meetings, Zoom panels with experts, and collaboratively reading and discussing timely, purposeful, and relevant content.
        - Support for administrators/first- and second-year teacher partnerships: First- and second-year teachers and their administrators participate in viewing and discussing 5-minute videos on best practice teaching strategies for teaching mathematics.

Both interventions are designed to not overburden the participants with large time commitments, to be feasible for national implementation with little funding, and to support the first-year teachers in a way that positively impacts job satisfaction and ideally teacher retention.

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At the 2019 MTE-Partnership Conference:

- Mentoring sub-RAC
  - Finalized evaluations/surveys (beginning, middle, end) for participants in the 2019–20 pilot year
  - Created a list of potential participants/contacts/recruitment strategies for participation in the 2019–20 academic year
  - Detailed a list of monthly interventions, set monthly meeting dates/times, set Zoom date for fall semester
  - Brief discussion on funding
- Admin sub-RAC
  - Noyce discussion
  - Looked at data collected from the 2018–19 pilot year in Knoxville and used it to guide the discussion on changes for the 2019–20 academic year of interventions, also considered follow-up opportunities with this group
  - Made changes and a plan for the second pilot year such as removing principal surveys, revamping questions, variations on future intervention design, and delegated tasks among members
  - Video discussion and selection of topics/videos, decisions on monthly intervention sessions (dates, topics, etc.)
  - Discussion on dissemination of data
  - IRB discussion
- Full STRIDES group
  - Group gained four new members who made valuable contributions during the RAC work time
  - Set meeting dates for fall semester
  - Created and submitted our Plan-Do-Study-Act Cycle
  - Brief discussion on funding
  - Lengthy discussion on equity and social justice and how we can integrate the charge of the working group into our RAC
  - IRB discussion and plan for each sub-RAC

Opportunities for Engagement

- Join us for larger scale intervention implementation for the 2019–20 academic year.
- Help us analyze the data and revise the interventions based on lessons learned in the 2018–19 pilot year and ongoing feedback during the 2019–20 academic year.
- Work with us to secure funding to support our work.
- Create a team to join the MTE-Partnership.
- Contact the RAC leader, Lisa Amick, by email at Lisa.Amick@uky.edu or by cell at 217-417-8605, for more information.

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RESEARCH PRESENTATIONS

Mathematical Modeling Lesson Adaptation for Increasing Local Relevance

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Abstract

How can local issues be effectively integrated into a mathematical modeling situation, particularly in the development of curriculum intended for widespread adoption? This report summarizes qualitative improvements made to student and instructor materials of the MODULE(S^2)[1]: Mathematical Modeling for Secondary Teaching curriculum after implementation at the Utah State University Blanding Campus STEAM Expo—an event that provides enriching experiences in science, technology, engineering, arts, and mathematics every spring for the community in rural Southeastern Utah. At this event, we engaged in-service mathematics teachers in two lessons pertaining to water consumption and conservation from the mathematical modeling course. We gathered resources to take a contextualized approach to the water impoverished region of Southeastern Utah, and adapted lessons accordingly. To guide our work, we applied a Plan-Do-Study-Act Cycle and collected qualitative data from the instruction and interaction with participants to improve our materials.

Introduction

The *Common Core State Standards for Mathematics* (CCSS-M) defines mathematical modeling as "the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions" (National Governors Association Center for Best Practices and Council of Chief State School Officers, 2010). Mathematical modeling is thought to be a critical process in learning mathematics because it allows for real-world interaction with mathematics, making appropriate assumptions, and justifying and reporting results. Kartal, Dunya, Diefes-Dux, and Zawojewski (2016) assert mathematical modeling to be a necessary part of STEM professions and an influencer to show if students can think creatively about complex problems. With mathematical modeling shown as an important part of mathematics education today, teacher preparation programs should ensure proper instruction of these skills. Teachers' mathematical knowledge for teaching (MKT) predicts gains in student achievement, and general pedagogical knowledge is highly related to instructional quality (Ball, Hill, & Bass, 2005; König & Pflanzl, 2016). Well-designed and facilitated mathematical modeling activities have great potential for addressing MKT and general pedagogical knowledge.

Inside teacher education, research has shown that courses taught with mathematical modeling tasks lead to increases in mathematical modeling performance (Bal & Doğanay, 2014; Karaci Yasa & Karatas, 2018). Studies have shown that mathematical modeling performance has a positive relationship with mathematical modeling conceptions (Anhalt & Cortez, 2016; Son, Jung, & I, 2017). Mathematical modeling conceptions have been positively tied to effective mathematical modeling instruction (Son et al., 2017). Instructing teachers using mathematical modeling tasks can improve performance, conception of mathematical modeling, and effective mathematical modeling instruction.

[1] Mathematics of Doing, Understanding, Learning, and Educating for Secondary Schools (MODULE(S^2)).

Smith, W. M., Callahan, K. M., Strayer, J. F., Jones, R. S., & Augustyn, L. C. (Eds.). (2019). *Proceedings of the eighth annual Mathematics Teacher Education Partnership conference*. Washington, DC: Association of Public and Land-grant Universities.

While mathematical modeling tasks are needed in teacher education, successful engagement in mathematical modeling requires that the tasks and their cultural contexts must have relevance. For this reason, we consider how mathematics and culture are linked, and aim to incorporate sociocultural modeling perspectives in this work. Sociocultural modeling is described as emphasizing “critical thinking about the role of mathematics in society, the role and nature of mathematical models, and the function of mathematical modeling in society” (Kaiser, 2017, p. 274). This idea of sociocultural modeling has close ties to D’Ambrosio’s (1985) *ethnomathematics*. Ethnomathematics focuses learning on the cultural experiences students have with mathematics to enhance understanding. Ethnomathematics utilizes students’ vast cultural background and experience in order to guide them through the modeling process. Sociocultural modeling emphasizes the idea of enculturation (Bishop, 1988) and students utilizing this knowledge of their culture to understand a problem with mathematics.

Problem and Purpose

Across the world, there are mathematical modeling prompts that are created for important local issues. Unfortunately, prompts may promote acculturation (Bishop, 1988) if not relevant to the culture in the area where these prompts are being used. Our goal is to improve the relevance of lessons for two water-related tasks by including local context and leading teacher participants to develop novel modeling tasks. Our specific research questions were:

1. What evidence of MKT development emerges during the implemented lessons from teachers in Southeastern Utah?
2. Do local teachers take away an appreciation for mathematical modeling, and see mathematical modeling as a productive approach for addressing water quality and water conservation issues?
3. What does this localization of a modeling task involve?

The implementation of the lessons involved in-service teachers, the participants. We, the researchers, believe that study results will address and inform the goals of preparing pre-service mathematics teachers. The in-service teachers’ knowledge of their students and their students’ cultures, interests, and motivations only enhance the study’s results for improving teacher preparation.

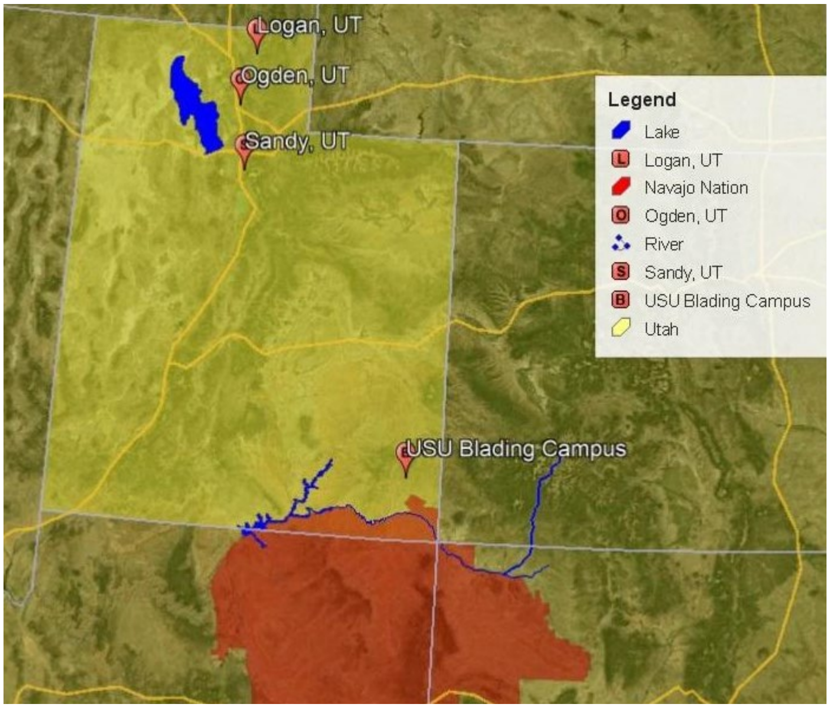


Figure 1. Map of Utah detailing the major locations pertaining to the study.

Conceptual Framework

A couple of viewpoints shaped the investigation of this study. First, the researchers used a Plan-Do-Study-Act (PDSA) Cycle developed by Deming (1993) to provide guided feedback for improvements throughout our investigation. The Mathematics Teacher Education Partnership's (MTE-Partnership) Research Action Clusters (RACs) frequently use this framework in curricular development studies.

Second, the researchers wanted to ensure local information about headlining issues were integrated into the workshop. Since the researchers were from an area in Northern Utah and the workshop was being held in Southern Utah (see Figure 1), we asked local collaborators for help gathering information and identifying speakers. A professional from USU Blanding pointed us toward numerous news articles that described various water-related issues in the area. We also connected with a local pre-service teacher from the Navajo Nation who shared some of her personal experiences regarding the effects of droughts, along with cultural and spiritual perspectives about water.

Third, the researchers applied the framework discussed in Aguirre, Anhalt, Cortez, Turner, and Simic-Muller (2019) that combines the process of mathematical modeling and discussions of social justice issues. After a broad social issue is addressed, a more specific situation can lend itself to mathematical modeling. The traditional cyclic mathematical modeling process is then engaged by making sense of the situation, researching information needed, making assumptions and choices, formulating and then solving a mathematical model, interpreting the solution, validating the outcome, and reporting results. After reporting the model's results re-contextualized in this specific social situation, participants build civic awareness and a sense of action. For example, our workshop used the Flint Water Task. The researchers introduced the social issue of water contamination and access to clean water in Flint, Michigan, and then raised the specific issue of water donations by several corporations. The participants then engaged in the modeling task, contextualized their solution to the social issue, and built their civic awareness and sense of action.

Additionally, our workshop was designed with equity-based teaching practices described by Aguirre, Mayfield-Ingram, and Martin (2013) to "strengthen mathematical learning and cultivate positive student mathematical identity" (p. 43). These equity-based teaching practices consist of going deep with mathematics, leveraging multiple mathematical competencies, affirming mathematics learners' identities, challenging spaces of marginality, and drawing on multiple resources of knowledge (Aguirre et al., 2013). The researchers framed this workshop as a collaboration. Our program included opportunities for participants to draw upon multiple resources, leverage multiple mathematical competencies, and go deep with mathematics. The workshop activities engaged participants in mathematical discourse and presented tasks with high cognitive demand with no set solutions and many solution strategies. The researchers recognized that a benefit of using mathematical modeling coupled with social issues is that multiple approaches emerge from the various backgrounds of participants.

Methods

The framework for the methods is a PDSA Cycle for improvement. In this setup, the researchers extensively planned the professional development, constructed viable research questions, made predictions, and noted what data would need to be collected in order to answer questions (see Appendix A). The researchers looked at participant work samples, attended to discussions during the sessions, kept notes as presenters and observers, and collected information through pre- and post-surveys to answer the research questions highlighted in this study. In each of the sessions outlined in Figure 2, the researchers placed opportunities to look at participant work samples. In the first two sessions, the participant work consisted of posters made for gallery walks to report their results (see Figure 3). One person presented material while the other acted as an observer and recorded noteworthy observations. These notes collected were structured to look for mathematical and pedagogical connections, evidence of motivation and reflection, focus in the modeling process, and general observations on how the group work proceeded. The researchers constructed a pre- and post-survey to provide additional information to the discussions. They consisted of a combination of open-ended questions and questions with a Likert scale. The two surveys were administered at the beginning and end of the workshop (see Appendix B). The questions on both the pre- and post- survey were used to track potential evolution of conceptions of mathematical modeling, beliefs on modeling social issues, and comfortability of using mathematical modeling tasks in their classrooms. The pre-survey contains questions informed the researchers on the background of the participants. The post-survey contains questions helped the researchers plan for future workshops.

The Do phase of our PDSA Cycle included 17 workshop participants. In the Study phase, the researchers compiled and synthesized the data to answer research questions. Due to the qualitative nature of this study, researchers found relevant information in the observation notes to answer research questions. Surveys were interpreted in a similar manner for all open-ended responses. Participants' work was looked at for evolution of mathematical modeling practices and evidence of other areas. This data was used to answer our research questions and to direct future actions.

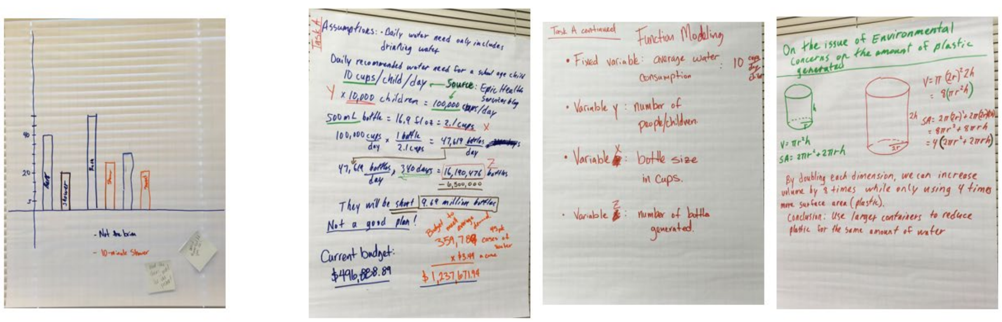
Workshop Plan

Water Conservation Session
- Shower v. Bath
  Task involves analysis of whether it is more water efficient to take a shower or a bath.
- Localizing Questions
  a. Where does your water come from?
  b. How much is water conservation an important concern locally?

Water Contamination Session
- Flint Water Task
  Tasks involves assessing the amount of water bottles needed to support the youth of Flint for one year as well as eventually creating a strategy to lessen the environmental impacts by all of the plastic waste created.
- Localizing Questions
  a. What are the clean water daily needs in your community?
  b. What other contamination issues have you heard of?

Local Water Issues Session
- Introduce Cultural Perspective on Water in the Area
- Present Summary of Headline Issues
- Create Modeling tasks
  Participants create localize modeling tasks to fit the needs of their students.

Figure 2. Outline of workshop plan, showing the major tasks and activities.



Session 1 Session 2

Figure 3. The participants' work during the sessions consisted of posters made for gallery walks to report their results.

Findings and Discussion

Figure 4 shows a shift in comfort level from the pre- and post-surveys. More than three-fourths of the participants stated that they were either certain or almost certain they would use these materials in their own classrooms. Almost three-fourths of the participants self-reported only occasionally or rarely using culturally relevant word problems.

Three excerpts from the surveys showed appreciation for mathematical modeling, and its help in understanding social issues:

- "Mathematical modeling is difficult and time consuming to plan, but the outcome and affect on students seems to greatly justify the effort."
- "… mathematical modeling can begin to normalize and focus the conversation [about community issues]."
- "Models help to judge the validity of proposed solutions and their potential effectiveness."

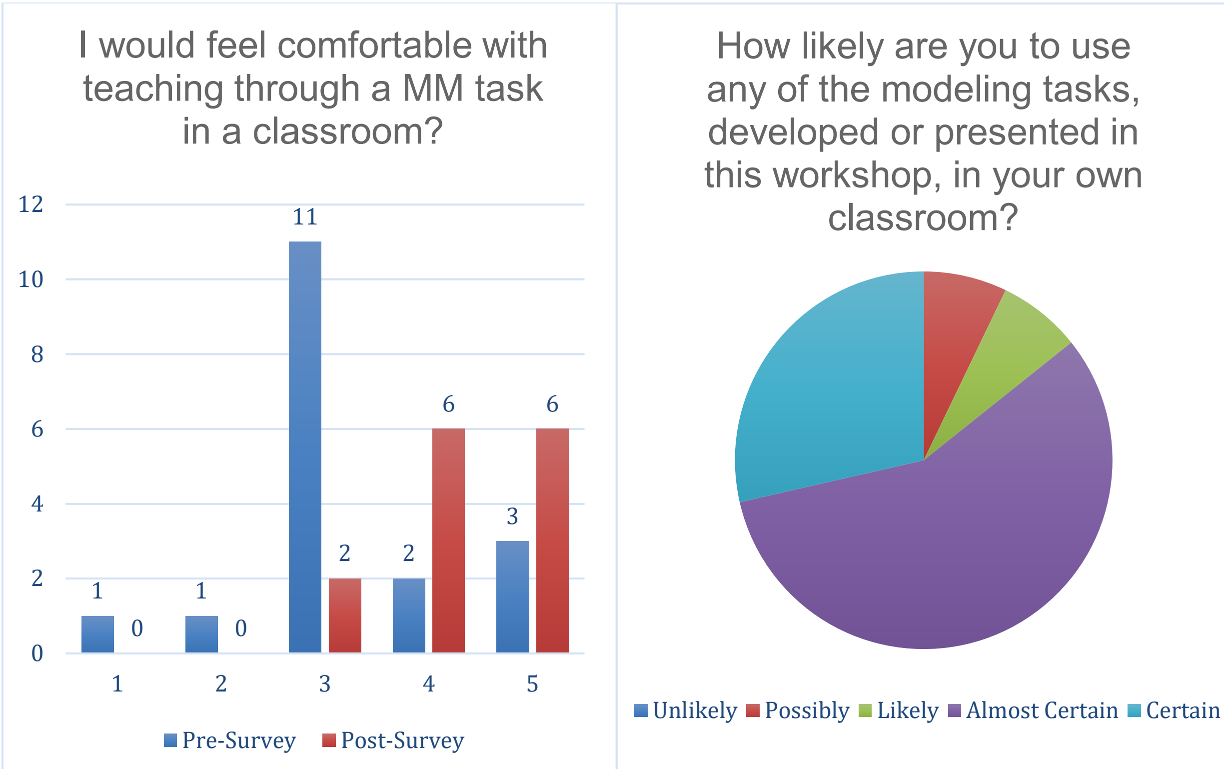


Figure 4. Survey questions regarding comfort level with modeling tasks.

The discussion of our findings also can be summarized by revisiting the three research questions:

1. *What evidence of MKT development emerges during the implemented lessons from teachers in Southeastern Utah?*

The evidence of MKT development during the implemented lessons included using mathematics in new ways. Participants used unit analysis, functions of two variables, scientific notation, surface area/volume, estimation, and error. For example, in order to discuss the surface area of a plastic bottle, the participants had to assume certain constraints over the bottle making it look like a can more than a bottle. Over the progression of the workshop, the evidence of tracking their work with gallery walks showed that the participants improved greatly in reporting their results to the group. They evolved from presenting simple calculations, tables, and charts to including assumptions, methods, sources, and conclusions. Participants showed increased emphasis on how choices were made for modeling with clearer articulation of assumptions.

2. *Do local teachers take away an appreciation for mathematical modeling, and see mathematical modeling as a productive approach for addressing water quality and water conservation issues?*

Evidence from the pre- and post-surveys indicated an increased awareness for water conservation and quality issues. In the pre-survey, one-third of the participants identified mathematical modeling to be useful in addressing social justice issues. Almost the entire group of participants did so in the post-survey. Additionally, the survey responses jumped from five responses dealing with water-related issues to 12 responses—showing that water conservation and quality were important social issues in their community. Lastly, participants showed growth in their appreciation and understanding of mathematical modeling. This evidence supports the research that performing mathematical modeling prompts will improve the participant appreciation of mathematical modeling (Anhalt & Cortez, 2016; Bal & Doğanay, 2014; Karaci Yasa & Karatas, 2018; Son, Jung, & I, 2017; Stohlmann, Maiorca, & Olson, 2015).

3. What does this localization of a modeling task involve?

Participants worked in groups to formulate mathematical modeling tasks of their own. They followed a four-step process when designing mathematical modeling tasks with local relevance. First, the participants made connections to people in the community with access to information and resources, which parallels the researchers' approach of contacting local experts in planning the workshop. Second, participants decided which information to use, choosing what they perceived as meaningful and engaging. Participants came up with tasks related to the following issues: mercury levels in fish, the geometry of recreational spaces, access of culinary water on the Navajo Nation, high levels of arsenic in well water, and the relationship between air pollution and automobile idle time—reflecting diversity in environmental and social issues beyond water-related topics. Deciding how to present the prompt was perhaps the most difficult aspect. Participants had to develop a question that would lead to multiple mathematical approaches. Third, participants sought more resources. They were already familiar with various sources of information, and searched the internet for reliable sources of additional facts and figures pertinent to their themes. Fourth, participants formalized and shared the modeling task and models they developed. They used a template document for mathematical modeling lessons provided by the researchers reflecting the general structure of lessons in the MODULE(S^2) Mathematical Modeling for Secondary Teaching course. Groups worked collaboratively to understand some different routes students might take when solving these problems. This fourth step helped in finding appropriate standards to align the modeling task to placement in curricula and appropriate implementation into the classroom.

Conclusion

We found that certain pedagogical techniques in the workshop helped develop mathematical modeling proficiency. When participants were able to see others' work and critique the work, this process helped them to structure reports of their own mathematical modeling work, and, ultimately, create novel mathematical modeling tasks. The researchers will add references to some specific water issues addressed in this workshop to increase awareness about native lands in Southeastern Utah. The materials also will be revised to include tools and suggestions in the lessons for how to increase the relevance for the community in which the lessons are implemented. Since multiple participants noted that the whole-group discussions about the community issues were just as important as doing the mathematical modeling, lesson plans within materials for pre-service mathematics teachers will include more time for discussing the social issues.

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Appendix A: The PDSA Cycle Overview

1) PLAN		
Questions:		
What evidence of MKT development emerges during the implemented lessons from teachers in Southeastern Utah?	Do local teachers take away an appreciation for mathematical modeling, and see mathematical modeling as a resource for addressing water quality and water conservation issues?	What does this localization of a modeling task actually involve? How much work is it to adopt materials to make them relevant for a specific community?
Predictions:		
We expect to see application of proportional reasoning, algebraic representations of functions, attention to units of measurement and conversions. We also expect to see growing sophistication in participant understanding of math modeling over the course of the workshop.	We expect enthusiasm for mathematical modeling as a means for addressing community issues to increase. We do not know whether participant willingness to teach through mathematical modeling activities will change.	Our planning for the workshop included the following stages: -Making connections to people in the community with access to information and resources. -Deciding how and what to present. (Our choices were based on anticipation of what may be impactful to the audience. We need to assess how effective these choices were.) -Creating ways for participants to seek and investigate resources on their own and make use of the resources we provided.
Data:		
Posters for mathematical modeling work and observing participant discussions. Presenter notes.	Open-ended survey with questions pertaining to this. Discussions and presenter reflections. Possibly follow up after the workshop.	Materials from sessions. Tasks and lesson plans generated by participants. Questions/discussion from after each session. Presenter notes and reflections.

2) DO
Recording data during the sessions was a challenge, since so much was going on, it was hard to parse the essential information. On the other hand, that richness of the data added to the depth of the results and the experience for the people involved. The audience was very enthused about the conversations that occurred regarding social and political issues that were raised through the modeling task. The questions raised and the discussions that arose motivated the participants who felt these discussions were just as important as doing the mathematical work in the task. The only obstacle that we dealt with was time—however, we did have sufficient time to get the plans started. Preparing a complete set of mathematical tasks would take a longer amount of time.

3) STUDY		
What were the results?		
Mathematics used by participants: Unit analysis, functions of two variables, scientific notation, surface area/volume, estimation and error. Reporting on modeling changed from showing simple calculations/tables/charts toward showing assumptions, calculations/tables/charts, methods, and conclusions. Discussions increasingly emphasized how choices were made for modeling with clearer articulation of assumptions.	The survey showed great evidence, as did discussions with participants. Participants were very productive in finding and sharing resources, but needed more time to develop the modeling tasks in detail. They liked having a short template to follow. Participants gained appreciation for social-cultural perspective of mathematical modeling.	They did make connections to their local water resources in their communities. On the question, “Where does your water come from?” we got everything from “I have no idea,” to various sources including wells, reservoirs, etc. After participants made connections, they grappled with deciding on what issues would be personally meaningful to their students. They created lessons on the following topics: Mercury levels in fish, Geometry of recreational space, Culinary water access in Montezuma Creek, Arsenic in well water, Air pollution and automobile idle time. This variety surprised us; clearly it wasn’t just limited to water issues.

4) ACT
What changes will we make to the MODULE(S^2) lessons? Include tools and suggestions for how to increase the relevance for the community near you. Raise awareness about water issues on native lands in Southeastern Utah. Include descriptions of how the gallery walk procedure will benefit the discussion and growth of reporting mathematical modeling results as the MODULE(S^2) lessons are implemented. What is our summary of steps involved in creating meaningful tasks for addressing community issues locally? 1. Making connections to people in the community with access to information and resources. 2. Deciding how and what to present. (Our choices were based on anticipation of what may be impactful to the audience.) 3. Creating ways for participants to seek and investigate resources on their own and make use of the resources we provided. 4. Formalizing and further sharing the participant-generated tasks.

Appendix B: Survey Instrument

Survey Questions

Both Pre- and Post-Survey	Pre-Survey Only	Post-Survey Only
What does mathematical modeling mean to you? Think about issues that are going on in your community, what are some community or social issues that are important to you? What is the value of mathematical modeling when addressing community issues? I would feel comfortable with teaching through a mathematical modeling task in a classroom? 1-5	How frequently do you use mathematical modeling tasks in your teaching? How frequently do you use culturally relevant word problems in class? Both with options of Never, Rarely, Occasionally, Frequently, and All of the Time	What are your takeaways from this workshop? What will you remember and use after this workshop? How likely are you to use any of the modeling tasks, developed or presented in this workshop, in your own classroom? With options given as Unlikely, Possibly, Likely, Almost Certain, and Certain.

Finding their Voice: Support Mechanisms to Engage and Empower Future Mathematics Teachers[1]

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Abstract

The NebraskaMATH Omaha Noyce Partnership Scholarship awards scholarships funded by the National Science Foundation (NSF) to undergraduate students interested in mathematics education at the University of Nebraska at Omaha (UNO). Scholars, who are dual mathematics and secondary education majors, are engaged and supported by Noyce faculty to not only excel in their college coursework and career preparation, but also to serve the university and community through teaching assistantships and STEM community outreach. The main goal of this program is to strengthen and expand the pipeline for preparing high-quality teachers of mathematics to better meet the responsibilities and demands of local school districts, particularly those serving students with high-need. Cross-campus collaborations between the departments of teacher education and mathematics co-constructed the Noyce infrastructure to emphasize and share the development of future, high-quality secondary mathematics teachers (Mathematics Teacher Education Partnership, 2014). This paper describes our program's unique design and implementation features aimed to empower, engage, and extend the talents of our undergraduate students. We share lessons learned and recommendations from faculty and participants regarding decisions and facets of the program considered to be most influential in STEM teacher and leadership development.

Introduction

At the national level, the focus on what it means to effectively teach high school mathematics is shifting (NCTM, 2018). Teachers are expected to collaborate with one another on instructional issues to provide high-quality, engaging learning experiences for all students (NCTM, 2014). University teacher preparation programs and mathematics departments must develop pre-service teachers who not only understand current research-informed instructional practices, but also who have first-hand experiences learning, teaching, and collaborating in student-centered environments (CBMS, 2016). This framework begs the question: Beyond undergraduate coursework and coordinated field experiences, what other components of teacher preparation support pre-service teachers to thrive as mathematicians, teachers, and leaders?

This paper describes unique features of our NSF-funded Robert Noyce Teacher Scholarship grant (DUE-1439796). The grant's purpose is to empower, engage, and extend the learning experiences for dual major pre-service mathematics and secondary education teachers. We will discuss the overall project infrastructure and underlying benefits of engaging pre-service teachers with leadership opportunities in a variety of diverse teaching

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and learning environments. Furthermore, this paper includes lessons learned and recommendations from the participants and from faculty. These lessons lead to programmatic changes and activities that have been instrumental in developing engaged and empowered pre-service teacher leaders.

Background

In 2013, the NSF awarded UNO with a Phase I Robert Noyce Teacher Scholarship Program grant. The aim of the grant was to address the growing concern that the United States is not producing enough science, technology, engineering, and mathematics (STEM) professionals for the needs of our nation, especially as compared to many other countries around the world (Ingersoll & Perda, 2010). This diminishing STEM competitiveness directly threatens our economy. The instruction of mathematics is of special concern, since it is often at the foundation of learning other STEM disciplines.

Within this context of concern for national competitiveness in STEM, school districts across the country (especially those in areas with at-risk student populations) are finding it increasingly difficult to fill mathematics teacher vacancies with qualified candidates. Vacancies for mathematics teachers in Omaha, Nebraska, mirror these national trends.

The NebraskaMATH Omaha Noyce Partnership, our Phase I Noyce Teacher Scholarship Program that is now in its fourth year, continues to build upon ongoing collaborative efforts between UNO and the Omaha Public Schools (OPS), to strengthen and expand the pipeline for preparing mathematics teachers at UNO. In particular, the partnership aims to better meet the high demand for STEM teachers in local school districts, especially those at-risk student populations. The Omaha Noyce Partnership is an initiative that aligns with NebraskaMATH, a statewide partnership, led by the University of Nebraska–Lincoln (UNL) that works to improve achievement in mathematics for all students and to narrow achievement gaps of at-risk populations. The program, which includes new coursework and complements the existing pathways in the College of Education, was developed by the UNO mathematics and education faculty in collaboration with school district partners. The goal of the program is to develop highly skilled secondary mathematics teachers who are committed to teaching in high-need schools by providing targeted support for students enrolled in and graduating from the mathematics/teacher preparation program. By *highly skilled*, we mean future teachers who will be strong in both mathematical and pedagogical content knowledge, as well as leadership skills that also will allow them to further build strong mathematics programs within their schools.

Program Structure

Participants

We have two types of program participants: (a) Noyce Scholars, undergraduate juniors or seniors, on full scholarship for a dual major; and (b) Noyce Interns, freshman through juniors, who are: paid hourly, potentially interested in mathematics and/or education, and are actively exploring the option of mathematics education as a career. All Noyce Interns (freshman or sophomores) are academically eligible and/or on track to pursue the Noyce scholarship. Unlike Scholars, Interns are paid as hourly workers, from $12 to $15 per hour, with several lump sum payments during the year. Due to the less formal commitment to a future career as a secondary mathematics teacher, approximately half of our Interns pursue their original STEM major or non-dual mathematics/education major. Table 1 shows the number of Scholars funded through our Noyce program since 2015.

Requirements

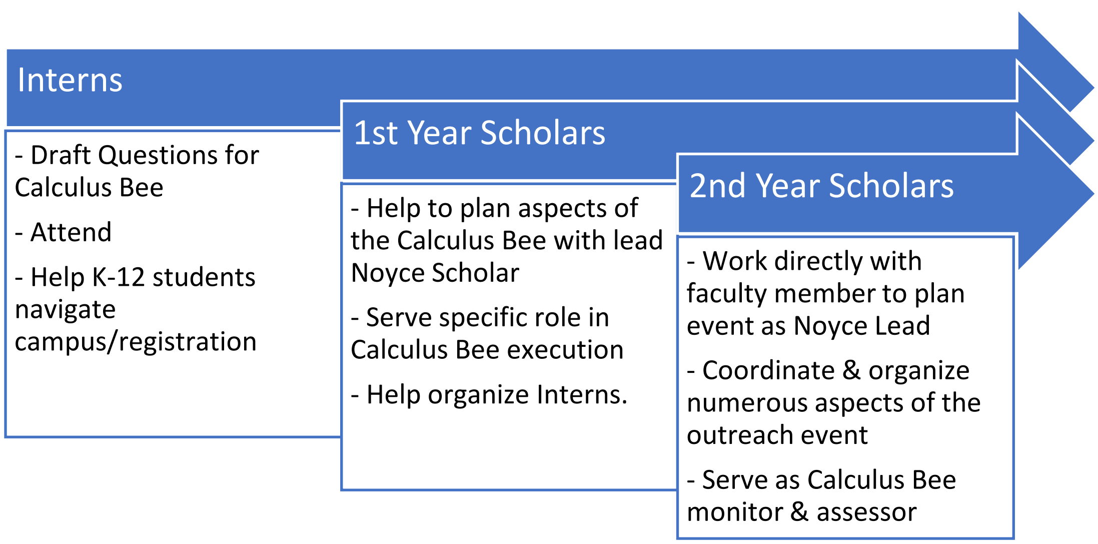
The Omaha Noyce Partnership program is designed to develop highly skilled secondary mathematics teachers who are committed to teaching in high need schools, defined as schools where more than 50 percent of students receive free or reduced lunch. This definition is not limited to schools in Nebraska. One Scholar-graduate is completing her Noyce obligation with a school district in Ohio, her home state. Our Scholars are required to obtain employment in said schools for a minimum of two to four years, depending on the amount of scholarship received (up to $16,000 per year), to fulfill the expectations of the Noyce scholarship program. If they opt not to teach or to do so in a high needs school, then the scholarship automatically becomes a student loan with repayment structures.

Table 1

Number of Scholars Funded, by year, in NebraskaMATH Omaha Noyce

Year	**Number of 1st-Year Scholars**	**Number of 2nd-Year Scholars**	**Total Scholars**
2015	4	0	4
	1 of the first-year Scholars did not apply for a second year of funding but did graduate with a teaching certificate and has fulfilled her teaching requirements.		
2016	4	3	7
	1 of the first-year Scholars was a senior and graduated.		
	1 of the second-year Scholars graduated but is not yet fulfilling her teaching requirements and will have to start repayment soon.		
2017	6	3	9
	All 6 first-year Scholars returned in 2018.		
2018	4	6	1
	2 of the first-year Scholars put their college career on hold for personal reasons.		
	1 of the first-year Scholars dropped out of the program to pursue a full mathematics degree.		
	1 of the second-year Scholars put her college career on hold because of a pregnancy and will graduate in 2020.		
	1 of the second-year Scholars will student teach Fall 2019.		
2019	6	1	7
		Total Scholarships:	**24**

UNO's longstanding partnership with OPS plays an important role in helping our Noyce graduates locate full-time teaching assignments that fulfill their scholarship requirement. Both mathematics and teacher education faculty from UNO work closely with our partners in OPS. To date, all 11 dual-degree completed Noyce participants are grant compliant, working in high-need schools as first, second, or upcoming third year teachers, which aligns



with the first and third of the MTE-Partnership's *Guiding Principles for Secondary Mathematics Teacher Preparation Programs* (2014).

Figure 1. Example of trajectory of involvement in outreach from Interns to second-year Scholars.

Over the course of our program, the hours and expectations for Scholars and Interns have evolved based upon periodic program review and recruitment considerations. Currently, Scholars and Interns dedicate approximately 12 and 8 hours per week, respectively, to Noyce-related activities. Such activities include, but are not limited to, a regular mentoring-meeting with faculty, serving as a teaching assistant, coordinating outreach activities, tutoring mathematics students, attending professional development, and meeting with their Noyce peers. We intentionally provide a variety of activities designed to build on the individual talents and skills of our Scholars and Interns so that they each have tailored leadership and instructional experiences. Scholars and Interns are required to reflect in weekly online journals on their engagement hours related to their development as a mathematician, teacher, and/or teacher leader. Figure 1 diagrams how Interns and Scholars, at a variety of levels, are engaged in one particular outreach event, a Calculus Bee for 900 high school students.

Translating our Mission into Practice

Teaching is a complex task and involves a variety of considerations and experiences to expose pre-service candidates to the realities and opportunities of their future career responsibilities. Dunlap and Hansen-Thomas (2011) found that when pre-service teachers engaged in targeted leadership experiences and were supported by faculty mentors, their self-efficacy increased. The authors suggested that novice teachers need to learn how to seek out experienced mentors, professional development opportunities, opportunities to network, and ways to take on leadership roles in their field to develop this self-efficacy over time.

The overarching mission statement of our Noyce program is to create better mathematicians, better teachers, and better leaders. Over time, our Noyce leadership team has operationalized how our program design aims to ensure our Scholars and Interns gain knowledge, skills, and professionalism through participation in our program. To achieve our program goals and mission, the Noyce faculty leadership team established avenues to leverage research on pre-service teacher development to empower, engage, and extend the learning experiences of our undergraduate Scholars and Interns.

Empowerment

Empowerment, in relation to teachers, occurs when teachers are given autonomy, gain knowledge, and see improved status in their field (Farrell & Weitman, 2007). Within the context of pre-service teacher preparation, we aim to empower our Noyce participants similarly. As faculty leaders we provide structure, without micromanaging. This supported autonomy ensures our Noyce participants have the opportunity to gain efficacy in their own abilities, accomplishments, and grow where they see there is the most need. To do so, we have integrated mentoring, active learning, and professional development experiences into the fabric of our program for all participating Noyce undergraduate students.

Mentoring. Mentoring within teacher education programs has been widely studied and supported by research based on its impact on teacher commitment, retention, and student achievement. University faculty members and advisors are identified as key players in STEM students' success (Marshall et al., 2011). Overall, strong mentoring programs have been proven to positively impact future student learning of the teachers who participate in the program and also significantly reduce the attrition rate of new teachers (Ingersoll & Strong, 2011).

Early in the grant, mentoring quickly surfaced as a crucial programmatic component to ensure Scholars and Interns were supported and retained in our program and the field. Faculty identified the need for very intentional and consistent mentorship of Scholars and Interns. Despite initial mentoring challenges, which included difficulty in getting mentors and mentees routinely together, over time the faculty mentor-mentee relationships have solidified to weekly contacts and empowered Noyce Scholars and Interns in not only their career goals, but also in navigating personal and academic success. Scholars and interns share that collaborating with their mentors and other Noyce leadership faculty has increased their confidence speaking to "authority/superiors" and also prompted them to take on more leadership responsibilities where they would need to "speak up more and take charge of my ideas." For instance, Noyce Scholars have presented at local, regional, and national conferences alongside Noyce faculty and their peers. They share how their Noyce experience has increased both their confidence speaking in front of others and also their perception of themselves as a professional. One Scholar stated, "I gained a voice and the confidence to speak to superiors as well as peers especially if there was something that wasn't right or that I needed to be successful."

Active learning experiences. Research suggests that coherent teacher development programs dedicated to active learning and focused on content knowledge result in positive effects on teacher practice (Garet et al., 1999). Noyce participants are encouraged to take courses taught by faculty, many of whom are Noyce mentors, who incorporate Inquiry-based Learning (IBL) instructional practices into their courses. In these IBL mathematics courses, students present problems, engage in collaborative group work and discussions, and experiment in hands-on, active learning environments (Ernst, Hodge, & Yoshinobu, 2017), which aligns with Guiding Principle 4 (MTE-Partnership, 2014). This environment aligns with best practices of mathematics teaching and learning (NCTM, 2014) taught within the teacher preparation program and therefore, serves as a model for Noyce Scholars' future classrooms. One participant said, "I now tend to speak up more and take charge of my ideas. I am also way more confident in jumping up in front of a class as well." These types of experiences as a learner can be powerful later as they develop more dispositions as teachers.

Noyce Scholars and Interns also actively participate in professional development training on concepts that are immediately relevant to their Noyce opportunities and degree program requirements. In particular, all Scholars and Interns actively participate in a required culturally responsive teaching (CRT) two-part seminar. CRT is a framework through which teachers learn to adapt curriculum in ways that create bridges between the communities and cultural identities of traditionally marginalized students (Ladson-Billings, 1995). Within these trainings, Noyce Scholars and Interns learn ways to bring their students' experiences into the mathematics classroom and the community, as asked in Guiding Principle 5 (MTE-Partnership, 2014). Noyce participants are able to immediately apply their professional, active learning opportunities to Noyce-related activities providing heightened meaning and empowerment as future teacher practitioners. CRT pedagogy embodies the necessary knowledge, skills, and disposition required of our Noyce Scholars and Interns who regularly work with diverse communities and students through outreach and other STEM programming.

Engagement

Scholars and Interns are actively engaged in authentic learning and teaching experiences related to mathematics. Our program provides a network of support including structured peer interactions, faculty mentoring in teacher education and mathematics, K–12 school district teachers, and other STEM-affiliated community members (e.g., non-profit directors, STEM coordinators). The primary network is the Omaha Citywide STEM Ecosystem, which is a community organization now over two years old, that engages more than 80 city organizations (including 13 public school districts) and has over 750 members who attend many different sessions and community events. One of just 58 citywide STEM Ecosystems as recognized by the STEM Funders Network, this organization has a full-time director that is a UNO-employee based at the Zoo. It offers many different engagement points for our Scholars and Interns, such as popular community-based teaching circles like Mathematics with an Architect, Mathematics with an Engineer, Mathematics with a Chef, and Mathematics at the Aquarium. Each of these teaching circle sessions are presented by community organizations, such as businesses, museums, and informal education organizations.

Teaching and service. UNO's traditional pre-service teacher trajectory includes over 150 hours of supervised field experiences within the teacher education program, but any additional teaching or service-learning opportunities are optional and not part of the formal programming in the College of Education. Unlike traditional secondary mathematics education students, Noyce Scholars and Interns have the opportunity to work with faculty as teaching assistants in IBL mathematics courses (Calculus and beyond). Over time, interested mathematics faculty now request Noyce participants, and some are even recruited into the program by faculty hoping to further engage students in the application of mathematical content through teaching. These experiences are unique to our participants in comparison to the traditional mathematics education programming on campus.

Scholars and Interns also help to coach and tutor students studying for the Praxis Core, a required exam for students entering formal programs in the UNO College of Education. Serving as a teaching assistant and peer tutor in mathematics are mutually beneficial engagement activities for all involved. Scholars and Interns are able to develop their mathematical and communication skills while having a tremendous impact on their UNO peers. In particular, the Praxis study group has been growing in attendance over the past year with an increase in students' Praxis mathematics scores after participating regularly in the tutoring sessions. In addition, the success of the program for helping elementary education majors is impressive, with the ability for the study group to help a student not only pass the Praxis, but also to do quite well on it. For example, one student started with two consecutive scores of 130, and then raised their score to 168 after study group participation, greatly exceeding the 150 cutoff score.

Community outreach. Another feature of the Noyce program that differs from traditional pre-service mathematics educational programming is that Noyce Scholars and Interns have numerous opportunities to lead STEM outreach events on campus, at local schools, and throughout the community. On campus, Noyce participants host one to two Student Math Circles per semester. The events are designed for Grades 6–12 students to use critical thinking and problem solving in mathematics while experiencing a college campus environment. The Scholars and Interns plan, coordinate, market, and facilitate events from start to finish with the guidance of faculty mentor support. These outreach events provide Noyce participants opportunities to engage with community partners beyond a structured classroom-based field experience and helps them to develop connections and experience with developing and hosting future outreach events of their own. One Scholar noted,

> The Omaha Noyce program has helped me develop my leadership skills among the group throughout the semester. For example, by the end of the semester I was able to volunteer to lead an activity/project and gather a group to help me with that project very easily. It has also become easier for me to reach out to people in the community and offer outreach ideas or assistance with outreach.

Our Noyce program also partners with Omaha Girls Inc., a national non-profit organization that focuses on supporting the academic, socio-emotional, and physical health of girls and young women, to offer a four-week summer STEM camp for 60 middle school girls each year. Noyce Interns work as instructional assistants during the camp and even lead mathematics and STEM-related sessions. They interact with students of diverse backgrounds and educational settings to provide instructional support and ultimately develop and teach their own STEM activities with the girls (also Guiding Principle 5).

Extension

Over the past five years, the Noyce leadership team has continuously reflected and responded to our programmatic and support structures to consider how we offer unique and enriching experiences and opportunities beyond the standard degree-program requirements. Through engagement and empowerment, we strive to not only recruit and retain participants in our program, but also to extend their network and capacity as 21st century STEM professionals.

Community of learners. Little (2002) describes a *community of learners* as a structured collaboration of teachers to critically examine aspects of their teaching practice, investigate alternative "conceptions of teaching and learning," and support one another's continued professional growth (p. 918). A major focus of the Noyce leadership team has been to help our Scholars and Interns develop as an active and robust community of learners while they are in our program. Communities do not always evolve naturally, and we found that some intentional structure was necessary for Scholars and Interns to collaborate regularly, both with faculty and one another, to develop a shared vision and mutually beneficial collaboration for participants' academic and professional growth.

Noyce participants meet with their mentor weekly and also hold a Scholar-led meeting each week where they plan upcoming events, participate in professional development and hear from speakers, and/or form study groups aligned to their mathematics coursework. Our intentional focus on community building has created a more consistent and quality experience for all of our Noyce Interns and Scholars. Instead of "isolated islands of excellence" based on each participant or mentor's experience or personality, the interconnected teaching, learning, and outreach has resulted in a more communal level of achievement, productivity, professionalism, and motivation (Guiding Principle 6).

Reflective practice. In education, the practice of reflection is viewed as an important and even critical mechanism through which teachers can examine and refine their teaching beliefs and practices (Schon, 1983). Isik-Ercan and Perkins (2017) suggest that effective teacher reflection includes reflection on meaning and on action. In our program, we incorporated a range of structures to help our Scholars and Interns reflect on their experiences and their beliefs about STEM teaching and learning.

Each week, Noyce participants create and post written reflections in three key parts: description, analysis, and implications. Beyond summarizing and justifying their weekly hours, Scholars and Interns use their reflections to grapple with their experiences in an open forum with other Scholars and Interns. Scholars and Interns are able to reflect with mentors in weekly meetings, yet also have the support of their peers within the online collaborative space. One Scholar shared her experience as a learning assistant in a Calculus course,

> That was the first time I had knowingly been in in an IBL classroom so it was cool to see how it ran. I love how you can actively watch the learning happen, and I was very intrigued by some of the discussions that took place.

Scholars and Interns are able to learn from and reflect upon how their peers interpret similar or different Noyce opportunities from their perspectives (Guiding Principle 6). After transferring our weekly reflections to the collaborative Canvas learning management platform, reflection completion rates increased to 92.5% (112 out of 121) during the last semester with little to no mentor interventions.

Discussion and Implications

During the first three years of our grant, the Noyce faculty leadership team attempted to find effective ways to operationalize our mission of creating better mathematicians, better teachers, and better leaders. We developed structures and opportunities to support our Scholars and Interns in becoming empowered future teachers and engaged and collaborative community members, and have attempted to extend these supports beyond their time as undergraduates and into their teaching practice.

Throughout their reflections and surveys, Scholars identified confidence, professionalism, and the ability to speak up and to superiors as leadership skills they have acquired through their Noyce participation. Beyond the traditional teacher preparation program experience of their non-Noyce peers, our Scholars have explicit experiences in the classroom, community, and on-campus, examining their ability to be teacher leaders. One Scholar expressed "being more comfortable stepping out of my comfort zone, which is an important part of being a leader." Another Scholar provided a holistic picture at how the Noyce program impacted him,

> The program has given me connections with people that I never imagined meeting, it's made me closer to the faculty and I've gotten to know the math/education department a lot better. As previously mentioned, it has also helped me develop leadership skills through experiences I wouldn't have otherwise. All the outreach events and meetings I feel make me a lot more prepared than I could ever be without the Noyce program.

Our collaborative leadership team has learned, through feedback and continuous refinement, the importance of developing structured mentoring relationships, offering professional development that is timely and relevant to the needs of our Scholars and Interns, and providing opportunities for them to engage with one another and other community stakeholders. It became very evident that ongoing reflection and program evaluation was critical in helping us to refine these supports in order to actively engage and retain Scholars and Interns over time. Through this reflection process, we made adjustments to our mentoring structures to ensure participants had consistent support from an engaged faculty member. We created weekly meetings to facilitate collegial relationships among student participants and developed a distributed leadership model by which Scholars helped to coach and mentor new Interns to address some of our programmatic challenges. Although we have made several such adjustments to said structures, we are still looking ahead for ways to be as intentional in the extension phase of our project, and we hope to learn more as additional Scholars and Interns graduate and begin their teaching careers. It is our hope that other teacher preparation programs can learn from the model presented in this paper and also hope to continue dialogue and research into how to effectively train future STEM educators and leaders.

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Sharing and Building Resources to Equip and Empower Mathematics Teacher Educators

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Abstract

Members of the Clinical Experiences Research Action Cluster (CERAC) were asked at the 2018 Mathematics Teacher Education Partnership (MTE-Partnership) Conference to complete a survey indicating how their university mathematics teacher education (MTE) programs addressed the National Council of Teachers of Mathematics' (NCTM) Mathematical Teaching Practices mentioned in *Principles to Actions: Ensuring Mathematical Success for All* (NCTM, 2014) in methods courses. Survey results indicated four cross-cutting themes for how the Mathematical Teaching Practices were implemented in MTE programs. Resources were categorized into videos, classroom interactions, readings, and clinical practices. The resources used by the CERAC members to address the MTP and Association of Mathematics Teacher Education's (2017) *Standards for the Preparation of Teachers of Mathematics* (AMTE Standards) are shared in the conclusion.

Introduction

In the summer of 2018, the CERAC began a Plan-Do-Study-Act (PDSA) Cycle to examine how CERAC members integrated the eight researched-based effective Mathematical Teaching Practices espoused in NCTM's *Principles to Actions* (2014) within their university mathematics teacher preparation programs. Clinical Experiences is one of five RACs in the MTE-Partnership created in 2012 to strategically target issues in the shortage of strong mathematics teachers throughout the United States. The CERAC is committed to strengthening clinical experiences for math teacher candidates; attending to the Mathematical Teaching Practices in methods courses should be a goal of CERAC members. A Google form survey was created that asked CERAC members to describe how they attended to teacher candidates' development of the Mathematical Teaching Practices. As part of this PDSA Cycle, the researchers collected and organized the data according to each of the Mathematical Teaching Practices. The following paper seeks to share the study portion of the process and describe ways in which those in the CERAC, the larger MTE-Partnership network, and how other teacher educators may initialize institutional and program actionable changes. The research aligns with the MTE-Partnership's *Guiding Principles for Secondary Mathematics Teacher Preparation Programs* (2014), particularly Guiding Principle 1: Building a national consensus on what effective secondary mathematics teacher preparation programs need to do in order to develop teacher candidates who promote mathematical excellence in their future students, and Guiding Principle 4: Helping to organize the identification, development, and dissemination of resources supporting effective secondary mathematics teacher preparation programs.

Theoretical Framework for Effective Mathematics Teacher Preparation

In 2010, the National Governors Association and the Council of Chief State School Officers initiated a state-led effort to provide standardization of what college- and career-ready students should know and be able to do and released the *Common Core State Standards in Mathematics* (*CCSS-M*). The math standards included eight *Standards of Mathematical Practice* that students should experience during their K–12 learning of mathematics:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning. (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010)

The *Standards of Mathematical Practice* provide a balance between procedure and understanding in learning mathematics (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). To support implementation of the *Standards of Mathematical Practice*, in 2014 NCTM published *Principles to Actions*, which included eight math teaching practices for effective teaching and learning. The Mathematical Teaching Practices provided a research-based framework for mathematics teachers to incorporate for effective student learning of mathematics. The Mathematical Teaching Practices are:

1. Establish mathematics goals to focus learning.
2. Implement tasks that promote reasoning and problem solving.
3. Use and connect mathematical representations.
4. Facilitate meaningful mathematical discourse.
5. Pose purposeful questions.
6. Build procedural fluency from conceptual understanding.
7. Support productive struggle in learning mathematics.
8. Elicit and use evidence of student thinking. (NCTM, 2014)

Additionally, *Principles to Actions* included guiding principles for school mathematics that included teaching and learning, access and equity, curriculum, tools and technology, assessment, and professionalism, and a focus on productive beliefs versus unproductive beliefs about learning mathematics (NCTM, 2014).

To support university math teacher educators' focus on preparing well-prepared math teachers, the AMTE Standards guide what math teacher candidates should be able to know, understand, do, and the dispositions they should develop. Teacher candidates should develop pedagogical content knowledge that promotes active mathematical learning, and they should collaborate and learn from their peers. They *must* connect with their individual students and cultural contexts and support every student's opportunity to learn mathematics. Teacher candidates should nurture positive mathematical identities in every single student (AMTE, 2017).

The researchers volunteered to create a survey to determine how the Mathematical Teaching Practices were being addressed by the CERAC members. The importance of sharing how the *Standards of Mathematical Practice*, Mathematical Teaching Practices, and AMTE Standards are implemented will be beneficial information for the MTE-Partnership community and teaching practitioners.

Clinical Experiences Research Action Cluster

The CERAC is a consortium of 26 universities and their school partners engaged in developing clinical experience models designed to build candidates' facility with the Mathematical Teaching Practices and other equitable teaching strategies (Aguirre, Mayfield-Ingram, & Martin, 2013; AMTE, 2017) to promote secondary school students' success in achieving college- and career-ready standards. The CERAC is composed of three sub-RACs: Co-Planning/Co-Teaching (CPCT), Paired Placement, and Methods. The Methods sub-RAC has worked to engage MTEs by developing modules designed to promote the standards of mathematical practice, high-quality lesson planning, and high-quality feedback. The CPCT sub-RAC has worked to develop teacher candidates' implementation of co-planning and co-teaching strategies with the use of online modules that attend to the co-planning strategies: one plan/one assist, partner planning, one reflect/one plan, one plan/one react, parallel planning, and team planning (Cayton, Grady, Preston, & Sinicrope, 2016); and co-teaching strategies: one teach/one observe, one teach/one assist, station teaching, alternative teaching, parallel teaching, and team teaching (Bacharach, Heck, & Dahlberg, 2010). The Paired Placement sub-RAC focuses on implementing a student teaching approach where two pre-service teachers are paired with a single mentor teacher. This type of placement allows the mentor teacher to provide purposeful coaching and mentoring, and the two teacher candidates to offer each other feedback, mentoring, and support (Leatham & Peterson, 2010; Conway et al., 2018). Each of these sub-RACs have purposefully attended to the Mathematical Teaching Practices through data collection such as the Mathematics Classroom Observation Protocol for Practices (MCOP^2; Zelkowski & Gleason, 2016); teacher artifacts; teacher candidate journals; Mathematical Teaching Practices surveys; focus groups; and completer surveys. Both the MCOP^2 and the Mathematical Teaching Practices surveys specifically target high-leverage teaching practices in mathematics education, Mathematical Teaching Practices, and equitable instruction.

Methods

Researchers from the CERAC completing the PDSA Cycle began the instrument design in the 2018 Conference work sessions. During the work session, the researchers worked from a shared Google platform in order to design questions that would attend to each of the practices. After creation, the researchers shared the survey with the CERAC team to ensure that questions were targeting the need. Questions were revised based on feedback and dispersed to the CERAC team for completion. The final version of the survey (see Appendix) consisted of eight open-ended questions. The survey was emailed to 20 CERAC members, representing 18 institutions. Eleven CERAC members responded for a 55% return rate.

Researchers analyzing the data used cultural domain analysis (Bernard & Ryan, 2010) to understand how CERAC survey participants addressed each Mathematical Teaching Practice in methods coursework. Cultural domain analysis seeks to understand how people in a group, such as the MTE-Partnership CERAC members, think about and use knowledge across a list of things such as the Mathematical Teaching Practices. A free list matrix (Bernard & Ryan, 2010) was created to count the frequency and relationship between the Mathematical Teaching Practices. After initial review of the surveys, four cross-cutting types of tools were found to be used among the CERAC members: videos, classroom interactions, readings, and clinical practices. These tools were then cross-analyzed to find patterns of association and dissociation using Table 1.

Results

A total of 11 institutions from the CERAC participated in completing the survey. Participants taking the survey may have mentioned multiple tools they use in their program to develop teacher candidates' understanding, development, and use of the Mathematical Teaching Practices. For this reason, the totals in this paper should be examined with a collective CERAC lens rather than as individual participants. A collective summary of how these 11 institutions responded is presented in Table 1.

Table 1
Cross-Cutting Types of Tools

	MTP1[a]	MTP2	MTP3	MTP4	MTP5	MTP6	MTP7	MTP8	Total
Readings	6 (.3)	5 (.19)	3 (.14)	5 (.23)	8 (.22)	7 (.39)	2 (.14)	5 (.29)	**41 (.23)**
Videos	4 (.2)	2 (.07)	2 (.1)	1 (.05)	2 (.05)	1 (.06)	2 (.14)	2 (.12)	**16 (.09)**
Classroom Interactions	1 (.05)	17 (.63)	13 (.62)	8 (.36)	13 (.35)	9 (.5)	9 (.64)	6 (.35)	**76 (.43)**
Clinical Practice	9 (.45)	3 (.11)	3 (.14)	8 (.36)	14 (.38)	1 (.06)	1(.07)	4 (.24)	**43 (.24)**
Total	**20**	**27**	**21**	**22**	**37**	**18**	**14**	**17**	**176**

[a] MTP is Mathematical Teaching Practices, from NCTM (2014).

Collectively, institutional responses were coded as videos, classroom interactions, readings, and clinical practices to produce a general organization of how the MTE-Partnership has addressed the Mathematical Teaching Practices. Frequencies in Table 1 represent the number of occurrences an MTE-Partnership institution used this tool to develop a Mathematical Teaching Practice. Proportions in parenthesis represent the proportion of occurrences a MTE-Partnership institution described a tool in developing each an individual Mathematical Teaching Practice. A brief review of the proportions may provide evidence for either further development of resources for a particular Mathematical Teaching Practice or a need for curriculum revision in partner institutions. The following four sections highlight responses that were categorized as each tool.

Videos

Participants in the CERAC noted a number of resources that were used that related to watching and reflecting on videos of teaching. These digital resources were described in ways that also connected with classroom interactions. However, the researchers decided to report digital resources separate from classroom interactions for readers' ease of implementation. As follows is a description of a survey response describing the use of a digital tool or video in a math methods course:

> Teacher candidates have previously worked the "Bike and Truck" task from the *Principles to Actions* toolkit lesson: Bike and Truck Task - The Case of Shalonda Shackelford. Teacher candidates watch video clip 1 in class, which is followed up with a "Think-INK-Pair-Share" formative assessment about observations from the first video clip. Teacher candidates watch video clip 2 in class, which is followed up with a Think-INK-Pair-Share on how Ms. Shackelford facilitate meaningful mathematical discourse in her classroom.

Other video sources mentioned include insidemathematics.org, ATLAS (https://atlas.nbpts.org/), the *Principles to Actions* toolkit, robertkaplinsky.com, and personally recorded videos for self-reflection. The use of technology to effectively notice and train future teachers is becoming an invaluable resource for teacher educators. Further analysis on the use of videos in teacher preparation follows in this paper.

Classroom Interactions

A number of different descriptions were categorized as classroom interactions from participant responses and represented the largest percent (43) of tools used to develop teacher candidates' development, understanding, and use of Mathematical Teaching Practices. A participant stated,

> To help teacher candidates make connections between physical, symbolic, verbal, graphical representations, tasks are carefully selected to provide opportunities to solve them in multiple ways. Additionally, teacher candidates are required to purchase a manipulative kit for their methods course.

A statement such as this directed toward multiple representations was categorized as classroom interactions. Similarly, respondents also described activities they performed in class to highlight MT Mathematical Teaching Practices such as facilitating meaningful mathematical discourse:

> Solving problems in teams requires meaningful mathematical discourse between team members. I also model real problems with mathematics every time I can during regular lectures. The courses I teach lend themselves to it wonderfully. Each small group must report out to the class. One member of each group will be chosen at random to present. This ensures that all members make sure that all other members are ready. Then, they use large-group discussion to compare and contrast group-solution strategies.

Respondents described activities used in class such as discussions, mathematical tasks, reflections, sorting activities, course assignments, assessment using the National Assessment of Educational Progress (https://nces.ed.gov/nationsreportcard), Advanced Placement Central (https://apcentral.collegeboard.org/) and MARS resources (https://mars.nasa.gov/participate/marsforeducators/), and teaching demonstrations by teacher candidates. Further analysis on how classroom interactions play a part in teacher preparation programs follows in the analysis section of this paper.

Readings

Resources were described in surveys that require teacher candidates to read materials in different ways. One participant stated,

> The math books I use in my two courses have a variety of problems suitable for promoting these goals. What I do is assign some of these problems for solving in class by groups of 3 or 4 students, and then have team members explain their solutions and conclusions.

Sometimes these descriptions intersected with other categories such as classroom interactions of discourse and reflection. Participants from the CERAC discussed the use of NCTM's (2014) *Principles to Actions*; *5 Practices for Orchestrating Productive Discussions* (Smith & Stein, 2011); Smith and Stein's "Creating Mathematical Tasks" (1998); EdTPA handbooks; *CCSS-M* Learning Progressions; estimation180.com; Pesek and Kirshner (2000); *Mathematical Mindsets: Unleashing Students' Potential Through Creative Math, Inspiring Messages and Innovative Teaching* (Boaler, 2015); Bass and Ball (2015); *Questioning Our Patterns of Questioning* (Herbal-Eisenmann & Breyfogle, 2005); and *Decoding the Common Core* (Dillon, Martin, Conway, & Strutchens, 2017). Readings are a powerful component of a mathematics teacher preparation program, but used alone they are not as effective. Analysis later provides structure for teacher preparation programs to consider professional reading in the success of their programs. However, clinical experiences are essential components of a teacher preparation program that should not be ignored.

Clinical Practice

Descriptions of teacher educator practice related to clinical experiences was the second-most described method to increase teacher candidates' understanding and use of Mathematical Teaching Practices. Descriptions from survey results categorized as clinical experiences related to debriefing sessions after teaching, lesson plan construction and implementation, task reflections related to portfolios such as EdTPA, modules, and field-based observation rubrics such as the $MCOP^2$ (Zelkowski & Gleason, 2016). Though clinical experiences are one of the most powerful tools for developing teacher candidates, they should be integrated with other tools to develop their understanding, which is described in the next section. One participant stated,

> The $MCOP^2$ does this for us. The requirement to engage students by teacher candidates during implemented lessons, the items that require teacher candidates to facilitate discourse and items that specifically assess student discourse make this evident. The quality of the discourse is addressed by planning two days in advance and receiving feedback prior to implementation.

As this participant described the clinical experience, it is important to highlight the connection of the clinical experience to feedback on the Mathematical Teaching Practices for analysis purposes.

Table 2

Innovation Configuration Tools Mapping

	Level 1	**Level 2**	**Level 3**
MTP1[a]: Goals to Focus Learning	25%	55%	20%
MTP2: Tasks that Promote Reasoning and P/S	58%	37%	7%
MTP3: Use and Connect Representations	38%	43%	19%
MTP4: Facilitate Meaningful Discourse	23%	50%	27%
MTP5: Pose Purposeful Questions	22%	56%	22%
MTP6: Building Procedural Fluency from Conceptual Understanding	33%	56%	11%
MTP7: Supporting Productive Struggle	14%	79%	7%
MTP8: Elicit and Use Evidence of Thinking	22%	60%	18%

[a] MTP is Mathematical Teaching Practices, from NCTM (2014).

Analysis

The tools used across survey participants suggested analysis that disaggregated instructional practices and teacher candidate experiences that are used throughout programs that promote the use of Mathematical Teaching Practices. In addition, these tools suggest the extent to which teacher educators provide opportunities for teacher candidates to apply Mathematical Teaching Practices in ways that provide explicit feedback and sustained implementation and support to ensure fidelity. For this reason, an innovation configuration was used to analyze the partnerships methodologies for program evaluation (Hall & Hord, 1987; Roy & Hord, 2004). An innovation configuration can be used to promote the implementation of evidence-based instructional practices in teacher education programs. Innovation configurations can be designed to assess teacher education programs as to the extent to which evidence based practices are taught, observed, and applied along particular strategic practices (Hall & Hord, 1987; Roy & Hord, 2004).

Researchers adopted the innovation configurations suggested by the Collaboration for Effective Educator Development, Accountability and Reform (CEEDAR) Center (http://ceedar.education.ufl.edu), an organization committed to improving the implementation of evidence-based instructional practices in teacher preparation programs. This group has suggested mapping tools and methods for addressing culturally relevant pedagogy and other teacher educational goals across three levels. Level one contains readings, lectures, presentations, discussions, models, quizzes, tests, and demonstrations of the Mathematical Teaching Practices. Level two includes observations, projects, case studies, reflective assignments not related to teacher candidates' teaching, and lesson plans. Level three was coded as K–12 student tutoring, small-group teaching, whole-group instruction, and tasks such as reflections, related to these experiences. Table 2 summarizes the analysis using this innovation configuration for each Mathematical Teaching Practice.

Limitations, Implications, and Suggestions

Results and analysis from this survey provide initial evidence that each Mathematical Teaching Practice is being addressed at varying levels across the MTE-Partnership network, and showing some general consistency in levels for each Mathematical Teaching Practice. Generally, the majority of instruction related to Mathematical Teaching Practices is at level 2 in CERAC survey participants. It is hypothesized that this is true based on the time spent in courses developing understanding of Mathematical Teaching Practices before participating in clinical experiences where these practices are expected to be implemented in with K–12 students. In addition, the large percent of level two IC codes is likely to correlate with the partnerships connection with alternative uses of assessment to gauge student development of Mathematical Teaching Practices. This first PDSA Cycle by the CERAC focused on the Mathematical Teaching Practices serves as a tool for other programs to critically examine their own strengths, weaknesses, and development of students' Mathematical Teaching Practices across institutional course progressions.

However, findings must be analyzed with caution. The current survey and analysis was administered and completed voluntarily by CERAC members. Participant involvement did not require deep synthesis or reflection by teacher educators. During analysis, the researchers noted shortened responses by participants as they described how their programs addressed the Mathematical Teaching Practices. Because of this fact evidenced in the data, it is advised for the MTE-Partnership to consider a broader survey administration with potential stipends for adequate survey completion.

An important component of this survey administration was the sample, which presents potential limitations, implications, and suggestions for future CERAC and MTE-Partnership research. The surveys' limited sample of CERAC participants makes findings difficult to generalize to other MTE-Partnership institutions. However, results provided opportunity for reflection by the network as a whole and a potential framework to begin tracking materials being implemented across the partnership. Using the framework presented in this paper or a similar one allows for the partnership to conceptualize their work and its progression of development inside and outside RACs for continued improvement.

Lastly, the innovation configuration framework provided in these proceedings can serve as a guide for other partner schools to critically examine their program goals toward teaching Mathematical Teaching Practices with fidelity. As programs begin to analyze their own student progressions, it is advised for the partner schools to attend to the level of understanding and demonstration of the Mathematical Teaching Practices at each innovation configuration level as students develop in their programs. A recommendation for programs using this innovation configuration for program improvement are encouraged to track levels of Mathematical Teaching Practice implementation as they relate to each course and their progression through their mathematical programs. Earlier courses would be expected to have more level one and two instances of innovative practices, while later courses would be expected to have more at levels two and three—largely because students need to know about

Mathematical Teaching Practices before being asked to implement them in experiences. It would be advised for program directors to utilize this innovation configuration framework to identify areas of need in courses by having discourse with the instructors and/or reviewing course syllabi.

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Appendix

MTE-Partnership Clinical Practices Implementation of Mathematics Teaching Practices from *Principles to Actions* Survey

1. What is your name?

2. How are you implementing the Mathematics Teaching Practice: Establish Mathematics Goals to Focus Learning with your PSTs?

3. How are you implementing the Mathematics Teaching Practice: Implement Tasks That Promote Reasoning and Problem Solving with your PSTs?

4. How are you implementing the Mathematics Teaching Practice: Use and Connect Mathematical Representations with your PSTs?

5. How are you implementing the Mathematics Teaching Practice: Facilitate Meaningful Mathematical Discourse with your PSTs?

6. How are you implementing the Mathematics Teaching Practice: Pose Purposeful Questions with your PSTs?

7. How are you implementing the Mathematics Teaching Practice: Build Procedural Fluency to Promote Conceptual Understanding with your PSTs?

8. How are you implementing the Mathematics Teaching Practice: Support Productive Struggle with your PSTs?

9. How are you implementing the Mathematics Teaching Practice: Elicit and Use Evidence of Student Thinking with your PSTs?

Developing a Framework for Equitable Mathematics Instruction

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Active learning and inquiry based instructional practices, as enacted in university Precalculus through Calculus 2 courses (referred to as the P2C2 sequence), have been shown to be beneficial in numerous ways. Freeman et al. (2014) found that active learning was associated with increased exam scores and concept inventory results, as well as significantly decreased withdraw and failure rates to a degree that calls into question whether traditional lecturing should continue to be used as a control in research studies at all and that supports “active learning as the preferred, empirically validated teaching practice” (p. 8410). Inquiry-based learning (IBL) in college mathematics has been shown to result in benefits described as learning gains in cognitive, affective and collaborative areas, for both women and men, while also decreasing the gap between the genders (Laursen, Hassi, Kogan, & Weston, 2014).

However, students who are members of underrepresented groups in mathematics, including but not limited to women and students of color, describe their experiences in active learning P2C2 courses as less inclusive, less positive and more marginalizing as compared to their white male peers (Voigt, 2017). Rasmussen and Ellis (2013) found that, within institutions that were part of the *Characteristics of Successful Calculus Programs* study, students who planned to continue on to a second semester of calculus, but changed their minds and stopped taking calculus after one semester, included significantly more women than men. While women make up approximately half the students in college Calculus 1 courses, their numbers drop off precipitously after the first semester of calculus (Rasmussen & Ellis, 2013). Students of color and students who are poor often face a different set of challenges in that they are more likely to experience high-school learning environments that are under resourced and that may not provide access to high-level mathematics courses (Berry, 2015). These learning experiences can lead to placement into precalculus rather than calculus in their first semester in college, and this scenario places an additional barrier to continuing in mathematics or mathematical fields of study. Responding to experiences of students of color in undergraduate mathematics courses Jett (2013) cites Leonard & Martin (2013) in his call for college mathematics instructors to take up culturally responsive pedagogy to “enact the brilliance that African American students bring to the mathematics space” (p. 102). Taken together, the research and evidence lay out a clear call for further investigation into the nature and characteristics of mathematics instruction that supports positive and inclusive learning experiences in undergraduate P2C2 courses, particularly those students who are members of underrepresented groups in mathematics.

This paper presents a theoretical model for equitable mathematics instruction, which I consider to be instruction that supports more positive and inclusive learning experiences for all students, especially for students who are members of underrepresented groups in mathematics. This framework for equitable mathematics instruction is based on a review of research in mathematics education that demonstrates the critical role of identity as a doer of mathematics in contributing to students’ decisions to participate in mathematics (e.g., Boaler & Greeno, 2000), while also showing the degree to which students who are members of underrepresented groups in mathematics frequently persist in spite of marginalizing racialized and/or gendered experiences in mathematics communities (e.g., Joseph et al., 2017; McGee, 2015; e.g., Solomon, Radovic, & Black, 2016). Research on Black women and girls’ experiences in mathematics has described robust mathematics identity as “an aspect of selfactualization that is needed for persistence, engagement and sustained success" in mathematics (Joseph, Hailu, & Boston, 2017, p. 203). McGee's (2015) research on the experiences of high-achieving Black college students defines robust in the context of mathematics identity as "the strength and agency that students develop in spite of their racialization to maintain self-motivated mathematics success" (p. 604).

The framework put forth in this paper suggests that mathematics instruction that values students' personal identities, enabling them to enact their fully intact cultural, racial and gender identities while doing mathematics, may support sense of belonging in mathematics. The model is built on the premise, which is in alignment with mathematics identity research, that a student who develops an identity as a doer of mathematics and a sense of belonging in mathematics in concert with each other – i.e., what I refer to in this paper as a robust mathematics identity – may be more likely to persist in mathematics. Equitable mathematics instruction is thus conceptualized as instructional practices and instructor beliefs that cultivate positive and inclusive mathematics learning environments that support students, particularly those who are members of underrepresented groups in mathematics, to develop robust mathematics identities and to persist in mathematics because of, rather than in spite of, their experiences learning and doing mathematics.

Theoretical Perspective

Learning is a social process (Wenger, 1998) and students' learning in mathematics classrooms is highly influenced by their ways of "engaging in and contributing to the practices of their communities" (Wenger, 1998, p. 7). Students who become participants (Wenger, 1998, pp. 55-56) within their mathematics learning communities take up a form of learning that has been shown to support their ability to envision themselves pursuing further study in mathematics or other STEM fields (Boaler & Greeno, 2000; Cobb & Hodge, 2002). Students' development of an identity as a doer of mathematics depends on the development of a vision of themselves doing mathematics that aligns with their perception of what it means or involves to do mathematics (Boaler & Greeno, 2000). In other words, students who experience roles as central participants in mathematics learning communities, and whose experiences of doing mathematics in those communities align with their visions of themselves as people, have been shown to be more likely to develop identities as doers of mathematics and to persist in mathematics or STEM fields of study.

Boaler & Greeno (2000), citing Holland, Lachicotte, Skinner, & Cain (1998) state that they are building on the theory that "identities develop in and through social practice" and use the term "'positional identity' to refer to the way in which people comprehend and enact their positions in the worlds in which they live" (p. 173). This attention to people's positions in the world points to the importance of drawing from research on the sociopolitical nature of mathematics and mathematics learning experiences (Gutiérrez, 2013; Nasir & McKinney de Royston, 2013; Valero, 2004). Aguirre et al. (2017) state that "a sociopolitical approach allows us to see the historical legacy of mathematics as a tool of oppression as well as a product of our humanity" (p. 125). Research demonstrates that members of certain populations (e.g., students of color and women) experience reduced opportunities to participate in the learning and doing of mathematics due to structures that are systematic and pervasive (e.g., Berry, Ellis, & Hughes, 2014; Langer-Osuna, 2011). Taking a sociopolitical perspective enables us to attend to the ways that "people's positions in the worlds in which they live" (Boaler & Greeno, 2000) are related to their positions in the mathematics worlds in which they take part.

What is Equitable Mathematics Instruction?

Equitable mathematics instruction refers to classroom practices—including instructional practices and instructor beliefs—that lead to development of students' robust mathematics identities through the cultivation of learning environments that support: (a) identity as a doer of mathematics and (b) a sense of belonging in mathematics. Robust mathematics identities develop when students' identity as a doer of mathematics and sense of belonging in mathematics are both fully supported by their experiences in mathematics learning communities. This paper theorizes that active and inquiry-based instruction, enacted in ways that are student centered and genuinely focus on students' ideas, experiences and ways of understanding mathematics, are more likely to support students' positive identities as doers of mathematics. My research further suggests that instructors' focus on students' assets or strengths, and instructors' awareness of mathematics education as sociopolitical, are likely to support students' sense of belonging in mathematics. Taken individually each of these components may function as valuable improvements to instruction, but they may or may not be enacted in ways that are entirely equitable or that lead to improved outcomes for students who are members of underrepresented groups in mathematics. When these instructional practices and beliefs are enacted together in complementary ways, they may represent a powerful and reliable model of equitable mathematics instruction.

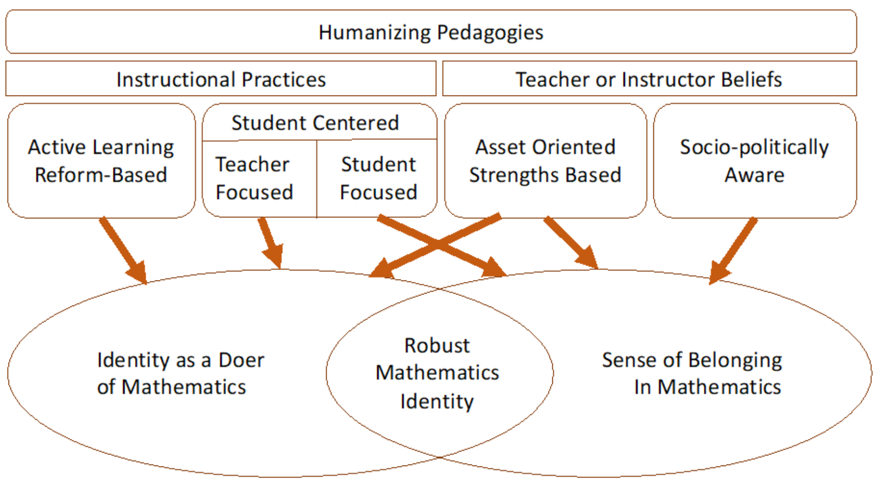

Figure 1. A conceptual model of equitable mathematics instruction.

Supporting Students' Identities as Doers of Mathematics

A student's identity as a doer of mathematics depends on their development of a vision of themselves doing mathematics that aligns with their perception of what doing mathematics means or requires (Boaler & Greeno, 2000). In other words, students who take on roles as central participants (Wenger, 1998) in mathematics learning communities, and whose experiences of doing mathematics in those communities align with their visions of themselves as people, are more likely to develop identities as doers of mathematics and to persist in mathematics or STEM fields of study. Instructional practices that support development of identities as doers of mathematics do so by supporting students' as sense-makers and problem-solvers in mathematics learning communities. Instructor beliefs that encompass an asset orientation, or that are strengths-based, may manifest as instructors' ability to take up instructional practices that anticipate student success with complex mathematical sense-making and problem solving.

Active and inquiry-based learning. Design principles for active learning were developed by the Active Learning Mathematics Research Action Cluster (ALM RAC) of the Mathematics Teacher Education Partnership (MTE-Partnership) and reflect a synthesis of research on active and inquiry-based learning. These principles encompass instructional practices that engage students in active meaning-making and sense-making and that support students to share "partially developed conjectures, explanations and representations of solution strategies" (Webb, 2016, p. 2). Active and inquiry-based learning have been shown to support students' development of identities as doers of mathematics (e.g., Boaler & Greeno, 2000) and contribute to increased success and persistence in university mathematics (Freeman et al., 2014; Laursen et al., 2014).

Student-centered instruction. Introduction of the terms "teacher-focused" and "student-focused" allow us to draw a distinction and intentionally highlight differences between these two forms of student-centered instruction. The term "student-centered" refers to learning environments in which students' voices are heard more than the instructor's voice and/or students are actively participating in the doing of mathematics. However, student-centered instruction that is teacher-focused allows for and frequently refers to learning environments in which the students are at the center of activity, but the focus remains on the teachers' ideas, understandings, and methods. Questions, problems, and solution strategies are still conceived of and posed by teachers. This paper uses the term "student-focused" specifically to describe contexts in which students are participating in doing mathematics in ways that begin with focusing on their mathematical ideas and understandings and proceed by valuing their perspectives, experiences, and relevant issues in their lives. In student-focused instructional contexts, the students have a meaningful degree of agency over the questions, problems, and solution strategies.

Supporting Sense of Belonging in Mathematics

Gutiérrez (2018) states "beyond being seen as a legitimate participant (a "doer" of mathematics), a student should be able to feel whole as a person—to draw upon all of their cultural and linguistic resources—while participating in school mathematics" (p. 1). Bringing this feeling to fruition requires that mathematics learning environments support both students' development of identities as doers of mathematics and also their ability to "feel whole as a person" in mathematics—i.e., their sense of belonging in mathematics. Instructors whose belief systems are asset-oriented or strengths-based and reflect sociopolitical awareness may be well positioned to interact with students in ways that facilitate students' development of a sense of belonging in mathematics. Instructional practices that center and build off of students' ideas and experiences, i.e., those that are student-focused, may further contribute to students' sense of belonging in mathematics.

Asset or strengths-based orientations. In order to clarify what is meant by the terms "asset orientation" and "strengths-based," this paper first describes the nature of deficit thinking, which is more prevalent in society and education, and which is particularly strong in mathematics. The term "deficit orientation" refers to ways of viewing students' knowledge, learning abilities, accomplishments, communities and/or students themselves that are focused primarily on students' shortcomings,or are predominantly concerned with what students do not know, seem unable to learn, or are unable to do, as well as resources that are unavailable in communities or the ways in which the students themselves seem to fall short of some ideal. Jett (2013) states that "deficit-oriented ideological paradigms and treatises, such as achievement gap discourse, often frame students of color, particularly African Americans, as mathematically deficient" (p. 103).

In contrast, an asset orientation, or a strengths-based view of students, is one that attends closely to identifying what students *do* know, *can* learn, and *can* do, as well as what resources *are* available in their communities and more generally the strengths and assets that the students possess. An asset orientation is likely to inform interactions that support a greater student sense that their mathematical ideas and experiences are valued and that their personal identities are fully welcomed and appreciated in mathematics classrooms.

Sociopolitical awareness. "People's positions in the worlds in which they live" (Boaler & Greeno, 2000) are related to their positions in the mathematics worlds in which they take part, and social contexts have been shown to impact the ways that people learn and do mathematics (Nasir, Hand, & Taylor, 2008). This attention to people's positions in the world points to the importance of attending to research on the sociopolitical nature of mathematics and mathematics learning experiences (Gutiérrez, 2013; Nasir & McKinney de Royston, 2013; Valero, 2004). Aguirre et al. (2017) state that “a sociopolitical approach allows us to see the historical legacy of mathematics as a tool of oppression as well as a product of our humanity” (p. 125). Sociopolitical awareness is likely to enable instructors to attend to the realities of how students’ experiences, ways of being, and interactions in society impact their experiences, ways of being, and interactions in mathematics classrooms. This awareness extends to instructors being aware of factors that originate outside the mathematics classroom that may affect the ways that students choose to participate in mathematics classrooms. This research suspects that sociopolitical awareness may serve to help instructors broaden their conceptions of what are accepted, desirable ways of participating in mathematics classrooms.

Furthermore, with regard to language in mathematics classrooms this paper claims that there is a need to broaden what are considered to be valid ways of communicating about mathematics—i.e., to expand what is valued as the “language of mathematics”—and that this represents a crucial component of validating students’ own ideas, experiences, ways of understanding, and ways of communicating about mathematics. In other words, broadening the nature of accepted communication in mathematics classrooms may be one way of enacting instruction that is student-focused and asset-oriented. Furthermore, for bilingual students in particular, inviting, supporting and encouraging students to make sense of mathematics and share their personal experiences with mathematics using the full range of their linguistic resources (i.e., translanguaging) is necessary to students’ development of a sense of belonging (i.e., their ability to be fully themselves) in mathematics communities.

Supporting Students’ Development of Robust Mathematics Identities

As previously described, identity as a doer of mathematics is crucial to students’ decisions to persist in mathematics. However, research describing the experiences of Black women and men who have persisted in mathematics (e.g., Joseph et al., 2017; McGee, 2015) makes clear that marginalized students are persisting in spite of racialized and gendered experiences in mathematics learning communities. It is possible that experiences that center students’ personal identities, enabling them to participate in mathematics while also bringing their full cultural, racial, and gender identities into those mathematics spaces, may support sense of belonging in mathematics. A student who develops an identity as a doer of mathematics and a sense of belonging in mathematics in conjunction with each other, i.e., a robust mathematics identity, may be more likely to persist in mathematics because of, rather than in spite of, their experiences learning and doing mathematics.

Envisioning Equitable Mathematics Instruction in Action

It may be easier to imagine equitable mathematics instruction through consideration of three scenarios. In each scenario a potential instructor response is first described in a way that is quite common in mathematics classrooms, but which may fall short of supporting all students to engage positively with that particular learning opportunity. This paper then offers a contrasting alternative response that may be more likely to support the student’s identity as a doer of mathematics and sense of belonging in mathematics, i.e., their robust mathematics identity, by reflecting an asset orientation or sociopolitical awareness or by enacting student-focused instruction.

<u>Scenario 1</u>: A student contributes an idea or a strategy for solving a mathematical problem in class. As described the strategy is not likely to produce a correct solution to the problem.

- The instructor might identify misconceptions or misunderstandings and correct them. This approach focuses on what is incorrect about the student’s ideas, and then positions the instructor’s way of thinking about the relevant concepts at the center of the conversation.
- Alternatively, the instructor could identify and acknowledge what aspects of the student’s suggestion are likely to be useful or productive and facilitate learning that builds off of those ideas. This instructional approach positions the student’s ideas as valuable, demonstrating an asset orientation toward the

students' mathematical ideas. It also keeps student thinking at the center of the classroom conversation reflecting a student focused approach.

Scenario 2: A student describes a mathematical idea using linguistic practices that differ from those of the instructor, perhaps using words and phrases from a language other than English or speaking in a dialect that is associated with a particular race, economic class, or geographic location.

- The instructor might respond less positively to this student's idea, or quickly move on to another student, because the structure or nature of the language practices are unfamiliar. This sends the message, intended or unintended, that this student's ideas aren't worthy of further consideration, or that it is not worth the time to come to clarity about the mathematical nature of these ideas. This approach may reflect a deficit orientation toward the mathematical ideas of students whose language practices differ from those of the instructor, or a lack of sociopolitical awareness about the significance of different language practices. Students whose ideas are responded to in this way lose the opportunity to access roles of central participation in the mathematics classroom.
- Alternatively, the instructor could consider this student's idea to be just as valid as others coming from students whose language practices are more similar to those of the instructor. The instructor may need to spend time and/or effort ensuring the student's idea is understood and acknowledged. That time reflects an asset orientation toward the student's mathematical thinking and a sociopolitical awareness about the use of language to express ideas.

Scenario 3: A student shares a problem-solving strategy that is unconventional or unfamiliar to the instructor.

- The instructor could dismiss the suggestion because they are unfamiliar with the student's approach. This instructional approach might be reflective of a deficit orientation based on an assumption that the idea has limited merit, or it could be related to lack of sociopolitical awareness of differing ways of communicating about and/or doing mathematics.
- Alternatively, the instructor might support the community to engage with and explore the idea to reveal its value and relevance. This strategy draws upon an asset orientation by positioning the idea as important and useful, and might also indicate sociopolitical awareness of how students' varied experiences in life can influence their ways of thinking about and doing mathematics.

Finally, a fourth example of how instructors can pose open-ended questions that open space in the classroom conversation for students to interact with mathematics in ways that make the most sense to them, can be achieved by simply changing the word "the" to "a":

Scenario 4: An instructor wishes to ask students to graph a relationship.

- This task could be presented as, "draw **the** graph," implying that there is one correct answer. Presumably the correct answer will reflect a teacher -centered approach since it will be the one the instructor has already thought of or the one the instructor deems to make the most sense.
- Alternatively, the instructor could ask students to "draw **a** graph" to invite a broader range of responses that reflect students' ideas. The answers may vary somewhat based on the ways that students set up their graphs, their choice of units, or their decisions about what dimensions to display on each axis. It's also likely that students who understand the scenario presented and the mechanics of graphing will provide graphs that are mathematically accurate and that reflect their own way of thinking about important characteristics of the scenario. This approach reflects a student-focused approach since students are not required to reproduce the instructor's way of thinking about the mathematics in order to earn credit for their work.

In each of these cases the alternative approach represents a way of enacting active learning that is not only student-centered but also student-focused, that reflects an asset orientation to students' ideas and contributions, and that demonstrates an awareness of the ways that sociopolitical factors can influence student participation in mathematics classrooms. Each of these examples reflects the instructor's genuine interest in understanding the student's contributions as well as use of instructional practices that model the importance of valuing and taking up students' ideas.

Conclusion

Successful implementation of equitable mathematics instruction depends partially on the affordances of mathematics department contexts that provide especially supportive structures and exhibit particular community characteristics, which include structural supports and community influences (e.g., tutoring centers, strong personal relationships, and curriculum adjustments that meet the needs of particular populations of students). Research has shown that these community structures play an important role in supporting Black women and girls, in particular, to develop robust mathematics identities and to persist in mathematics (Joseph et al., 2017).

The proposed model of equitable mathematics instruction contributes to the goal of increasing our understanding of how mathematics instruction might embrace and value students' full cultural, racial and gender identities in mathematics learning spaces. In so doing equitable mathematics instruction may play an important role in rehumanizing mathematics, mathematics instruction and mathematics communities. It is theorized that equitable mathematics instruction, through supporting development of robust mathematics identities that include both identities as doers of mathematics and sense of belonging in mathematics, may contribute to increased participation among students who are members of persistently underrepresented and historically marginalized groups in mathematics.

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Inculcation of Pre-service Mathematics Teacher Social Consciousness Through Social Justice Mathematical Modeling Projects

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Abstract

The challenge for teacher educators of ensuring pre-service mathematics teachers develop sensitivity to the social-contextual factors that affect the teaching and learning of mathematics looms large. One way this can be addressed is through the incorporation of mathematics for social justice projects into existing curriculum. In this presentation, we discuss the results of the test enactment of a mathematics for social justice modeling project with pre-service teachers of mathematics and statistics. Participants engaged in the mathematical modeling process by making assumptions, identifying variables, and creating, refining, and implementing a mathematical model intended to identify inequities created and perpetuated by "tracking" in school mathematics. Participants documented the development and results of their model in a technical report wherein they provide education policy recommendations intended to dismantle inequitable structures they identify in the system of mathematics education. We present evidence from student work samples for a productive evolution of social consciousness among participants.

Introduction

During recent years, the mathematics education community has seen an outpouring of articles, books, and position papers by mathematics teachers, mathematics education researchers, and professional teacher and teacher educator organizations discussing the imperative for attending to issues of equity in school mathematics (see, for example, National Council of Teachers of Mathematics, 2014, 2018; National Council of Supervisors of Mathematics & TODOS, 2015; Association of Mathematics Teacher Educators, 2017). This call for equity is also embedded throughout the Mathematics Teacher Education Partnership's (MTE-Partnership) *Guiding Principles for Secondary Mathematics Teacher Preparation*, and indicators 5-E and 6-C have special relevance to this report. Guiding Principle 5 refers to candidates' knowledge and use of educational practices, and indicator 5-E states that pre-service teachers need to finish their teacher preparation program with the recognition that "all students in their classes—including low-performing students; gifted students; students of different racial, ethnic, sociolinguistic, and socioeconomic backgrounds; English language learners; students with different sexual orientations; and students with disabilities—have the potential to make important contributions," (p. 5) and that teacher candidates must maintain high expectations for all students. Guiding Principle 6 refers to professionalism, advocacy, and leadership, and indicator 6-C states that candidates require preparation as agents of change "so that through their actions, behaviors, and advocacy, [they] demonstrate a dedication to equitable pedagogy that promotes democratic principles by holding high expectations for *all* students, while recognizing and honoring their diversity" (Mathematics Teacher Educator Partnership, 2016, p. 5). In this paper, we report preliminary results suggesting the cultivation of *social consciousness*, that is, an *awareness* of and *sensitivity* to the social and cultural contexts in which students learn and do mathematics and the *belief* in their importance for equitable mathematics teaching practice, in 12 pre-service teachers of mathematics by way of a social justice-oriented mathematical modeling project.

Problem and Purpose of Study

There is little, if any, disagreement among stakeholders in school mathematics with the idea that all students deserve access to mathematics with a high level of cognitive demand from a capable teacher, at least in principle. Unfortunately, this consensus has done little to diminish many mathematics teachers' views of the practice of tracking. According to several sources, *tracking*, the "practice of dividing students into separate classes for high-, average-, and low-achievers," (a practice the Association of Mathematics Teacher Educators (AMTE) refers to as "oppressive") seems to count teachers among its most ardent supporters (AMTE, 2017; Brentnall, 2016; Burris & Garrity, 2008; Keller, 2011; Oakes, 2005). The 2018 National Survey of Science and Mathematics Education appears to support this conclusion. Of middle school and high school mathematics teachers polled, 66% and 70% respectively, agreed with the statement "Students learn mathematics best in classes with students of similar abilities," and only 36% and 33% consider themselves well-prepared to "Differentiate mathematics instruction to meet the needs of diverse learners." Finally, the results from the 2008 *MetLife Survey of the American Teacher* indicate that 43% of respondents agreed with the statement, "My class/classes in my school have become so mixed in terms of students' learning ability that I/teachers can't teach them" (Markow & Cooper, 2008).

Dispositions toward tracking are not the only concerning result from the survey. Only 49% of middle school and 46% of high school math teachers consider themselves prepared to "encourage participation of all students in mathematics," 37% of middle school and 38% of high school math teachers feel able to "encourage students' interest in mathematics," 23% of middle school and 26% of high school math teachers feel adequately prepared to "provide mathematics instruction that is based on students' ideas," and 13% of middle school and 17% of high school math teachers believe they can "incorporate students' cultural backgrounds into mathematics instruction" (Banilower et al., 2018). Put differently, teachers do not feel prepared to utilize equity-based teaching practices (Aguirre, Mayfield-Ingram, & Martin, 2008).

How can we ensure that students leave our teacher preparation programs understanding that "the social, historical, and institutional contexts of mathematics affect teaching and learning and know about and are committed to their critical roles as advocates for each and every student" (AMTE, 2017, p. 21)? The answer is certainly multifaceted, but one point of interest with the potential to address a host of other issues is that of teacher beliefs. A large body of research suggests that teacher beliefs about effective pedagogical practices, about which students should or are capable of learning cognitively demanding mathematics, and about what it means to do mathematics, have a profound impact on their teaching practice and on the achievement levels of their students (van den Bergh, Denessen, Hornstra, Voeten, & Holland, 2010; McKown & Weinstein, 2008; Rubie-Davies, Hattie, & Hamilton, 2006; Staub & Stern, 2002; Rosenthal & Jacobson, 1968). Far less understood than the impact of teacher beliefs are the mechanisms underlying their formation and eventual acceptance (Bell, Halligan, & Ellis, 2006).

The State of Research into Teacher Belief Inculcation

Studies where researchers attempted to alter teacher beliefs are far fewer than those attempting to understand their impact, but what research has been performed provides us with a starting place for identifying effective means to the inculcation of social consciousness. For example, explicitly challenging the beliefs of pre-service teachers and requiring reflection seems to be an integral component of effective attempts to inculcate new beliefs and alter teaching practice (Caudle & Moran, 2012; Conner, Edenfield, Gleason, & Ersoz, 2011; Turner, Warzon, & Christensen, 2011). Furthermore, teacher candidates who receive instruction in the way they are expected to teach, who are witness to the successful implementation of innovative teaching practices, are more likely to use innovative teaching practices and possess stronger belief in their own capacity to teach (Jao, 2017; Katz & Stupel, 2016; Lloyd, 2013; Lloyd, 2018; Loucks-Horsley, 2010). While successful, these attempts have been relatively disparate, making comparison a challenge and severely complicating, but also highlighting the need to address the problem of developing a general theory of teacher belief inculcation. Only further complicating the situation is that there appears to be no common definition of "belief" underlying and unifying existing studies of teacher belief. Recent efforts in the realm of cognitive psychology provide a promising roadmap to a framework of belief that might be co-opted by teacher educators.

The Potential in Social Justice Mathematical Modeling

It is our belief that not only should teacher educators not shy away from discussing sensitive topics (e.g., issues regarding race and/or politics) in the classroom, but that they should be encouraged to do so, and that such "hard topics" can serve as the basis for mathematically rich, cognitively demanding tasks that can, and should, be incorporated into public school mathematics classrooms. This form of mathematics pedagogy, known as mathematics for social justice or teaching mathematics for social justice, seeks to situate students of mathematics as agents of change by engaging them in cognitively demanding mathematical tasks that require them to analyze issues of fairness in society (Gutstein, 2005). While related, mathematics for social justice should not be confused with the ongoing struggle for social justice in mathematics education. The distinction between the two may be illuminated by a (particularly relevant) example: eliminating tracking in school mathematics is an issue of social justice in mathematics education while providing students with the opportunity to critically analyze the inequities created and perpetuated by tracking using mathematics is an example of mathematics for social justice.

In addition to providing pre-service teachers with an opportunity to engage in a mathematical task possessing a high level of cognitive demand, the use of a mathematical modeling project provides additional benefits by incorporating several of the components that contribute to a change in teacher beliefs, including the successful utilization of innovative teaching practices and ongoing reflection. To this end, this study attempted to change pre-service teachers' beliefs and inculcate social consciousness by engaging students in a social justice mathematical modeling project during eight weeks of a methods course.

Project Design Framework

In this study, pre-service teachers were tasked with answering the question "How does 'tracking' in school mathematics affect students' achievement?" through the lens of equity, by way of developing a mathematical model. Note that we employ the term *mathematical modeling* in the manner of the Consortium for Mathematics and Its Applications (COMAP) and the Society for Industrial and Applied Mathematics (SIAM) as described in their joint effort publication *Guidelines for Assessment & Instruction in Mathematical Modeling Education* (GAIMME) (COMAP & SIAM, 2016). The project was structured after the HQPBL framework, which requires adherence to six criteria: intellectual challenge and accomplishment, authenticity, a public product, collaboration, project management, and reflection (Buck, 2018). Students were randomly placed in groups using the automatic group creation utility on the Canvas Learning Management System and were required to meet several times throughout the semester.

The project was enacted in a course titled Methods of Secondary School Mathematics Teaching, for pre-service mathematics teachers, and facilitated by an outside student researcher. Participants had received general instruction regarding the mathematical modeling cycle, as envisioned in the GAIMME report, shortly before being introduced to the project. The facilitator introduced the project by providing the pre-service teachers with a framing narrative, where participants were imagined as education policy researchers for a fictitious think-tank called the "MacGuffin Institute," by way of an initial slideshow presentation and discussion on the difference between equality and equity. Following a general discussion of equity, the concept of tracking in school mathematics was introduced and participants were encouraged to share relevant experiences. Participants then took a few minutes to answer some prompts regarding what information they deemed important for the central project question. Then teams were set to work on the initial task sheet after which they were directed to begin the work of producing a mathematical model intended to explore differences in student outcomes. One week later, participants were contacted electronically with the next set of instructions, wherein they were directed to develop an initial mathematical model and reflect on their progress. Three weeks later, they received in-class time to work on their models and to ask questions of the project facilitator. In addition, they were provided with another task sheet that included directions to: read excerpts from the National Council of Teachers of Mathematics (NCTM) publications *Principles to Actions: Ensuring Mathematical Success for All* (2014) and *Catalyzing Change in High School Mathematics: Initiating Critical Conversations* (2018) that discuss tracking; revise their mathematical models based on what they learned from the reading assignments; complete and submit their team's technical report; and write and submit a final reflection following the submission of their team's report. In their technical report, teams were directed to explain the process of developing, testing, and revising their model, and to provide policy recommendations for education policymakers intended to address the inequities they identify. Participants were encouraged to seek assistance outside of the classroom from the project facilitator. Following a brief look at the different approaches each team took in the development of their model, and the different results they produced, the teams participated in a final discussion of tracking that highlighted the ways in which it acts as a barrier to learning mathematics and disproportionately negatively impacts certain demographics of students. This discussion was planned to follow the structure outlined by Smith and Stein's *5 Practices for Orchestrating Productive Mathematics Discussions* (2018). Following the discussion, the participating pre-service teachers were directed to write down two to three specific actions they intended to take to challenge inequitable structures in their future careers as secondary mathematics teachers.

Conceptual Framework for Analyzing Belief Inculcation

Belief is only beginning to receive the consideration it is due among cognitive neuropsychologists (Bell et al., 2006a). This being the case, it should come as no surprise that a cognitive account of mathematics teacher belief formation has yet to be suggested in the literature. However, attempts by researchers to influence teacher beliefs taken together with what information cognitive psychology offers about belief formation can provide us with a starting point to account for and, consequently, better inculcate productive beliefs in teachers.

For this research we adopt a specific definition of *belief*, namely "the mental acceptance or conviction in the truth or actuality of some idea" (Connors & Halligan, 2015, p. 1). In an attempt at systematizing our efforts of social consciousness inculcation, we chose to use the five stages of belief as proposed by Connors and Halligan as an underlying framework for understanding the cognitive processes in which individuals engage when encountering new beliefs (see Figure 1).

Belief formation, by their account, begins with a *precursor*, an internal or external trigger, which is immediately followed by a *search for meaning*. During this stage, the individual explains or accounts for the precursor and then situates it within their existing "web of beliefs" (Connors & Halligan, 2015, p. 7). This process results in one or more "proto-beliefs" (Connors & Halligan, 2015, p. 8) intended to account for the trigger. Next comes the *candidate belief evaluation*, wherein the individual evaluates the proto-beliefs in terms of their "observational adequacy" (i.e., how well it/they explain the trigger) and "consistency with pre-existing beliefs" (Connors & Halligan, 2015, p. 8). Should a proto-belief survive this investigation, the next stage, that of *accepting and holding the belief*, is initiated, though the conviction with which the belief is held may vary significantly depending on the details of the evaluation that resulted in its acceptance. The final stage describes the *consequential effects of holding the belief*. These effects can be far reaching, as the individual will "perceive the world in a way that is consistent with the new and congruent existing beliefs" (Connors & Halligan, 2015, p. 10). In other words, just as the individual's existing beliefs prior to the precursor to the newly accepted belief influenced its formation, so too will the formation of *future* beliefs be influenced by the newly accepted belief (Connors & Halligan, 2015).

The creators of the stages of belief framework, recognizing the limits of their account of belief, advise caution with respect to the operationalization of their model, though they do "suggest that a complete theory of belief will need to account for at least these five stages" (Connors & Halligan, 2015, p. 10). Following their recommendation, while we attempt to align student work with the framework to gain insight into the process of pre-service teachers' belief development, we recognize the complexity of belief formation and the limits of the stages of belief framework and, therefore, do not employ the framework as a strict coding system.

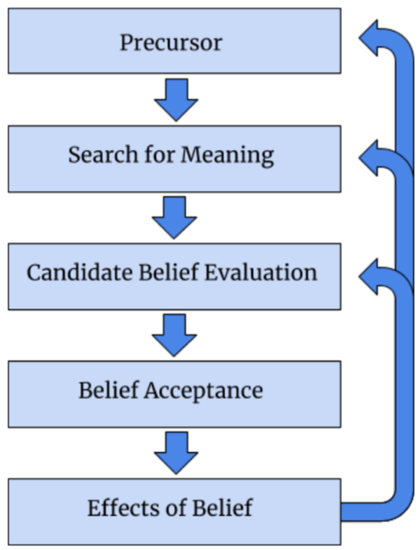

Figure 1. The five stages of belief formation, with arrows indicating their interdependence. Adapted from Connors, M. H., & Halligan, P. W. (2015). A cognitive account of belief: a tentative road map. *Frontiers in psychology*, *5* (1588). doi:10.3389/fpsyg.2014.01588.

Methods and Procedures

Data collection made use of written assignments including: an initial task sheet; two reflection journals, one written near the beginning of the project and one following the submission of the pre-service teacher team's technical report; and an action plan written immediately following the final in-class discussion of the modeling approaches taken by different teams and the social justice implications of tracking.

The initial task sheet included a prompt asking students to reflect on and list their own perspectives and biases regarding the issues of tracking. The first reflection journal included prompts addressing multiple facets of the project, but only one intended to elicit student responses for gaining insight into their beliefs.

1. *Do you agree with the organizations that claim tracking creates and reinforces social inequities and is actually oppressive? Why or why not?* (For example: https://amte.net/sptm/chapter-6- elaborations-standards-preparation-middle-level-teachers-mathematics/standard-c4)

The second reflection journal also included two such prompts.

1. *Write down what you consider to be the most important thing you have learned about the practice of tracking in school mathematics.*
2. *Write down a specific plan you intend to enact to ensure that you are working to challenge inequitable structures like tracking in your future career.*

The action plan involved a single prompt, issued verbally by the project facilitator, to write down, in two or three sentences, a plan for how they will promote equity in their future classroom and challenge inequitable structures within their own school.

Of the 12 pre-service teacher participants, eight submitted the initial task sheet, 10 submitted the first reflection journal, 11 submitted the second reflection journal, and 11 were present to complete the action plan writing assignment. In addition, it is unclear what number of students completed the reading assignment, and therefore the degree to which they incorporated the ideas found therein in a revised mathematical model.

Pre-service teacher writing assignments for each individual were read and situated within the five stages of belief formation to gain insights into the evolution of students' beliefs and, therefore, of social consciousness. Evaluation of student work was performed by the researcher/project facilitator, and the course instructor and researcher reviewed and reflected on the in-class discussions associated with the project. As this is a preliminary study, no formal instrument was utilized for evaluating student work with respect to changes in belief.

Data Analysis

The initial task sheet prompt, wherein participants were asked to list their own perspectives on and biases regarding the issue of tracking, failed to consistently provide clear insights into students' views of tracking. The reflection journal entries provided the clearest insights into the students' changing (or nonchanging) beliefs over the course of the project. The action plan provided some evidence of an evolution of social consciousness in the participating pre-service teachers by way of the commitments they chose to make. However, none of the participants made specific mention of the issue of tracking itself, opting instead to focus on specific related issues like maintaining high expectations for all students, advocating on behalf of students, and challenging the practice of *teacher tracking*, i.e., the widespread practice of placing inexperienced teachers in charge of remedial mathematics courses.

All 12 participants appear to have experienced some degree, however small, of productive evolution of social consciousness; though, as is the case with any research relying on a form of self-reporting and given that participants were aware that their writings would be used for research purposes, we cannot discount the possibility that changes in participant rhetoric regarding the use of tracking in school mathematics may be due to the Hawthorne Effect, that is the tendency of the participants to behave differently than they normally would because they know they are taking part in a study (Corsini, 2001; McCambridge, Witton, & Elbourne, 2014). In addition, it must be pointed out that while we situate pre-service teachers' written submissions within the five stages of beliefs formation described above, it is likely that participants experienced multiple iterations of stages of belief formation, with numerous beliefs, and at multiple times throughout the project. That is, even if the statements found in the student work examples below do in fact align with the stage of belief development we suggest, they may come from entirely different iterations of belief development, or from the development of different beliefs altogether.

Of the 12 participants, three appeared to have begun the project already believing that tracking creates and perpetuates inequities, and so no belief inculcation may have been necessary or possible. For example, one student wrote in their first journal entry that, "I don't actually think that the benefits that it [tracking] provides are enough to make up for the bad effects on the other students." Their existing web of beliefs may have either already included the belief that tracking is harmful, or was well-equipped to quickly situate the precursor, quickly leading to the acceptance of the new belief.

In one of the starker examples, one participant admitted early on that their past experiences in school mathematics have contributed to their holding views that

> Students in AP Classes/Honors are smarter, work harder, are better. Students in Remedial classes aren't as smart, hard working [sic], or dedicated as those in AP classes. Students who get dropped down for AP or honors classes didn't try hard enough. Students at the 'normal' level are equivalent to those in remedial classes.

The participant provides us with some valuable insights into their "web of beliefs" on the nature of how they understand success in school mathematics. Four weeks later, the same participant wrote,

> When I was first introduced to this topic, I thought that tracking was a good thing, because it challenged students. As the class and I have engaged in discussion, I was able to get a different view from the experiences of my classmates. It made me think critically about the downsides of tracking. As to whether I agree or not with the claims that tracking reinforces social inequalities and is actually oppressive, the verdict is still out for me. I hope that it is a good thing but based on the data I've seen and the experiences my classmates have shared, I'm starting to believe that it may not be good [sic] thing.

The participant appears to be evaluating proto-beliefs as possible explanations for the concerning data they encountered and is at the same time reevaluating the original unproductive beliefs about tracking. An additional three weeks later, the student appears to have accepted one of the proto-beliefs, concluding,

> I think the most important thing that I have learned about tracking is how harmful it can be. I've never considered it to be harmful, but I was always in the 'advanced' classes. This project helped me see the opposite side, when students are placed in 'remedial' classes, and the damage that it can do to the students. It helped me to see the side that I was unfamiliar with and has actually helped me see that tracking is harmful to many students. I thought it was a good thing when we started this project, but now I think that it is a harmful thing to most students, remedial and/or advanced.

One week later, in the action plan written immediately following our final classroom discussion of tracking, this pre-service teacher expressed the intention to "be an advocate for students who aren't receiving the support they need." Without a longitudinal study we will be unable to determine the conviction with which the newly accepted belief is held, and therefore whether it will actually shape the actions and future beliefs of the participant. However, this response may indicate what form *consequential effects of holding the belief* might take.

Another two participants, while possibly exhibiting a weakening of resolve in their conviction that tracking benefits students more than it harms them, never appear to have accepted a productive belief about tracking. In their first reflection journal entry, one of these students wrote,

> I don't agree with the organizations that claim tracking creates and reinforces social inequalities and oppresses students. I believe that tracking schools actually help students develop math skills at their own level. If they need remedial help for mathematics then they should take remedial math classes to help them catch up to the regular level students are at. Then when they no longer need remedial classes they should be able to take regular classes. When the students' skills surpass their peers in regular classes then they should take advance [sic] honors math classes. Tracking schools [sic] goals are to improve each student's mathematical skills and to teach each student at their own level.

This participant appears to have accommodated the information of the precursor, i.e., that tracking creates and perpetuates inequities, within their existing system of beliefs. We cannot be certain whether the participant evaluated other candidate beliefs or immediately situated the new information within their web of beliefs in an initial search for meaning. Their second reflection journal entry appears to show no change in belief, with the student writing,

> The most important thing I have learned about the practice of tracking in schools is that it is supposed to help students learn at their own level. Every student is at a different level of learning and some students need more help than others so tracking is a way of incorporating equity in schools. This purpose is to give students an opportunity to learn math concepts at their own level.

The participant may have evaluated other beliefs, but none appear to have been accepted. In the same entry the participant appeared to exhibit at least a willingness to remain open to alternative conclusions regarding tracking, expressing their commitment to "gather data from student performance based on this knowledge and after years of teaching and collaborating with teachers of different schools that are tracking or non-tracking, I will come to a conclusion about the effectiveness of tracking." The student does appear to have undergone some evolution of social consciousness, however slight, with the student expressing their intent to employ "differentiated learning techniques" to accommodate students of varied achievement levels.

Of the remaining six participants, two began with a neutral view of tracking and later concluded the practice is harmful, three began with a neutral view of tracking and retained that perspective for the remainder of the project (though two of them committed to differentiating instruction for diverse learners while the other to continuing a critical analysis of tracking in their action plans), and one only submitted the action plan wherein they made a commitment to hold all students to high expectations.

Results

The Connors and Halligan (2015) stages of belief served as the underlying theory for understanding how the use of mathematical modeling projects may serve to help inculcate productive, equitable beliefs. Long-term mathematical modeling projects provide teachers with multiple opportunities to introduce precursors intended to challenge pre-service teachers existing beliefs. They also provide pre-service teachers with candidate beliefs in the form of model results and conclusions from their own work and that of their peers, with teacher feedback and questioning regarding their assumptions and choice of variables, and with time to evaluate the candidate beliefs they entertain. Should student groups produce models that do not lead them to the intended candidate beliefs, the final presentation and discussion serves as a forum where model results that may promote unproductive beliefs, and therefore with the potential to reinforce inequities, can be challenged by not only their teacher or project facilitator, but also by their peers.

Belief formation is still far from well understood, and the degree to which a belief is accepted and held, the factors leading to its acceptance, and even whether a belief is accepted at all, are difficult to measure. Most relevant to this study, the number of factors influencing the development of pre- and in-service teacher beliefs about the learning and teaching of mathematics, significantly complicates the challenge of isolating the source of a new belief and the mechanisms involved in its acceptance (see Figure 2).

Through the presentation and reflection in the final class discussion on the models and methods, evidence emerged that all three groups evolved in their understanding of the mathematical modeling cycle, and their understanding of mathematics for social justice. While several students were expecting to come up with a system of equations to solve, or a differential equation to analyze leading to a neat answer to a complex question, they instead found that the open nature of the problem required them to mathematize a complex issue and critically evaluate the effects of assumptions. They expressed surprise regarding the utility of mathematical approaches for addressing social issues.

As was expected by the researcher/project facilitator and the course instructor, some of the student groups (2 of the 3) relied on proportional reasoning for their mathematical model. Struggling to locate specific tracking data, they both relied on the assumption that high schools that offer calculus are schools that employ tracking, the rationales for both groups being that calculus courses only appear at the end of an accelerated or honors course track. They both gathered information available for individual schools that offer calculus from the Department of Education Civil Rights Data and compared the proportion of students enrolled in the schools, to those taking calculus, both of which were disaggregated by race. They both argued that if tracking were equitable, each student demographic would be represented in calculus courses proportional to overall student populations' demographic makeup. One group concluded that tracking is inequitable, while the other group's model appeared to suggest the opposite. This contradictory outcome was discussed as a class, and the differences in outcome were attributed to the possibility of flawed assumptions, which then produced contradictory results when applied to different schools.

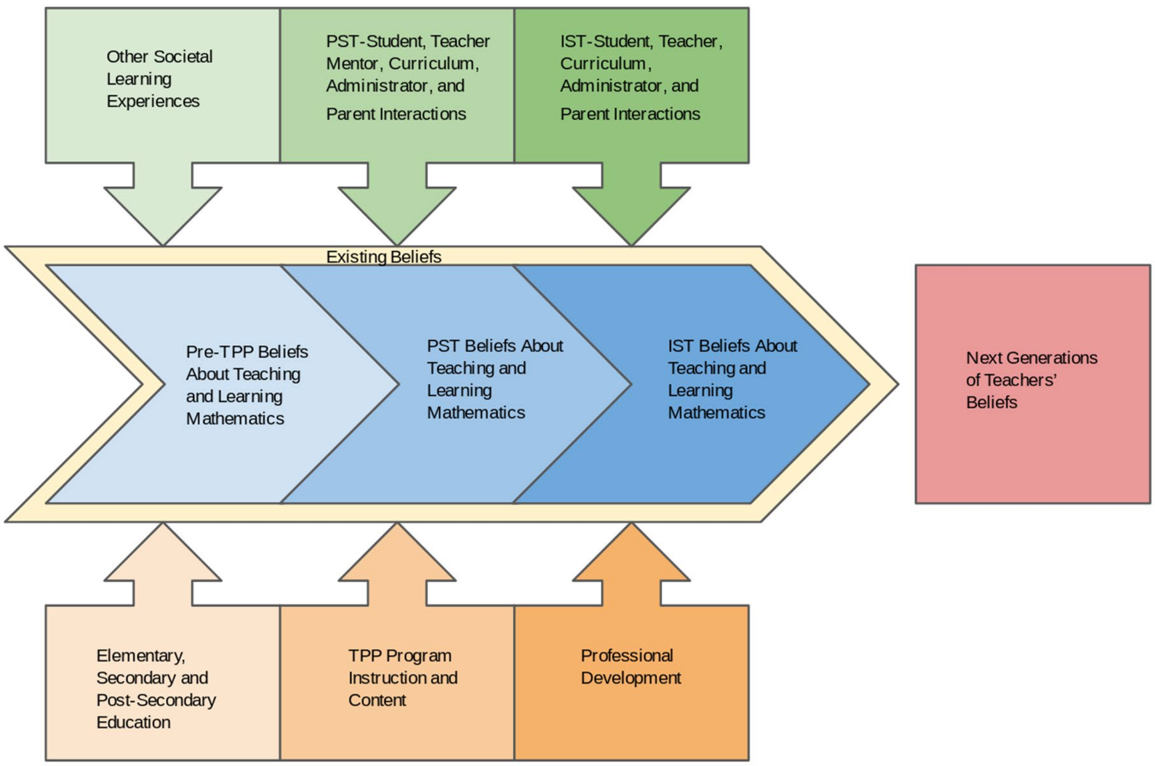

Figure 2. Factors influencing the development of pre- and in-service teacher beliefs, and the web of existing beliefs through which they are filtered. TPP refers to the teacher preparation program in this diagram, and IST refers to in-service teacher.

In an unexpected approach, the remaining group used NAEP data plots, comparing the percent of students tracked in eighth grade to AP course participation among high school students, AP and NAEP test scores, and child poverty rates. The model was inconclusive.

Conclusions

The impact of teacher beliefs, productive or otherwise, on their teaching practice and their attitudes toward inequitable practices like tracking cannot be overstated, and the results of this study provide additional evidence for the need of a comprehensive theory of teacher belief inculcation. While the stages of belief formation may serve as a useful starting place for understanding pre-service mathematics teacher belief formation, it must be integrated with existing research into influencing teacher beliefs.

This study also suggests a number of ways in which the project we utilized can be improved. Additional, and more specific, writing prompts eliciting students' beliefs about tracking will significantly improve the ability of researchers to gain insight into pre-service teachers' belief formation. In particular, the project would benefit from a simple prompt following the initial description of the practice of tracking, but prior to informing students of the many teacher organizations' statements condemning the practice, asking for their opinion of the practice. An additional reflection journal following the final social justice discussion of tracking also would provide better data on the participants' beliefs following the project than relying solely on the action plan, as we did for this study. In addition, students were expected to locate their own data for this project so as to avoid suggesting any specific model for them to develop, but the frustration pre-service teachers experienced and expressed in located the information they sought suggests additional scaffolding or a repository of relevant data may be beneficial. Furthermore, while this project was initially designed to be facilitated primarily outside of normal class meeting time, additional meetings dedicated solely to discussing and working on the project would be beneficial, not only to alleviate students' stresses arising from open-ended tasks like this project, but also to provide the project facilitator with additional opportunities to question student groups regarding the choice of variables and assumptions, and the limitations and implications of their models under development.

In summation, while improvements are still warranted, mathematical modeling projects appear to be a promising means of inculcating and influencing the beliefs of pre-service mathematics teachers.

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Paired Placement Internships: Clinical Teaching Becomes a Collaborative and Empowering Model for Ongoing Professional Development

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Abstract

The paired placement model for clinical teaching places two teacher candidates with one mentor teacher (Leatham & Peterson, 2010). This clinical teaching model has encouraged collaboration, pedagogical risk-taking, increased reflection, and better classroom management (Mau, 2013). Members of the Mathematics Teacher Education Partnership's (MTE-Partnership) Clinical Experiences Research Action Cluster (CERAC) have formed a sub-RAC called Paired Placement, which has implemented the paired placement model across multiple institutions for five years and have used Plan-Do-Study-Act (PDSA) Cycles to collect data before, during, and after the clinical teaching experience. The PDSA Cycle incorporates data from structured and unstructured interviews, surveys, teaching evaluations, reflective journals, and focus groups. Secondary mathematics teacher candidates who were enrolled in university-based teacher preparation programs, their mentor teachers, and their university supervisors participated in the PDSA Cycles. In this report, members from the Paired Placement sub-RAC share results from individual PDSA Cycles. Results describe the progress of each institution in their use of the paired placement instruments and various implementation protocols.

Overview

Clinical experiences are opportunities for teacher candidates to enact what they have learned in coursework via teaching, observing, and collaborating with effective mentor teachers. Furthermore, clinical experiences have been cited as more influential on long-term teaching practices than program coursework (Wilson, Floden, & Ferrini-Mundy, 2001). Moreover, the 2017 Association of Mathematics Teacher Educators' *Standards for Preparing Teachers of Mathematics* (AMTE Standards) state:

> An effective mathematics teacher preparation program includes clinical experiences that are guided on the basis of a shared vision of high-quality mathematics instruction and have sufficient support structures and personnel to provide coherent, developmentally appropriate opportunities for candidates to teach and to learn from their own teaching and the teaching of others. (p. 26)

The Clinical Experience Research Action Cluster (CERAC) of the Mathematics Teacher Education Partnership (MTE-Partnership), a subsidiary of the Association of Public and Land-grant Universities (APLU), is one group that has taken on the challenge of transforming secondary mathematics teacher candidates' field/ clinical experiences. The MTE-Partnership is a consortium of over 104 U.S. universities and colleges, along with partner school districts, focused on improving the initial preparation of secondary mathematics teachers. The MTE-Partnership uses a networked improvement community (NIC) design that incorporates improvement cycles to develop adaptable interventions across contexts to support comprehensive program improvement (Martin & Gobstein, 2015). Rather than addressing a single dimension of a secondary mathematics program, the MTE-Partnership is undertaking parallel lines of research in multiple areas. The CERAC consists of representatives of 26 university-led teams that have employed improvement science methods to develop resources that support improved models for both student teaching and early field experiences, as well as professional development for mentor teachers. In order to support the AMTE Standards, as stated previously, the CERAC focuses on a problem that is two-fold:

1. There is an inadequate supply of quality mentor teachers to oversee clinical experiences. Too few teachers are well-versed in implementing the *Common Core State Standards for Mathematics* (*CCSS-M*) and other college- and career-ready standards, and teachers are especially inexperienced with embedding the *Standards for Mathematical Practice* into their teaching of content standards (National Governors Association & the Council of Chief State School Officers, 2010.)
2. Bidirectional relationships between the teacher preparation programs and school partners in which clinical experiences take place are rare. Such relationships that reflect a common vision and shared commitment to the vision of *CCSS-M* and other college- and career-ready standards, and other issues related to mathematics teaching and learning are critical to the development and mentoring of new teachers.

One of the sub-RACs of Clinical Experiences focuses on the paired placement model, which places two teacher candidates with one mentor teacher (Leatham & Peterson, 2010). This model has been credited as a model of learning to teach that encourages collaboration, pedagogical risk-taking, increased reflection, and better classroom management (Mau, 2013), and thus can serve as a mechanism for helping to address the problem of the quantity and quality of mentor teachers. Members of the Paired Placement sub-RAC have implemented the paired placement model across multiple institutions for five years and have used PDSA Cycles (Bryk, Gomez, Grunow, & LeMahieu, 2015) to collect data before, during, and after the clinical teaching experience to improve the implementation of the model and to monitor the model's effectiveness. The PDSA Cycles incorporate data from structured and unstructured interviews, surveys, teaching evaluations, reflective journals, and focus groups.

In these five years, a number of different PDSA Cycles were used to refine and improve the effectiveness of the paired placement model. For example, a combined workshop/orientation meeting among the mentors, teacher candidates, and university supervisors at the beginning of the placement were not included in the original implementation of the model but were later added and refined to ensure each person involved in the collaboration understood their roles, responsibilities, and expectations, collectively. As a result of another PDSA Cycle, interview questions were developed for teacher candidates and later refined to include both mentor teachers and university supervisors in order for researchers to understand the effects of the paired placement model across the institutions.

Team members use these questions to help understand potential successes and obstacles of implementation across multiple contexts. These findings have been shared at annual MTE-Partnership conferences (2017, 2018) and at the 2017 National Council of Teachers of Mathematics (NCTM) Research Conference. Our partnerships across multiple institutions in the U.S. have allowed researchers to use PDSA Cycles to effectively enact the paired placement in ways that fit multiple contexts (Conway, Erickson, Parish, Strutchens, & Whitfield, 2017; Strutchens et al., 2019). Conway et al. (2017) provided a synthesis of the benefits and potential pitfalls of the paired placement from multiple contexts. Conway et al. found that the paired placement model increased teacher candidates' collaboration, sense of community, pedagogical risk-taking, reflective practice, and accountability. The model also decreased the number of cooperating teachers needed for placement in the field, which increased the quality of mentors in field placements (Conway et al., 2017). In 2019, Strutchens et al. described the implementation of the paired placement and co-planning and co-teaching field experiences across multiple contexts. Furthermore, Strutchens et al. (2017) described the use of the PDSA Cycles in creating and revising the Mathematics Teacher Practices Survey, the Mathematics Classroom Observation Protocol for Practices ($MCOP^2$), and the Program Completer Survey. Teacher candidates across the different institutions also were asked to complete the Mathematics Teacher Practices Survey every two weeks, and the teacher candidates were observed by a project member using the $MCOP^2$ developed by Gleason et al. (2017) at least twice throughout the semester.

This year, four institutions attempted to implement the paired placement model. In this report, members from the institutions share how they used one of the PDSA Cycles in their own institutional context. Researchers at Auburn University focused on how the university supervisor, mentor teachers, and two teacher candidates worked together; the Columbus State University research team examined observational tasks; and the researchers at the University of Hawai'i at Manoa investigated the co-planning and co-teaching strategies used by their pairs.[1]

Auburn University: Interviews and Focus Groups

The research (Leatham & Peterson, 2010; Mau, 2013; Peterson & Leatham, 2018; Strutchens et al., 2019) related to the paired placement model purports that the model enables teacher candidates to become student-centered, reflective, and more collaborative practitioners. Given these predicted attributes of the model, we used PDSA Cycles early and later during Spring 2019 to examine whether the teams composed of a mentor teacher, a pair of teacher candidates, and a university supervisor were working in a manner that would lead to the teacher candidates developing these traits. At Auburn University, we emailed the questions in Appendix A to the team members around Week 3 of student teaching. The team member emailed their responses to the project leaders; the project leaders then read the team members' responses to determine how well the team was working toward the aforementioned attributes. Teacher candidates also were required to keep journals related to their growth as teachers throughout the semester, and their growth also was monitored and measured with Auburn University program assessments. Project leaders held a focus group with the team members at the end of the semester to determine the growth of the teacher candidates and the effectiveness of the paired placement model (see questions used in the focus group in Appendix B). Thus, several data sources were used to determine teacher candidates' growth toward the aim of "during student teaching, teacher candidates will use each of the eight Mathematics Teaching Practices (NCTM, 2014) at least once a week during full time teaching" (Stutchens et al., 2017) and the predicted traits afforded by the paired placement model.

For this report, we focused on the PDSA Cycle that was given after two weeks of student teaching during Spring 2019 and the focus group that took place at the end of the student teaching experience during the same semester. Figure 1 contains the types of questions that were asked of each team member during the PDSA Cycle, and Figure 2 contains the types of questions that were asked during the focus group session.

1. How are the members of the team interacting?
2. How are the students responding to team members?
3. What is going well?
4. How can we better support you?
5. What are your strengths and weaknesses related to implementing equitable teaching practices?
6. At this point what do you see to be the benefits and challenges of the paired placement model?

Figure 1. PDSA Cycle focus on the veracity of the paired placement model.

[1] All names in the reports are pseudonyms.

1. What are attributes of the paired-placement model that contribute to successes in your current teaching practice?
2. What are attributes of the paired-placement model that contribute to challenges in your current teaching practice?
3. In what ways, if any, did the paired placement internship impact your working or collaborating with other teachers to support student learning?
4. In what ways did the paired placement internship impact your teaching practices? For example, did the emphasis on the mathematics teaching practices influence your own teaching of mathematics?
5. Did the paired placement internship encourage you to take more risks and try new teaching approaches in your mathematics classroom? If so, will you share an example?
6. What would you say are the components of the paired placement internship that positively impacted the interns' teaching practices? What components could be added to the model to make it more effective for future candidates?
7. Have the interns contacted you about a time in their current teaching career where they felt the experiences in the paired placement internship impacted their actions?
8. Have you served as a mentor teacher in a 1-1 internship previously, if so, which model do you think best serves the interns? Explain.

Figure 2. Focus group interview questions related to the veracity of the paired placement model. Note that each member of the team was asked questions appropriate for his/her particular role during student teaching and beyond.

Findings from PDSA Cycle

Since we had two sets of paired placements for Spring 2019, we labeled them Team A and Team B.

Team A. The mentor teacher on Team A has been teaching since 2003. She has the following degrees in mathematics education: a Bachelor of Science, Master of Science and a Ph.D., as well as an Ed.S. certification in mathematics education. She also has attended several professional learning workshops held by faculty members at Auburn University and has served as a presenter at mathematics education workshops and conferences. Dr. Gold has hosted several traditional teacher candidates and three pairs prior to this semester. The teacher candidate pair placed with her were secondary mathematics education majors. Miss Bold can be described as caring, easygoing, and average academically. The other member of the teacher candidate pair, Miss Wells, can be described as intense, driven, caring, and above average academically. The university supervisor, Dr. Merit, is a senior professor of mathematics education who is well versed in inquiry-based instruction and current reforms in mathematics education. He also served as an instructor for the secondary mathematics teaching course and an integrating mathematics with technology course, which had practicum experiences attached to them. Miss Bold and Miss Wells were in the secondary mathematics teaching course the semester before their student teaching semester and did their practicum experiences with Dr. Gold. Placing the pair with Dr. Gold the semester before their student teaching semester enabled the pair to become familiar with Dr. Gold and her students the semester before, so that they could quickly ramp up into teaching during student teaching. Based on the PDSA Cycle and the focus group session, the four worked well together.

The major themes that emerged from Team A were collaboration focused on learner needs and support for the designated lead teacher. The PDSA Cycle that was given around the third week of the student teaching semester revealed that the team worked well together. Miss Bold stated the following about how the members of the team were working together:

> Some things that are going well include being able to discuss ideas with Dr. Gold and Miss Wells, having more than one set of eyes monitoring the classroom at all times, and splitting up the daily duties. No one ever feels too overwhelmed because there is always someone else backing you up and helping out. If one of us forgets something, odds are that someone else remembered it and already took care of it. If a student gets too rowdy and needs to leave the classroom, there is still two other teachers in the room with everyone else.

Dr. Gold echoed the teamwork that took place among her and the teacher candidates:

> There is an incredible amount of teamwork. When I have to discuss lesson plans or a classroom issue with one intern, the other is available to help students with tutoring, etc. We all contribute during lessons led by one intern by helping with seat work or effective questioning during group investigations. We spend time most days allowing each of the three of us to give feedback on the day's events. One intern and I have acted as students to allow the other intern to practice a part of a lesson when unsure of how it may go. We also have the ability to pull kids into the hall and work one on one as needed as the other remains in charge of the class.

Miss Wells also agreed with Miss Bold's and Dr. Gold's assessment of how the three were interacting together. Dr. Merit also felt that they were working well together and were focused on meeting the needs of their students. The group's comments at the end of the semester were very positive and reiterated that each of them valued having three teachers in the room who really wanted the students to succeed. Dr. Gold made the following statements about the paired placement model:

> There is no comparison, and I doubt that I would accept a single intern again after participating in this model.
>
> There is more time to collaborate about and perfect lessons before they are taught because there is always another teacher to help out as needed.
>
> Students benefit from the extra help that is available when in a traditional internship, there is sometimes a decrease of one-to-one-interactions with the teacher because the teacher and intern need time to collaborate about lessons, etc. This takes away from the availability of the teacher.
>
> Cooperating teachers are supposed to leave the interns alone for periods of time and thus cannot continuously give feedback when not observing but a second intern in the room can provide feedback in these cases.
>
> Another set of eyes and another opinion or constructive criticism is helpful.
>
> They learn from observing the strengths and weaknesses of each other.

Team B. The mentor teacher on Team B has been teaching for 13 years. She has a bachelor's degree in elementary education and master's degree in library media. She became a certified secondary mathematics teacher through an alternative route. She also has attended several professional learning workshops held by faculty members at Auburn University and has served as a presenter at mathematics education workshops and conferences. Mrs. Jewel has hosted several traditional teacher candidates and one pair prior to this semester. The teacher candidate pair placed with her were secondary mathematics education majors. Mr. Winn can be described as caring, easygoing, and average academically. The other member of the teacher candidate pair, Miss Mack can be described as intense, driven, caring, and above average academically. The university supervisor, Dr. Weston, is a retired mathematics teacher educator who is well versed in inquiry-based instruction and current reforms in mathematics education. Dr. Weston has served as a supervisor for Auburn University for several years. Mr. Winn and Miss Mack were in the secondary mathematics teaching course the semester before their student teaching semester and did their practicum experiences with Mrs. Jewel. Placing the pair with Mrs. Jewel the semester before their student teaching semester, enabled the pair to become familiar with Mrs. Jewel and her students the semester before, so that they could quickly ramp up into teaching during student teaching. Based on the PDSA Cycle and the focus group session the four worked well together.

The major themes that emerged from the PDSA Cycle for Team B were collaborating around lesson planning and ensuring that all of their students felt challenged and supported. Mrs. Jewel stated the following about how they worked together early on in the semester:

> The three of us are interacting very well. We are able to spend time co-planning, co-teaching, and evaluating our lessons. While one educator is leading the lesson, the other two are helping students. We seem to have common goals that lead us to effective planning, teaching, and assessing our students.

Mrs. Jewel also stated the following about their team:

> A strength is in lesson planning. Our schedule includes two inclusion classrooms. The teacher candidates and I believe the inclusion classes should be given the same opportunities as the non-inclusion classes. We maintain high expectations for all students' learning and performance.

Both Mr. Winn and Miss Mack agreed with Mrs. Jewel's assessment of their teamwork. For example, Miss Mack stated the following:

> One of the biggest benefits is for the students, they have three teachers in the classroom which is very helpful. Another benefit is the co-planning that takes place each day and the fact that there are more opinions when we debrief.

In agreement, Mr. Winn made the following statement:

> The benefits are that students always have a teacher to work with. Also, it is super helpful to have two other teachers to collaborate with every day. The benefits of this is crazy. Mrs. Jewel and Miss Mack impact each lesson that I teach as they give me input for every lesson. This helps me improve as a teacher while improving student learning too.

Dr. Weston also agreed that the team worked well together. During the focus group, the team talked about how the paired placement provided the opportunity for the mentor teacher to grow in her pedagogical practices as well as the teacher candidates. Each member of the team felt supported and encouraged.

Earlier in the semester the teams worried about becoming dependent on each other and then in the future being in the classroom without a support system, but these worries had subsided by the end of the semester. Another weakness that was highlighted and will be resolved in the next version of the syllabus is that the observation protocols for the teacher candidates that were not the lead teacher were not being used well by either team. In the future we plan to put the protocols in the appendix of the syllabus and provide time frames for implementing them.

Lessons Learned: Questions and Focus Group Protocols to Gauge the Veracity of the Paired Placement Model

These protocols intended to support the paired placement model seem to be serving their purposes. Sending out the questions to the teams early on helped AU to decide if they needed to do any interventions to help the pair of teacher candidates and the mentor teacher to work together in a more cohesive manner. Moreover, the Paired Placement sub-RAC's protocol help us learn if the teachers are focusing on student growth together. The focus groups have continued to help the subRAC determine how well the paired placement model was implemented and if it met the needs of all of the stakeholders (mentor teacher, teacher candidates, and students).

Columbus State University: Observational Tasks

Columbus State University (CSU) is housed in an urban area servicing a population of approximately 200,000 in Columbus, GA. CSU's undergraduate mathematics education certification route is modeled after the UTeach program in Austin, Texas, with slight modifications to include a course focused on pedagogical content knowledge of mathematics and science students. CSU also offers a traditional undergraduate degree certifying students in Grades 4-8 in two different content areas. Teacher candidates from both of these programs have opportunities to take place in the paired placement model. Since teacher candidates in clinical experiences are often required to observe others demonstrate teaching when in apprentice and paired placement models, observational tasks are strongly recommended. Because of different contextual situations including state requirements on the number of days of lead teaching, teacher candidates who are not in lead teaching roles need direction on initializing and focusing on important pedagogical processes. Thus, one of the teacher candidates observes and reflects using the instrument "Observation Tasks for the Teacher Candidates When they are Not Teaching" prompts (see Appendix C) while the other takes a lead role teaching.

The prompts from the instrument use short questions requiring teacher candidates to observe the class through specific lenses of: equity, learning, assessment, management, and tools or technology. The other teacher candidate is asked to observe the lead teacher during the lesson and take notes related to the closed-ended prompt questions. Before and/or after the lesson, the teacher candidate and lead teacher are asked to discuss these questions and the associated lens.

Observational Tasks PSDA Overview

During the 2018–19 academic year, Columbus State University implemented a PDSA Cycle as a part of the Paired Placement sub-RAC research protocol. CSU re-evaluated its implementation and the instruments' effectiveness to prompt teacher candidate reflection and understanding of these lenses during clinical experiences. In the summer of 2018, the questions were shared among the team and revised to meet current needs in each program. The PDSA form (see Appendix D) was completed in the fall of 2018 and shared among the members in order to facilitate discourse on the forms' intended use, which included expectations for student reflections. The PDSA form helped the Paired Placement sub-RAC track, predict, and hypothesize how the instrument would increase learning in Fall 2018 and Spring 2019 clinical experiences. In the spring of 2019, student reflections were analyzed using the framework and predictions of the PDSA form. After analyzing student responses, the PDSA form was completed in anticipation for what would be modified in the summer of 2019 and enacted the following academic year 2019–20.

Findings from PDSA Cycle

Teacher candidates did not generally reflect with a lens of equity about instructional task creation and implementation as a form of differentiation or other tool to ensure equity. Most teacher candidates noticed class participation and the ability or lack of the teacher to engage disengaged students as places of equity and inequity.

One teacher candidate stated that her mentor teacher "feels [some students] are a 'lost cause' [and] were left alone." While another teacher candidate reflected that "All students are engaged," and "monitors and gives individual assistance as needed, [she] circulates [and] does not sit down." As teacher candidates reflected with a lens toward learning, they failed to relate this back to student evidence; however, teacher candidates noticed whole group instruction and a focus around procedures repeatedly during reflections. Problem solving, conceptual understanding, and sense-making discussions were not present in teacher candidates' reflections, which likely was due to the lack of these interactions in the classroom setting. Teacher candidates often noted the lack of involvement of some part of the class, and many described the effort or lack of effort of the teacher to encourage learning even in questions that were not addressed toward equity. When teacher candidates used a lens of assessment, reflections disclosed teacher's and teacher candidates' discourse as it relates to summative assessment. Reflections provided insight to teaching moves made by the lead teacher that do not go beyond what could be seen by observation. However, the prediction was correct that teacher candidate focus was geared toward summative assessment rather than formative. Little evidence was found in teacher candidates' reflections that disclosed a deeper level of teacher and teacher candidate discussion of technology. Reflections highlighted a lack of TPCK in lesson planning orchestration. Technology was discussed as a pedagogical tool that could be used in any class. Technology also was discussed with a lens toward assessment and as a tool for presentation.

Lessons Learned: Observation Tasks

The instrument provided an excellent tool to prompt student thought around critical areas of mathematics education instruction. Using the prompts simultaneously with methods courses provided deeper connections to concepts and practical applications from the field by students. Students also found the prompts useful in helping them focus on certain skills for teaching or high leverage practices. The prompts also provided a structure and opportunity to bring up different topics with leader teachers and potentially taboo topics such as equity.

Not all teacher candidate reflections exhibited a discourse between both the observer and the lead teacher. Though the instrument provided a structure to start conversations between the teacher candidate and the lead teacher, it failed to prompt the need for discourse between the two. Clearer instructions need to be developed that distinguish one lens from another and provide clearer instruction to their implementation in clinical experiences. It is also suggested that these prompts are separated to make clearer connections between course objectives during methods courses.

Potential rewording or inclusion of new prompts for each lens may also be useful for future uses of the instrument. For a lens toward assessment, the Paired Placement sub-RAC looks to move the focus of teacher candidates toward teacher moves that were made in selection of problems, transitions from small group to whole group, etc., that are not evident as primary portions of a lesson. During technology prompts, we would like teacher candidates to seek clearer connections and discussions around the use of tools and technology to implement the observed lesson as it relates to mathematics. Equity seemed to cross-cut many of the students' reflections, but when attending to this specifically prompts that require discourse around task creation and implementation will need to be developed. In addition, a new learnings prompt might provide a more explicit way to connect student learning to assessment evidence. Lastly, prompts encouraging students to reflect on the lead teacher's management might also include reflections related to micro-messages, equitable instruction, and/or the growth mindset.

University of Hawai'i at Manoa (UHM): Co-Planning/Co-Teaching

The UHM Institute of Teacher Education secondary mathematics program comprises four courses offered across two semesters, typically during the teacher candidates' final year of their program. During the fall semester, students take one secondary mathematics methods course (ITE 404G) and an accompanying observation practicum course (ITE 402G). In the spring semester, students are enrolled in the student teaching course (ITE 405G) and the accompanying seminar course (ITE 406)[2]. The secondary math program supports both undergraduates and post-baccalaureate students to earn a secondary mathematics teaching license in Hawai'i. During the fall semester, teacher candidates are in their mentor teachers' classrooms a minimum of 200 hours. During the student teaching semester, candidates are in their mentors' classrooms full time and typically take on half of the teaching workload of their mentor teacher starting at the beginning of the semester.

Each member of the UHM pair involved in this project worked with its own primary mentor most of the day, but co-taught two classes (one from each mentor-different grade levels/ subjects). Thus, UHM's paired placement model is a modification on the original paired placement model described in this paper's overview. Both the mentors and the student teachers attended a one-hour orientation session approximately one month before the student teaching semester began. During this session, some of the research related to the common benefits and pitfalls of the paired placement model were shared, and the team brainstormed strategies for preventing and/or overcoming some of the pitfalls mentioned in the research. In addition, the group developed a group Google calendar and strategies for sharing documents via the cloud. Finally, during this session, the co-planning/co-teaching models were introduced to the teacher candidates, as the mentors already were introduced to these models at the beginning of the observation practicum course.

Co-planning/Co-Teaching PSDA Overview

As UHM is in its first year of implementation, we sought to gain insight into candidates' perceptions of the implementation of the paired placement model. In addition, UHM used the PDSA Cycle to learn about teacher candidates' perceptions of the value of co-planning and co-teaching with their peer and mentor. The first goal was particularly valuable, as UHM had to establish a new pair in the middle of the school year, once the original pair was terminated due to student illness (and subsequent withdrawal from the licensure program and the research project). The second goal emerged during the student teaching semester and is influenced by two important discoveries: 1) other institutions (see Columbus University's PDSA report above) have reported that during co-taught lessons, the non-lead teacher candidate often had no focus or structure to their classroom support; and some members of the Co-Planning/Co-Teaching sub-RAC have noticed that pairs who rely solely on the One Teach/One Assists Co-Teaching model tended to score lower on the MCOP[2] (R. Sears, personal communication, June 26, 2019).

Co-planning/Co-Teaching PDSA Cycle. In addition to the common data collection efforts described in the overview of this paper (i.e., interviews, MCOP[2]), UHM initially planned to informally interview and formally observe teacher candidates every three weeks throughout the student teaching semester (final semester of the licensure program) to learn about their experiences with the paired placement model. However, although during the first few weeks each member of the paired reported that the paired placement was going well, the researcher noted that the candidates primarily worked alone to plan and only used the One Teach/One Assist Co-Teaching model. This left the team wondering whether candidates were possibly using other strategies for their non-observed lessons, and curious as to the candidates' reasons for not using other co-planning and co-teaching

[2] It is important to note that the researcher acts as both the instructor and field supervisor for all the secondary math program courses.

models in their lesson planning and lesson implementation. Thus, the PDSA Cycle involving a Co-Planning/Co-Teaching Survey was conceived (see Table 1).

Data sources. During this PDSA Cycle, the researcher (acting as the student teaching seminar instructor) assigned the pair of teacher candidates to co-plan and co-teach two lessons that would be formally observed. Thus, the primary data collection tool was the teacher candidate lesson plans and lesson plan and observation notes of the researcher/instructor. Teacher candidates also were asked to complete a survey (via Google forms) to provide more insight into how they were engaging in the co-planning and co-teaching practices.

Table 1

Co-Planning/Co-Teaching Survey Prompts

Co-Planning Prompts	Co-Teaching Prompts
How have you planned with your peer and/or mentor?	How have you taught with your peer and/or mentor?
Have you used any of the Co-planning strategies? If so, which co-planning strategies have you used? Why those?	Have you used any of the Co-teaching strategies? If so, which co-teaching strategies have you used? Why those?
What are some benefits of co-planning?	What are some benefits of co-teaching?
What are some drawbacks of co-planning?	What are some drawbacks of co-teaching?

Findings from the PDSA Cycle

Both candidates continued to report that the paired placement model had been a positive experience for them. One candidate shared,

> We found that since we are colleagues with a high level of trust and have had experience working together prior to this placement that we can count each other for doing our fair share while we work. We also enjoy passing ideas back and forth in real time to help mold them into what we want faster

Co-planning models used. Both candidates reported that they primarily used the “one plans, one assist” and “one plans, one reflects” models. One candidate shared his reasons for using these two models:

> We tend to use these two strategies more often because we both have more expertise with our respective math content that we've worked with since the beginning of the year, so we each take the lead in planning for those classes. However, we are also trying to move closer towards the partner planning or even the team planning model as we are also moving towards a team-teaching model in our classroom.

They also both mentioned that they frequently share their lessons with their mentors and share resources with their peer and their mentors. Additionally, one of the mentors, who is also the math department chair, mentioned that the teaching practices of the pairs have influenced the entire department. The entire department now uses one of the teaching strategies that the pairs used during their student teaching.

Co-teaching models used. Both candidates reported that they have used the one-teach/one assist and parallel teaching models. One candidate explained her choice to rely on the one-teach/one assistant strategy by saying:

> We have used one teach, one assists the most frequently. It lets one of us act as the main focus of attention while the other can float around and assist students who need a little extra help or have questions. We have also tried parallel teaching. This was useful in creating smaller, more focused groups in which we could conduct an activity with rich discussion and high cognitive demand efficiently.

In addition, during the fourth formal observation which occurred after the survey was disseminated, the university supervisor noted that the pair also used the station teaching model; although, this model was not mentioned by either candidate.

Survey as an intervention. The survey prompts (disseminated during Week 9 of a 16-week semester) and the explicit instructions to co-plan *and* co-teach two lessons may have served as a catalyst and reminder for the candidates to try out the other co-teaching and co-planning strategies. After the survey, candidates endeavored to use more of the co-planning and co-teaching strategies as is evidenced by the addition of a greater variety of co-planning strategies in their lesson plans and co-teaching strategies observed during the formal observations after the survey.

Lessons Learned: Early Professional Development Needed for Mentors and Teacher Candidates

At UHM, one candidate mentioned that he felt that the co-planning/co-teaching models were "sprung" on them, and that they would have liked more time to become familiar with the strategies. Other sub-RAC partners have verified similar discussions when implementing the model at their institutions, which indicates that candidates might need time during the previous semester to try out strategies either with their mentor or with their partner. Given that there is no guarantee that pairs will stay together throughout both semesters (as we experienced this year with our first pair), it might be worth considering ways to ensure that teacher candidates are engaging in co-teaching and co-planning with their mentors during their observation practicum in the previous semester. Prior to the fall semester, mentors were provided the co-planning and co-teaching model handouts from the Co-Planning/Co-Teaching sub-RAC and some tips on using the strategies throughout the semester (via email), and students were assigned to co-plan with their mentors during the fall semester. However, the previous comment demonstrates that this is clearly not enough to ensure that co-planning actually occurs between mentors and teacher candidates. The Paired Placement sub-RAC is now considering how we might provide a longer face-to-face workshop for all secondary math mentors and teacher candidates at the beginning of the observation practicum semester that will allow both groups engage in the strategies together (as was reported by Auburn University). In future methods courses, we will also seek to highlight the co-planning aspects of the assignments during the methods course.

Co-Planning/Co-Teaching Survey (via Google Forms). The survey as a data collection tools appeared to be appropriate for gathering the type and quality of the data that our institution was interested in during the first year of implementation. In response to a few of the co-teaching prompts, one of the teacher candidates responded that he/she had already answered the question before, stating, "see answer above." It might be worth considering whether it is necessary or important to keep the co-planning and co-teaching prompts separate. Also, as mentioned earlier, given that the survey seemed to push the candidates to use more strategies, it might be worth disseminating the survey earlier in the semester, so that candidates are encouraged to try out a greater variety of strategies sooner.

University of Montana—Not All Pairs Succeed

The University of Montana secondary mathematics education program comprises a 120-credit bachelor's degree in mathematics including approximately 33 credits of university general education course work; a 41-42 credit mathematics major, which includes one four credit mathematics methods course; 16 credits over three semesters of field experiences including the one semester student teaching semester; and about 30 additional credits of professional education course work.

Over the most recent 25-year period, on average, just over nine students graduated each year with a Bachelor of Science in mathematics with the mathematics education option. The state prepares 25 to 30 secondary (Grades 5-12) mathematics teachers each year, the majority coming from the two flagship universities in the state. Our professional education program is typically the junior and senior year, and, although, we have 15% of our students at the post-baccalaureate or graduate level each year, the 15-credit student teaching semester culminates the program.

Reflections on Implementing the Paired Placement Model

One common challenge to the traditional student teaching placement scenario is the ability of people to get along; to play fair (Wilkens, Ashton, Maurer, & Smith, 2015); and to establish a needed community of learners (Wenger, 1998) between the student teacher, cooperating teacher, and the classroom of students. This problem is intensified with the paired placement model because three adults instead of two must co-exist in the same classroom. At the University of Montana, student teachers, cooperating teachers, and university supervisors participate in a daylong in-service at the beginning of the experience, learning about what to expect, the desired teaming approach, and the need for open and honest communication. In this report we will share a PDSA Cycle of a failed paired placement. Whereas discussing this breakdown of community as a failure is harsh, we can learn from these relatively few situations.

Using an action research cycle, the PDSA, we investigated the relationship among the paired student teachers and the cooperating teacher. A university supervisor observed the pair co-teaching a lesson at the end of the first week of the semester. The paired student teachers functioned seamlessly, as though they had been team teaching for months, taking turns with lesson components, working independently with small groups, collaboratively with whole group work, and implementing the co-planned lesson with expertise. They shared time driving together to and from school as well. The university supervisor reported no problems as the math teaching practices were evident, the seventh grade students were engaged and demonstrated successful completion of the lesson objectives. No further observations were scheduled for the month, the assumption made that all was working well and would continue to do so.

This assumption was an error, as there were little issues that arose beginning in the second week, unknown to the university supervisor. By the sixth week, upon visiting with the pair—who were no longer even speaking to each other—an agreement was made to work together again, to push forward as each had unique strengths and the surfaced problem of not being at the appointed pick-up place at the designated time in the morning before carpooling to school was in the recent past. This pair, who knew how to work together, had demonstrated they were each proficient in teaching, and even carpooled to school together, but within another two days, decided to permanently split. The cooperating teacher found another teacher in the building to accept one of the paired student teachers and continued to work with the other.

Lessons Learned

At UM, our recommendation, based upon five years of successful paired placements and investigating this one broken pairing, is that one must push forward with working closely with all pairs, even those who seem proficient early on, and listen for minor differences that arise and not abandon these efforts during the first month of paired placements. There are differences that can be overcome, and future teachers become stronger for resolving those. University supervisors and cooperating teachers must listen carefully and frequently in this first month.

Second, there are differences that cannot be overcome, and these are primarily related to an individual's inner strengths, as well as their physical and mental health. Is the individual strong enough to see that although he/she is not as fast at responding to an individual student question, speed alone is not the issue? If a middle school student states she would rather work with the other student teacher, is one able to accept that does not mean one is not able to function in this classroom? Is one able to see that the other student teacher brings strengths to the classroom, but others in the room also bring strengths? This comparison of student teachers' abilities to perform in the role of a teacher is natural, and when one individual is unable to work past that and only desires to be the sole teacher in the room so as to avoid any comparisons, it is certain to fail.

Third, when one student teacher shows signs of paranoia and takes actions to sabotage the others in the room, it is best to separate the team and recommend mental counseling. Healthy individuals are essential for success in student teaching placements.

In conclusion, our recent case of a failure for two paired student teachers and one cooperating teacher to complete the semester allows us to reaffirm what others have found in investigating failure in placements (Bullough & Draper, 2004) and the lack of good characteristics of stakeholders in the student teaching placements, namely, the importance of mental and physical health (Koerner, Rust, & Baumgartner, 2002). The PDSA Cycle informs our practices in preparing successful paired placements—listen daily to your paired student teachers and provide reassurance that minor differences are to be expected and can and must be resolved.

Concluding Thoughts

Given this year's findings, we are renewed in our commitment to implementing the paired placement model at our various institution. The instruments we explored through our PDSA Cycles now will provide us additional tools to improve the implementation of the paired placement model across institutions by helping us examine implementation more critically. One of our sub-RAC's next steps will be to modify our tools to reflect what was learned through the PDSA Cycles reported in this paper. In addition, the members of our sub-RAC are exploring how we might incorporate a few of the same tools and protocols that were studied in this report at the individual institutions across all institutions in the 2019–20 school year. For example, given the findings that mentors and teacher candidates may have needed early experience with using the co-planning and co-teaching models and the observation tasks, it might be that all institutions include a workshop for all parties (similar to the one described by Auburn University) as a part of the paired placement model. Finally, our sub-RAC is now in the process of developing a system for collecting and sharing PDSA data across institutions so that data can be more easily analyzed and compared across institutions. The added benefit of this final step is that it will allow institutions who are considering adopting the paired placement model to have access to lessons learned and implementation tools all in the same place.

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Appendix A

PDSA Cycle Focus on the Veracity of the Paired Placement Model

Teacher Candidates PDSA Cycles

7. How are the three of you interacting?
8. How are the students responding to you?
9. What is going well?
10. How can we better support you?
11. What are your strengths and weaknesses related to implementing equitable teaching practices?
12. At this point what do you see to be the benefits and challenges of the paired placement model?

Mentor Teacher PDSA Cycles

1. How are the three of you interacting?
2. How are the students responding to the teacher candidates?
3. What is going well?
4. How can we better support you?
5. What strengths and weaknesses do the teacher candidates exhibit related to implementing equitable teaching practices?
6. At this point what do you see to be the benefits and challenges of the paired placement model?

University Supervisor PDSA Cycle

1. How are you interacting with the teacher candidates and the mentor teacher?
2. How are the students responding to the teacher candidates?
3. What is going well?
4. How can we better support you?
5. What strengths and weaknesses do the teacher candidates exhibit related to implementing equitable teaching practices?
6. At this point what do you see to be the benefits and challenges of the paired placement model?

Appendix B

Focus Group Questions

Interview Protocol for Graduates of the Program Who Were in Paired Placements

1. What are attributes of the paired-placement model that contribute to successes in your current teaching practice?

2. What are attributes of the paired-placement model that contribute to challenges in your current teaching practice?

3. How did the paired placement internship prepare you to work/collaborate with other teachers to support student learning?

4. In what ways did the paired placement internship enable you to design and implement lessons that build procedural fluency from conceptual understanding?

5. How did the paired placement internship prepare you to take risks and try new teaching approaches in your mathematics classroom?

6. Now that you have been teaching for _________ , what would you say are the components of the paired placement internship that have positively impacted your current teaching practice? What components could be added to the model to make it more effective for future candidates?

7. Tell me about a time in your teaching career so far where you felt your experiences in the paired placement internship impacted your actions?

Interview Protocol for Mentor Teachers Who Have Participated with the Paired Placement

9. What are attributes of the paired-placement model that contribute to successes in your current teaching practice?

10. What are attributes of the paired-placement model that contribute to challenges in your current teaching practice?

11. In what ways, if any, did the paired placement internship impact your working or collaborating with other teachers to support student learning?

12. In what ways did the paired placement internship impact your teaching practices? For example, did the emphasis on the mathematics teaching practices influence your own teaching of mathematics?

13. Did the paired placement internship encourage you to take more risks and try new teaching approaches in your mathematics classroom? If so, will you share an example?

14. What would you say are the components of the paired placement internship that positively impacted the interns' teaching practices? What components could be added to the model to make it more effective for future candidates?

15. Have the interns contacted you about a time in their current teaching career where they felt the experiences in the paired placement internship impacted their actions?

16. Have you served as a mentor teacher in a 1-1 internship previously, if so, which model do you think best serves the interns? Explain.

Interview Protocol for University Supervisor Who Have Participated with the Paired Placement

1. What are attributes of the paired-placement model do you think contribute to successes in interns' teaching practices?

2. What are attributes of the paired-placement model do you think may pose challenges in interns' future teaching practices?

3. What would you say are the components of the paired placement internship that positively impacted the interns' teaching practices? What components could be added to the model to make it more effective for future candidates?

4. Have the interns contacted you about a time in their current teaching career where they felt the experiences in the paired placement internship impacted their actions?

5. Have you served as a university supervisor in a 1-1 internship previously, if so which model do you think best serves the interns? Explain.

Appendix C

Observation Tasks for the Teacher Candidates When They Are Not Teaching or Co-Teaching

Suggestions for use: For each observation, teacher candidates should use only one set of questions below and record their observations in their journals. During the observation phase, teacher candidates should list specific activities and interactions happening in the classroom. After the observation, teacher candidates should debrief with their mentor teacher and the person whom they observed. During debriefing teacher candidates should reflect on and discuss their recorded observations.

1. Examine the Lesson through an Equity Lens
   - Are all students engaged in the lesson?
   - Is the teacher using multiple ways of approaching the topic in order to insure student understanding?
   - Is the teacher providing special accommodations for students who need them?
   - Is the teacher ensuring that all students are attaining the mathematical goal for the lesson?
   - Are all students being challenged to reason and make sense of the lesson?
2. Examine the Lesson through a Learning Lens
   - Are the tasks worthwhile for the students?
   - Is the teacher asking questions that help promote student engagement in the task?
   - Is the teacher maintaining a high level of cognitive demand?
   - Are the students making the connections that they need to make?
   - Does the teacher listen to student thinking and respond appropriately, such adjusting instruction or asking appropriate questions?
3. Examine the Lesson through an Assessment Lens
   - What evidence of students’ understanding of the concepts or skills did you see?
   - Is the teacher using a variety of ways to assess students’ understanding?
   - Did the teacher elicit and use evidence of student thinking?
   - Did the teacher use student feedback to modify instruction?
4. Examine the Lesson through a Tools and Technology Lens
   - What tools and technology are being utilized to help students understand the concepts?
   - What materials are being used in power points or on board legible and organized appropriately?
5. Examine the Lesson through a Management Lens
   - Does the teacher have clear routines for the students to follow?
   - Does the teacher facilitate cooperative learning groups well?
   - Does the teacher call on students systematically or does the class answer in concert?
   - Do the students know the consequences of their actions both good and bad?

Appendix D

PDSA FORM WITH PROMPTS

Test Title:	Observation Tasks for the Teacher Candidates When They Are Not Teaching or Co-Teaching			**Date:**	During Clinical Observations
Tester:	Basil Conway IV	**Cycle#:**		**Driver:**	
What change idea is being tested?	Observation Task Implementation				
What is the goal of the test?*	To learn candidates perception of the value of the observation tasks and any needed changes to the tasks towards their becoming an effective teacher of secondary mathematics.				

*Identify your overall goal: To make something work better? Learn now a new innovation works? Learn how to text in a new context? Learn how to spread or implement?

1) PLAN				**3) STUDY**
Questions: Questions you have about what will happen. What do you want to learn?	**Predictions:** Make a prediction for each question. Not optional.	**Data:** Data you'll collect to test predictions		**What were the results?** Comment on your predictions in the rows below.
1. How did the instrument help students highlight equity in the classroom? How did the instrument drive discussion in the classroom with the mentor?	Students will notice inequitable situations where students may be held to different expectations. Students will justify teacher actions based on student behavior.	1. Observation Reflections 2. Discussion around these topics during methods class or other connection points when implemented.	→	
2. How did the instrument help students highlight learning in the classroom? How did the instrument drive discussion in the classroom with the mentor?	Students will find teaching and learning focused towards procedures and performing well on standardized testing. Cognitive demand will be lowered through teacher and student interactions.	1. Observation Reflections 2. Discussion around these topics during methods class or other connection points when implemented.	→	
3. How did the instrument help students highlight assessment in the classroom? How did the instrument drive discussion in the classroom with the mentor?	Student will focus reflections more on summative assessment rather than formative assessment. Evidence will suggest unclear connections of modification of instruction to formative assessment.	1. Observation Reflections 2. Discussion around these topics during methods class or other connection points when implemented.	→	

PDSA FORM WITH PROMPTS

4. How did the instrument help students highlight tools and technology in the classroom? How did the instrument drive discussion in the classroom with the mentor?	Tools and technology will be discussed by students in terms of technological pedagogical knowledge with little to no emphasis on mathematical technological pedagogical knowledge.	1. Observation Reflections 2. Discussion around these topics during methods class or other connection points when implemented.	→	
5. How did the instrument help students highlight management in the classroom? How did the instrument drive discussion in the classroom with the mentor?	Students will focus on controlling misbehavior rather than encouraging behavior that is conducive to learning.	1. Observation Reflections 2. Discussion around these topics during methods class or other connection points when implemented.	→	
Details: Describe the who/what/when/where of the test. Include your data collection plan.				**What did you learn?**
The 5 different questions each relate to a different lens from the instrument. The student journal or reflections may be collected as they are assigned or at the end of the semester. It is encouraged to debrief each of these lens as they are assigned or implemented in the field.				What changes we should make to the survey administration. TBD

2) DO (Briefly describe what happened during the test, surprises, difficulty getting data, obstacles, successes, etc.)	**4) ACT** (Describe modifications and/or decisions for the next cycle; what will you do next?)
TBD	Based on the answers to the two questions about the survey, we may revise the questions or the frequency of administering the protocol.

Using Assessment Frameworks to Inform the Design of Classroom Assessment[1]

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Abstract

Goals for mathematics education are often communicated through a variety of policy documents and resources designed to illustrate how students and teachers might engage in mathematical activity. For example, the *Standards for Mathematical Practices* articulated in the *Common Core State Standards for Mathematics* (*CCSS-M;* National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010), provide a synthesis of reasoning goals, processes, and habits of mind that suggest how students should experience mathematics. To address these goals, teachers need to be mindful of how characteristics of mathematical tasks can elicit various aspects of student reasoning. The analysis of goals, tasks, and potential student reasoning is at the heart of classroom assessment design, which is described in the Mathematics Teacher Education Partnership's (MTE-Partnership) *Guiding Principles for Secondary Mathematic Teacher Preparation* (2014) under Guiding Principle 5: Candidates' Knowledge and Use of Educational Practices. Classroom assessment is more typically designed around familiar tasks that appear in instructional materials rather than designed to promote specific types of reasoning. To support productive formative and summative assessment design and use, a framework for what it means to assess student understanding is needed. The framework described and exemplified in this paper has been used by secondary mathematics teachers to analyze mathematical activities, interpret student work, and rethink approaches to scoring and grading summative assessments. This framework has also been used to analyze the reasoning goals addressed in exams used in the undergraduate calculus sequence.

Rationale

Education researchers repeatedly have asserted that to improve student learning, teachers need to give greater attention to their use of classroom assessment (Black & Wiliam, 1998; Wiliam, 2007). To effectively guide student learning in a manner that reflects more formative purposes and goals assessment, instructors must develop greater confidence in their own decision making and understandings of the role of classroom assessment. With respect to both secondary and tertiary mathematics education settings, the real benefit of understanding classroom assessment is not in the scoring and grading of quizzes, midterms, and final exams; rather, the value of developing professional knowledge about classroom assessment is that it informs: (a) the types of questions that are posed as part of instruction, (b) the ways in which student responses might be interpreted, and (c) the subsequent instructional moves based on student responses. The proposed assessment framework in this paper supports the use of tasks that are more open to students' ways of thinking, strategic competence, and student insight.

Guiding Principle 5 (MTE-Partnership, 2014) focuses on candidates' knowledge and use of educational practices, which includes the "knowledge, skills and dispositions needed to implement educational practices" (p. 4). Specific aspects of assessment are articulated in Guideline 5-C:

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> Assessment and Reflection: The teacher preparation program ensures that teacher candidates can assess the ongoing learning of their students using both formative and summative assessments, and appropriately and ethically use data from those assessments to promote the success of all students as well as to reflect on their own professional growth. (p. 4)

The purpose of this paper, therefore, is to argue for more substantive experiences with classroom assessment in teacher education and outline a practical framework for assessing student understanding that can be applied in these various professional preparation contexts. This framework has been applied in teacher education and professional development for over 20 years in primary, secondary, and tertiary mathematics education, confirming that this framework is, indeed, practical and can be used to inform emerging conceptions of what it means to assess understanding. However, while this framework has been used in these mathematics education contexts (e.g., Shafer & Foster, 1997; Webb, 2009; Webb, Stade, & Grover, 2014), knowledge of this framework among mathematics educators is limited. Even though more recent attention has been given to distinctions between summative and formative assessment, as well as the role of feedback (e.g., Chappuis, Stiggins, Chappuis, & Arter, 2012; Wiliam & Leahy, 2015), teachers lack opportunities to critically examine how tasks, questions, and activities promote students' mathematical reasoning. To what extent do my classroom assessments represent expectations for mathematical reasoning and important mathematical goals? To what degree is the mathematical reasoning elicited in instructional materials, classroom activities, and graded summative assessments aligned? The application of this framework can expand how future mathematics instructors interpret curriculum, enact instructionally embedded assessment, and use students' ideas to inform instruction.

A Framework for Classroom Assessment in Mathematics

To restate the relevant MTE-Partnership guideline in the form of a problem statement: *What knowledge, skills, and dispositions are necessary for productive and equitable use of classroom assessment*? An important starting point is to re-examining one's dispositions toward assessment, through a critical analysis of prior experiences with classroom assessment, as a mathematics student and teacher, and the conceptions of classroom assessment that were reinforced by such experiences. Classroom assessment is often thought of as testing, and the design and use of summative assessment. This is often the more visible and emotionally palpable experience, so it is more memorable. For this reason, programs designed to support mathematics educators' classroom assessment practices must necessarily challenge conceptions of assessment and how these conceptions actually influence their understanding of the discipline they teach. Assessment practices often reify what counts and what is valued. This powerful interaction between goals, as articulated by assessment tasks, and instructional goals was described by Schoenfeld (2007) citing Burkhardt as, " 'What You Test Is What You Get' (WYTIWYG)" (p. 72). As Schoenfeld went on to describe:

> The WYTIWYG principle operates both at the curriculum level and at the individual student level ... and if the test focuses on skills, other aspects of mathematical proficiency tend to be given short shrift. … Similarly, students take tests as models of what they are to know. Thus, assessment shapes what students attend to, and what they learn (p. 72).

Even though Schoenfeld was referring to standardized tests in this excerpt—and the need for greater innovation in format and design—this principle is still valid with respect to the design of classroom tests and other summative assessments, the more regular and common experience for mathematics teachers and students.

Beginning with Dispositions

Given the influence of assessment on practice, dispositions toward mathematics assessment deserve far greater attention in mathematics teacher education than simply acknowledging that somewhat negative experiences or limited dispositions about classroom assessment exist. As noted by Webb (2009):

> Facilitating change in teachers' assessment practice is not so much a resource problem as it is a problem of: (a) creating opportunities for teachers to reconceptualize their instructional goals, (b) re-evaluating the extent to which teachers' assessment practices support those goals, and (c) helping teachers develop a "designers' eye" for selecting, adapting and designing tasks to assess student understanding. (p. 3)

As a starting point, having pre-service (and in-service) mathematics teachers articulate their conceptions of what it means to understand mathematics, and how such understandings might be represented, is an important activity prior to introducing a new framework for assessment. Describing the various ways we are confident in what we know, and the extent to which this knowledge represents some degree of understanding, can help an instructor assess the degree to which various forms of reasoning are recognized. By situating the examination of a new framework with respect to one's prior experiences and conceptions, an instructor will have a better sense of the conceptual terrain and instructional experiences that should be included when discussing less familiar conceptions of classroom assessment.

Another way of thinking about dispositions related to classroom assessment is that we each develop a tacit framework for assessment by drawing upon our experiences as learners and teachers of mathematics. If most of that experience has been devoid of opportunities to engage in reasoning beyond recall to demonstrate success in mathematics, what it means to "get an A" on a test, then why should we expect any divergence from this self-developed, repeatedly reinforced framework? To invite the use of a different assessment framework, it is necessary to critique and reflect upon past practices and experiences to understand the need for a different approach that better exemplifies what it means to do mathematics.

The Dutch Assessment Pyramid

One assessment framework that was designed to support teachers' conceptualization of what it means to assess student understanding was based on an assessment pyramid, first developed in 1995 by researchers at the Freudenthal Institute to support the design of the Dutch National Option for the Third International Mathematics and Science Study (Dekker, 2007) and later adapted by Shafer and Foster (1997) for use in research studies in secondary mathematics classrooms in the United States (Webb & Peck, 2019; see Figure 1). The categories for the assessment pyramid also were used to define the mathematical competencies clusters in the initial version of the mathematics assessment framework for the Programme for International Student Assessment (OECD, 2003, pp. 41–49). The Dutch Assessment Pyramid illustrates the relative distribution for the different levels of thinking that are named: reproduction, connections, and analysis (described in more detail as follows)—the levels of thinking dimension. The other two dimensions are domains of mathematics, which is the range of mathematics content addressed, and the questions posed dimension, which can be thought of as level of difficulty. The dimension of domains of mathematics suggests that this framework for assessment is a domain general approach that does not integrate the notion of a learning trajectory to progressively develop students' understanding of specific domains of mathematics. That would call for a different, perhaps complementary, domain specific framework for how student reasoning in measurement, proportionality, solving equations, etc., develops over time (cf. Webb, 2017).

Before elaborating further about what is meant by connections or analysis, notice how levels of thinking and difficulty are not the same dimension. That is, a task that requires students to make connections between a problem context and possible solution strategies is not necessarily a more difficult task than a recall question. In fact, questions that involve the recall of multiple procedures (e.g., using trigonometry identities to solve a definite integral) can be more difficult than a problem context involving minimization of area where students decide between using a visual and/or symbolic representation to model the situation. It is also important to note that even though recall tasks still form the majority of assessment questions in this model, recall tasks cannot be the only type of assessment task used with students if the goal is to assess student understanding. Teachers need to move beyond a battery of questions designed to assess student recall and include tasks that allow students to show that they can relate mathematical representations, communicate their reasoning, solve problems, generalize, and mathematize context problems.

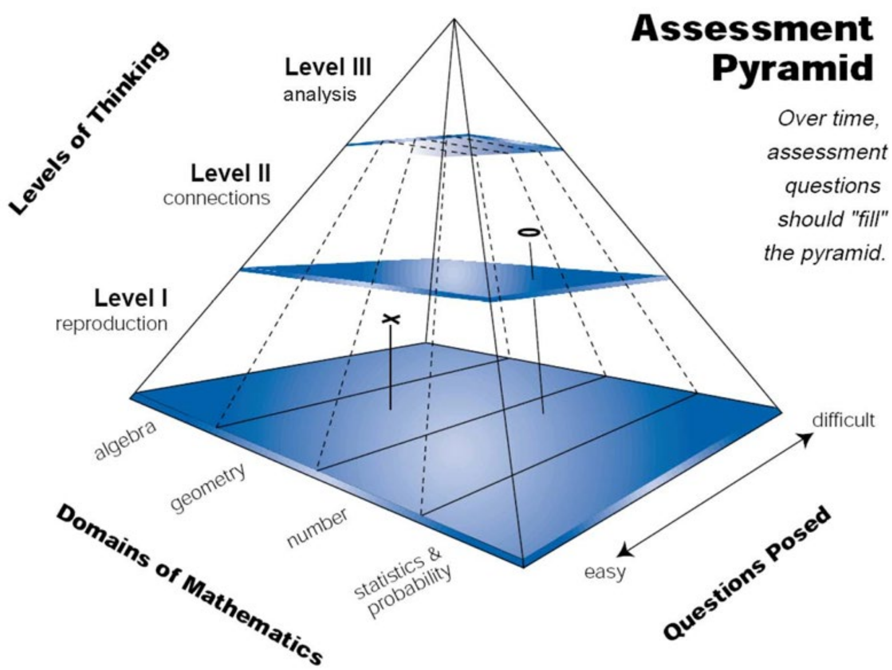


Figure 1. Dutch Assessment Pyramid (from Shafer & Foster, 1997, p. 3).

Using an Assessment Framework to Develop Knowledge and Skills

Assessment frameworks such as this are designed, in part, to serve as a lens for both approximating the potential reasoning elicited from tasks and interpreting students' responses to the tasks. Students reveal their understanding of mathematics when there are opportunities to do so in the tasks and activities that are selected that allow them to demonstrate their understandings. Even though this seems like an obvious statement, consider the extent to which there are opportunities in mathematics classes for students to demonstrate a range of reasoning in the questions asked during instruction or the tasks that are included on quizzes and tests.

To develop a *task-reasoning* lens and a greater sense of the similarities and differences among these three different types of reasoning, after an initial overview of each category and an explicit discussion of how higher level reasoning categories do not necessarily imply the task is more difficult (a different dimension in the pyramid), it is useful to allocate an extended period of time to review and discuss various exemplar tasks that represent each of the categories. In addition to categorizing each task's level of reasoning, which can often be stated as Level I, Level II or Level III, students should be prepared to justify the reasoning level they select. The process of reviewing mathematics tasks to identify the type of student reasoning that is likely elicited is an ideal activity for pairs or small groups so that different perspectives and opinions can be shared and deliberated. Since the Programme for International Student Achievement (PISA) mathematics framework (Organisation for Economic Co-operation and Development [OECD], 2003) includes many examples of tasks aligned with these three reasoning goals for secondary mathematics, after each description of the reasoning level, released items from the 2003 PISA are discussed to exemplify features of each category.

Reproduction (Level I). The thinking at this level essentially involves reproduction, or recall, of practiced knowledge and skills. This level deals with knowing facts, recalling mathematical objects and properties, performing routine procedures, applying standard algorithms, and operating with statements and expressions containing symbols and formulas in standard form. These tasks are familiar to teachers, as they are found in the most commonly used tasks on standardized assessments and instructional materials. These types of tasks also tend to be the types of tasks teachers have an easier time creating on their own, which is one of the reasons they are so prevalent on summative assessments in mathematics.

Mathematics Example 5

Solve the equation $7x-3 = 13x + 15$

Mathematics Example 6

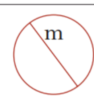
What is the average of 7, 12, 8, 14, 15, 9?

Mathematics Example 7

Write 69% as a fraction.

Mathematics Example 8

Line m is called the circle's: __________



Mathematics Example 9

1 000 zed is put in a savings account at a bank, with an interest rate of 4%.
How many zed will there be in the account after one year?

Figure 2. Examples of reproduction items from PISA (OECD, 2003, p. 43).

Figure 2 includes several examples of tasks that require recall of mathematical terms and skills. Even though all of these tasks exemplify Level I reasoning in the assessment pyramid, the level of difficulty is not the same. Example 8 involves recall of the definition for diameter (easier), while solving the equation in Example 5 requires recall of procedures, computation with integers, and multiple steps to find a solution. Consider how any of these items could be made easier or more difficult, by adding more numbers, more complex values, or additional steps to the prompt. Regardless of the shift in difficulty, the task will still be assessing the same type of reasoning.

Connections (Level II). The reasoning for this category can involve connections within and between different domains in mathematics. Tasks that elicit this type of reasoning motivate students to use different representations and strategies according to situation and purpose, without being asked to use a particular approach. Tasks that assess connections often involve a realistic context to assess connections made between the context and relevant mathematics. Tasks that reflect characteristics of this category often require students to make decisions about, and choose from, various strategies and mathematical tools to solve problems (e.g., strategic competence). Therefore, these tasks tend to be more open to a range of representations and solution strategies.

In the seal problem shown in Figure 3, even though this is a selected response item, the type of reasoning this task elicits involves making connections between a problem context and mathematics, perhaps even visualizing or drawing a diagram modeling the situation described in the task. The mathematics involved in this task is not necessarily difficult, but it is a different type of reasoning that involves identifying and extending a cyclical pattern; essentially, this task requires the choice of a solution strategy that exemplifies Level II reasoning.

Mathematics Example 15: SEAL

A seal has to breathe even if it is asleep. Martin observed a seal for one hour. At the start of his observation the seal dived to the bottom of the sea and started to sleep. In 8 minutes it slowly floated to the surface and took a breath.

In 3 minutes it was back at the bottom of the sea again and the whole process started over in a very regular way.

After one hour the seal was:

A. at the bottom

B. on its way up

C. breathing

D. on its way down

Figure 3. Example of connections item from PISA (OECD, 2003, p. 51).

Another example of a task that exemplifies the connections level for the assessment pyramid is shown in Figure 4. The rock concert task requires mathematization of a problem context and the identification of key information that will support a valid estimation of the number of people attending the concert. The use of another selected response task here as an example is intentional. The format of a task does not necessarily determine the type of reasoning that is elicited. Indeed, if student work is limited to selecting one of the choices, A through E, this does not provide sufficient evidence to the teacher of the reasoning the student used.

For a rock concert a rectangular field of size 100 m by 50 m was reserved for the audience. The concert was completely sold out and the field was full with all the fans standing.

Which one of the following is likely to be the best estimate of the total number of people attending the concert?

A. 2 000

B. 5 000

C. 20 000

D. 50 000

E. 100 000

Figure 4. Example of connections item from PISA (OECD, 2003, p. 90).

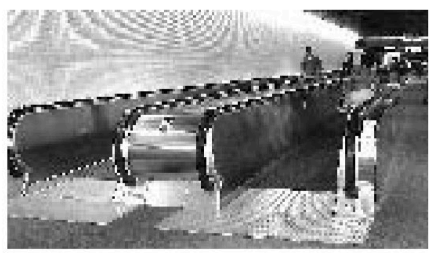
One could imagine adapting the rock concert task to make it a constructed response item without any choices shown. How might that change the difficulty of the task if a set of estimates is not provided? Another option would be to leave the choices for the estimated attendance, but add an additional prompt to explain or justify how the estimate was selected.

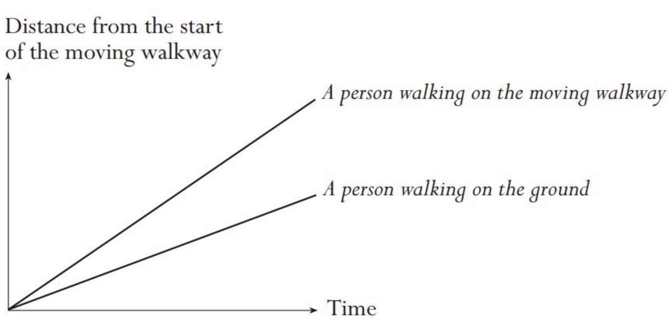
Analysis (Level III). For the analysis category, students are asked to mathematize unfamiliar situations; that is, to recognize and extract the mathematics embedded in the situation and use mathematics to solve the problem. Such tasks require analysis and/or the development of models and strategies. Mathematical arguments, generalization and proof reflect a deeper understanding of mathematical relationships and require some degree of synthesis and analysis to demonstrate this type of thinking. In this category, mathematics itself can be used as a context to promote further generalization of properties, relationships, patterns, and principles. Problems in the analysis category reveal students' abilities to plan solution strategies and implement them in less familiar problem settings that may contain more elements than those in the connections cluster. Having students create tasks can also involve this type of reasoning, as task design involves consideration of important aspects of a mathematics topic and the wording and representation of the task requires some degree of analysis about reasonable parameters, depending on the intended audience.

The moving walkway task (see Figure 5) involves the context of "people movers" often found at an airport. A simple adaptation of this task that might be more familiar to some students could involve an escalator with a stairway next to it. This task requires not only making sense of a mathematical representation of the situation, but reflection about the relative slopes of two linear functions and how a third situation compares to the two that are shown in the graph. This task also motivates students to consider various ways of making sense of graphs that illustrate distance versus time. It is an unfamiliar mathematical task presented for a familiar situation. To add a reasonable line to the graph for the third situation, students need to reflect on the situation and the ways in which the graph illustrates what is happening.

On the right is a photograph of moving walkways.

The following Distance-Time graph shows a comparison between "walking on the moving walkway" and "walking on the ground next to the moving walkway."





Assuming that, in the above graph, the walking pace is about the same for both persons, add a line to the graph that would represent the distance versus time for a person who is standing still on the moving walkway.

Figure 5. Example of a reflection item from PISA (OECD, 2003, p. 91).

Figure 6 is an adaptation of a task that is included as an active learning activity in the first semester calculus course at the University of Colorado Boulder. It is included here to exemplify another aspect of Level III reasoning in the domain of functions and graphing. The first two items are from a sheet of 16 similar tasks in which students use the information provided to sketch a reasonable graph that fits the list of properties. Students need to make sense of the information and how it, collectively, can be translated into a reasonable graph. This aspect of the task exemplifies Level II reasoning.

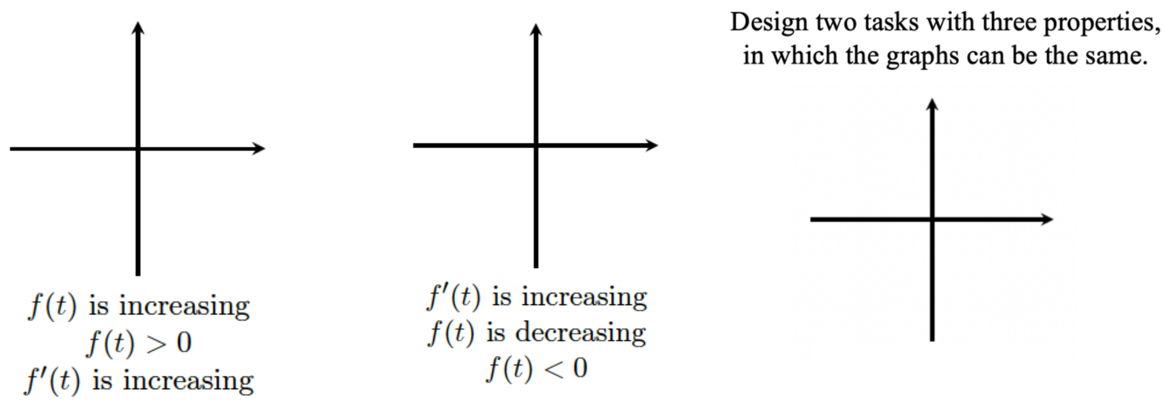


Figure 6. Sketching Snippets (BOALA, 2018).

The third part to the task asks students to design their own problems with a specific constraint: the two problems they design need to result in the same graph. The type of reasoning involved to create a task (or tasks) to fit specific parameters is not necessarily difficult, but it requires knowledge of how information about functions and derivatives influences a graph consistent with the information provided.

Future mathematics teachers should first grasp the meaning of the terms reproduction, connections, and analysis before they reference reasoning categories as Level 1, Level 2, and Level 3. The visual representation of a pyramid suggests that the distribution of tasks across different types of reasoning is not equal. As suggested by Dekker (2007), the ratio among Level I : Level II : Level III type tasks should be approximately 3 : 2 : 1. That is, at least half the tasks should elicit recall of knowledge and procedures, one third should elicit Level II type reasoning (connections), and one sixth should reflect Level III reasoning (analysis). As noted in Figure 1, “over time, assessment questions should fill the pyramid” (Shafer & Foster, 1997, p. 3). This does not mean that each assessment opportunity (e.g., quiz, test, assignment) should reflect this ratio; rather, the distribution of reasoning to be experienced occur over a course of study. It also is likely that some topics in mathematics are more conducive to a higher proportion of tasks that elicit reasoning beyond recall, but the goal should be to represent a range of reasoning for each topic. In a multi-year professional development project that focused on teacher application of the assessment pyramid in the design and use of assessments (Webb, 2011), we found that teachers did move to greater use of tasks that elicited reasoning beyond recall. In this study by Webb (2011), even though the adoption of the 3 : 2 : 1 ratio was not achieved, a significant increase in the percent of problems eliciting reasoning beyond recall was observed:

> Teacher use of tasks that elicit higher order thinking increased from a median of 11.8% in their baseline portfolios to 14.2% in Year 1 and 19.5% after Year 2. Overall, this represents a 65% increase from Y0 to Y2 in the median percentage of higher level tasks used by teachers in response to PD. (p. 8)

Applying the Framework to Support Planning for Formative Assessment

Reflecting on prior experiences with classroom assessment and engaging in critical analysis of status quo assessment practices is necessary to promote student-centered dispositions toward assessment. Knowledge and skills with respect to reasoning goals and how they can be addressed in classroom assessment can be developed through the discussion and analysis of mathematical tasks and activities, and how they reflect the different types of reasoning and difficulty illustrated in the assessment pyramid. The focus on tasks can appear to be an overt focus on resources and processes related to summative assessment. However, these emergent knowledge, skills, and dispositions should be extended to the questions and activities that included as part of instruction, in particular to inform formative assessment practices.

Formative assessment is an integral part of classroom assessment. While the purpose of summative assessment is for reporting and grading artifacts of what has been learned, the purpose of formative assessment is to gather information (through students' responses) to inform instruction. At the heart of formative assessment processes is the provision of feedback so students can gauge how their current understandings compare to instructional goals. Information (i.e., potential feedback) can be provided by instructional materials, classmates, math tutoring centers, instructors, online videos, etc. The information becomes feedback when it is actionable—i.e., when students use the information to make progress toward the instructional goals (Wiliam & Leahy, 2015; Hattie et al., 2016).

A framework for assessment can be used to expand opportunities for formative assessment when the instructor recognizes the potential reasonings that might be elicited from specific questions and tasks. The responses to Level I (recall) tasks are often limited to incorrect or correct answers, unless the task involves recall of a sequence of procedures that might demonstrate some partial recall of skills as in the "solve an equation" and "find the average" tasks shown in Figure 2. The actionable feedback that can be provided to such responses is often limited to error correction, reminders of valid strategies, revisiting the steps of an algorithm, etc. In contrast, Level II and III tasks typically involve a range of solution strategies—they are more open tasks. Students' responses to these types of problems can reveal relationships between different valid approaches, which promotes mathematical connections and relational reasoning. Such tasks also can invite justifications and mathematical argumentation, which can be probed and honed with additional questions, qualifications, counter-examples, and related representations. The type of feedback that is provided to this type of student reasoning can invite subsequent reflection and analysis. For Level II questions, the potential connections that are elicited can lead to feedback that develops those connections. That is, the feedback generated from these types of tasks also can lead to additional enhancement of the same type of reasoning.

Professional development that has focused on the application of the Dutch Assessment Pyramid (Webb et al., 2004; Webb, 2009) begins with the examination of summative assessments and the extent they collectively address the reasoning goals outlined in the framework. The critique and improvement of summative assessments then leads to re-articulating content goals *as well as the reasoning goals* for a unit or course so that assessments have a greater likelihood of eliciting evidence of student understanding. New tasks are selected, adapted, or designed so that summative assessments address more than Level I reasoning and are more closely aligned to the "reasoning beyond recall" goals. The value of this work is that when attention is given to analyzing and improving summative assessments, this aspect of goal-driven assessment design can influence instructional planning, including more informed selection of questions and tasks used in lessons (Her & Webb, 2004). This process of reflecting on, and planning for, possible student responses to instructional tasks supports more productive and focused formative assessment practices.

Summary

This paper offers an approach to achieve Guiding Principle 5, in particular, knowledge, skills, and dispositions related to classroom assessment. Since classroom assessment is often under-addressed in pre-service mathematics education, and classroom assessments are more typically designed around familiar tasks that appear in instructional materials rather than designed to promote specific types of reasoning, it is important for instructors to have a framework that can be used to justify the choices that are made. Even in cases where instructors have limited influence on the design of summative assessments, the approaches outlined here can be applied in instruction for more formative purposes. When mathematics instructors can justify their choice of questions and tasks that they use in lessons, and the potential student reasoning that might be elicited, they are better prepared to adapt instruction in ways that reflect students' interests and ideas.

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Presentation Abstracts

Transparency and the Big Ideas in Calculus: Using a Write/Feedback/Re-write Cycle to Improve Student Understanding

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Gauss said "Mathematics is the queen of the sciences … She often condescends to render service to astronomy and other natural sciences, but in all relations, she is entitled to the first rank." The dual role of queen and servant is keenly felt by STEM students as they progress through the calculus sequence. Many concepts lay the foundation for applications in physics, engineering, and business; however, to be successful, procedural fluency is not sufficient. Students must be able to solve problems, which is a complex task that includes deciphering from a context what is the appropriate tool to use, establishing functions/relations, executing calculus-based procedures, interpreting the results, and communicating and justifying the appropriateness of the solution(s). Each step in this process requires careful thought and metacognitive action by the student.

Writing assignments with the possibility of revision can help students persist in learning difficult concepts, raise the level of student understanding, and improve the quality of student work. Through the write/re-write feedback loop, students learn how to use mathematical notation and language appropriately. Developing writing assignments to address key concepts in a course also helps instructors clarify their expectations to students. This presentation describes how the Calculus 1 teaching team at Western Michigan University developed, utilized, and revised a series of writing assignments to promote and to assess problem-solving skills.

Using Co-Teaching to Develop Classroom Mathematical Discourse During Clinical Experiences

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In this presentation we will describe how utilizing co-teaching during clinical experiences can be used to enhance the nature of mathematical discourse within the classroom setting. We will utilize data garnered via surveys, field notes of classroom observations, and personal reflections to highlight that, by working collaboratively, instructional pairs can increase the likelihood students are afforded an opportunity to communicate mathematically and develop a conceptual understanding of secondary mathematics content.

Development of Equity Literacy in the MODULE(S^2) Statistics for Secondary Teachers Course

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This presentation is of a Plan-Do-Study-Act Cycle that has been conducted within the MTE-Partnership community. The MODULE(S^2) writing teams have discussed ways in which educational equity might be addressed in our courses. A teacher who is equity-literate can identify educational inequities present in the classroom and school and is equipped to begin to redress these inequities. In addition, we believe that mathematics teachers can play a role in empowering marginalized students to create change for themselves and their communities. For this to occur, teachers must first come to terms with the realities of racial and economic inequity that currently exist in the United States. To this end, we have included equity-related content in the MODULE(S^2) Statistics for Secondary Teachers course.

The course was piloted at Eastern Michigan University during Winter 2019. To evaluate the effectiveness of its equity-related content, a survey of pre-service teachers' views on educational equity was administered at the beginning of the course and again at mid-semester. The survey explored various key areas of pre-service teachers' equity literacy: their developing conceptions of the meaning of equity in education, their attributions for student success, and their level of comfort or discomfort with participating in conversations about educational equity. Pre-post analysis of this survey, along with interviews and classroom observations, have illuminated the ways in which the course contributed to pre-service teachers' emerging equity literacy. The study also revealed ways in which the course might become more effective.

The Power of Collaboration:
A Taste of Improvement Science

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Through a collaborative partnership between Georgia State University and Atlanta Public Schools, we sought to develop the practices of our secondary mathematics pre-service teachers to increase their students' conceptual understanding and engagement in the mathematical practices. Although our pre-service teachers passed edTPA, a performance-based, subject-specific assessment for teacher candidates, they did not always meet the standard on key rubrics that addressed conceptual understandings. Additionally, when observed with the MCOP^2, they also struggled to engage students and promote conceptual understanding (Gleason, Livers, & Zelkowski, 2015). Using the Plan-Do-Study-Act Cycle on co-planning and co-teaching during the internship (Sears, et al., 2017; Bacharach, Heck, & Dahlberg, 2010) to monitor the developments, we reflected on the following questions: *Where are the pre-service teachers struggling? What experiences are we providing through our current courses to support their development? What can we do better to improve our outcomes?*

Collaboratively, we decided to work with the Standards of Mathematical Practices and Co-Planning and Co-Teaching interventions. Our process involved:

1. Familiarization – identifying mathematical practices in sample teaching videos.
2. Internalization and Application – using new understandings to plan and implement in their own practice.
3. Reflection – view their own teaching to assess their practice.
4. Feedback – receive feedback from others, including the cooperating teachers, university supervisors, and professors. At a roundtable session, participants will be presented with a comparison of strategies used in the past and in this cycle. We will also discuss our next steps to refine our process for continuous improvement.

Comparing Secondary Teachers' MKT for Geometry Exposed in Video and Written Representations of Practice

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The MODULE(S^2) RAC is developing both video and written simulation of practice activities as one aspect of the materials being created to develop mathematical knowledge for teaching in upper-level mathematics content courses. These simulation of practice assignments engage prospective teachers in responding to mathematical thinking that is grounded in K–12 classroom practice through classroom vignettes and K–12 student work and are designed to expose the mathematical knowledge for teaching that prospective teachers hold. We have observed that differently designed activities (whether prospective teachers' responses are in video or written form) may give teacher educators different views on prospective teachers' assets and how to build on them. As an initial step in understanding these differences, we draw from data on 36 representations of secondary geometry teaching practice generated by prospective teachers, and address the question: *In what ways do video and written representations of practice expose different aspects of prospective teachers' mathematical knowledge for teaching?* We argue that attending to potential differences in video and written representations is critical to developing prospective teachers' mathematical knowledge for teaching from an asset-based perspective.

Learning to Teach Statistics for Social Justice

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The MODULE(S^2) RAC's statistics writing team has created secondary mathematics teacher education materials that develop pre-service teachers' statistical knowledge for teaching as well as their knowledge of social contexts of mathematics teaching and learning and ways to enact change in these contexts. In this presentation, we will take a deep dive into one lesson from the materials focused on teaching statistics for social justice. In this lesson, pre-service teachers integrate their learning of statistics with how to teach statistics for social justice. They learn how statistics can be used to discover and quantify inequities, both in society at-large and in their community, in ways that can lead to specific actions to address problems. The lesson materials, along with artifacts of practice from implementation of the lesson with pre-service secondary mathematics teachers, will be shared and discussed.

Pathways Through Calculus

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Inspired by and drawing inspiration from Project DIRACC and Project CLEAR Calculus, the text *Pathways to Calculus* (Carlson et al.) provides pre-calculus content designed to probe student reasoning and foster conceptual understanding of proportional reasoning, co-variational reasoning, the connection between slope and constant rate of change, and average rate of change. After teaching with these materials for several semesters, James Hart set about developing a series of calculus I investigations that build upon the notational conventions, habits of mind, and key ideas in *Pathways to Calculus*. Tentatively named “Pathways through Calculus” in honor of its progenitor, Hart’s collection of investigations aims to adapt key concepts from these three ongoing projects into a fairly traditional course layout. In this presentation, Hart will describe his philosophy and current state of the project, share some sample investigations, and discuss the benefits and challenges encountered while implementing them in his Calculus I course.

Preparing Secondary Mathematics Teachers in Statistics Through a Dynamic Repository

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Secondary mathematics teachers are typically not equipped to teach statistics, yet school districts assume teachers are qualified. Teachers who lack a statistics background in their mathematics education and teacher preparation programs struggle to make teaching these concepts exciting and enlivening. Some of these teachers are tasked with teaching concurrent enrollment introductory statistics, yet they are ill-equipped to bring a college course for university credit to their high school. At Utah State University (USU), we bridged this gap for our concurrent enrollment introductory statistics teachers to deliver a high-quality experience for high school students. We created a repository of teaching materials for the concurrent enrollment teachers to provide them both the content and pedagogical content knowledge to integrate active-learning experiences, use of technology, and quality assessments in alignment with the recommendations of the ASA GAISE Report (2016). This repository has led to collaboration between secondary teachers and USU instructors as well as improved coordination of objectives, lessons, and pedagogy at the secondary and college levels. Through this dynamic online repository, our teachers access teaching materials, class activities and ideas, sample assessments, and end their course with a common departmental final exam. This presentation will demonstrate the use of an online repository as a coordinating and as a training tool that enhances course quality at both the secondary and collegiate levels.

Program Recruitment and Retention (PR²)

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The work of recruitment and retention is often in addition to the required teaching, research, and service of our job. Yet, the issue of recruitment and retention is arguably one of the most important tasks that we do. This panel of PR^2 members will present projects that focus on recruitment and retention that are woven into their work of advocating for education, broadening institutional support, revising program structure, and supporting candidates into the field. This session will begin with a brief background on the priorities for recruitment as well as barriers to recruitment and retention of teacher candidates. Each panelist will provide a brief overview of the projects they are engaged in at their institution or across their state including how they implemented the framework of the Plan-Do-Study-Act Cycle. Participants will be given the opportunity to discuss similar work at their own institution or ask questions of the panelist.

Impacting Teacher Retention by Supporting Secondary Mathematics Teachers in their First Year of Teaching

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This study reports on the design and initial implementation of two yearlong (AY 2018–2019) pilot interventions, created through a design-based approach, intended to support secondary mathematics teachers in their first year of teaching. The first intervention is designed to support these specific teachers develop meaningful professional relationships with a school-based mentor and to create an online community of practice for support with other professionals. The intervention consists of first-year teachers and their mentors participating in monthly professional development sessions such as online meetings, Zoom panels with experts, and collaboratively reading and discussing timely, purposeful, and relevant content. The second intervention is geared toward strengthening the relationship between these teachers and their administrators. This intervention asks teachers and their administrators to view a series of five-minute videos on best practice strategies for teaching mathematics and then spend five minutes afterward discussing the content of the video in the context of their school site. Both interventions are designed to not overburden the participants with large time commitments, to be feasible for national implementation with little funding, and to support the first-year teachers in a way that positively impacts job satisfaction and, ideally, teacher retention.

Findings from the First Year of Implementation of the Modeling Practices in Calculus Curriculum

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A team of educators and researchers within the STEM Transformation Institute at Florida International University are developing and conducting research on the Modeling Practices in Calculus (MPC) curriculum. MPC is a student-centric design in which students are meant to emulate the authentic practices of mathematicians in the classroom in order to learn Calculus I by actively doing mathematics in a lecture-reduced classroom with a focus on students working with their peers. These authentic practices include students actively working in groups to develop modeling and problem-solving skills; engaging in a culturally responsive learning environment that features learning assistants, multiple representations, and argumentation; and building proficiency with mathematical terminology, language constructs, and symbols. Curriculum artifacts and highlights of the first year of implementation will be presented. Also, the ongoing process of evaluating, modifying, and implementing of the curricular materials and class structure will be discussed.

An Operational Framework of Active Learning

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Defining active learning for both research and practice is complex and challenging. Definitions emphasize that active learning is *not* direct instruction then reference aspects of student engagement (Freeman et al., 2014; Hayward, Kogan, & Laursen, 2016). John Dewey (1914) argued that "not" definitions limit sustained experiential educational reform and are difficult for teachers to implement. Research with novice teachers illustrates confusion between the facilitator's goal with active learning and the participant's goal with active learning (Rogers & Yee, 2018). Specifically, novice teachers struggled to recognize which students should be engaged with which type of active learning.

Overlaying Vygotsky's sociocultural theory (Steele, 2001) onto the active learning framework provides two dimensions to categorize methods of learning that clarify and distinguish factors of active learning without polarizing it as simply "not" direction instruction. The first dimension looks at who or what the instructor engages with while the second dimension looks at who or what the participants engage with. Table 1 clarifies and distinguishes critical aspects of active learning using a sociocultural lens. From this table of nine categories for learning activities, we see the limitations of direct instruction, while having a larger sense of what is possible with teaching activities. This framework provides the field with an operational frame to generate meaningful discussion around organizing teaching activities for novice teachers as well as code and analyze research data, fundamentally showing how multidimensional teaching can be. This framework can promote theoretical discussion within the Active Learning Mathematics Research Action Cluster of the MTE-Partnership.

Table 1

Sociocultural Dimensions with Examples of Active Learning

	Instructor Engages with Content	**Instructor Engages with Group(s) of Students**	**Instructor Engages with Whole Class**
Students Engage with Content	*Direct Instruction*	Concept Maps Application Card	Wait Time Muddiest Point Self-Assessment Quiz Brainstorming Set It Up
Students Engage with Instructor	*Student Questioning*	Role-Playing	*Teacher Questioning* Quick Poll Minute Paper Inquiry-Based Learning
Students Engage with Students	Find the teacher's mistake Application Card	Inquiry-Based Learning Peer Review Case Studies Jigsaw	Think-Pair-Share Muddiest Point Inquiry-Based Learning

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A Dialogue on the Liaising Structure of the Equity and Social Justice Working Group

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The session will be focused on a set of discussion questions relating to the ongoing efforts of the ESJWG's move into a more active liaising role supporting RACs in their efforts to transform secondary mathematics teacher education programs. Currently, ESJWG views the liaising structure as bilateral; we believe collaboration and learning will occur in both directions. This dialogue will inform more of the MTE-Partnership community about the liaising structure of the ESJWG and to get feedback on how we might improve that structure in dialogue between stakeholders about some of the most important and foundational issues of the MTE-Partnership. Questions such as the following will be the starting point of the discussion: *How might members of ESJWG support, extend, or refine the work of your RAC? What are critical equity issues your RAC has better understood that should inform work of the partnership more broadly? Can ESJWG help? What challenges are related to equity and social justice that could become a collaborative project with ESJWG, or something to be taken up by the ESJWG?* We will also dialogue about questions RACs might have such as: *When would it be best to ask the ESJWG for feedback on a project? What is an acceptable amount of support to request from the ESJWG? What roles are the ESJWG not willing to take?* Presentation time will consist of no more than 10 minutes with the majority of the time focused on discussion.

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